MP 472 Quantum Information Processing

Quiz questions:

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1) Quantum bit, or qubit, is an elementary unit of quantum information. Write down its general state in the Dirac notation and vector representation.

2) How large is the Hilbert space of n qubits?

3) Show whether the states $|\beta_{00}> = (|00>+|11>)/2^{-1/2}$, $|\beta_{01}> = (|00>-|11>)/2^{-1/2}$ are orthonormal.

4) Write down the standard computational basis states of a three qubit system.

5) H is the Hadamard gate. What is the matrix representation of H^2 in the standard computational basis?

6) X is a bit flip gate and Z is a phase flip gate. Prove HXH=Z.

7) Calculate HZH.

8) Show whether the Hadamard gate is unitary.

9) Show whether the bit flip gate is Hermitian?

10) What are the eigenvalues and eigenvectors of the phase flip gate?

11) Show which of the following states $|\psi> = (|01>+|11>)/2^{-1/2}$, $|\phi> = (|01>-|10>)/2^{-1/2}$ is a separable state? Explain what does it mean separable state.

12) Let two qubits be in the maximally entangled state $|\beta_{00}> = (|00>+|11>)/2^{-1/2}$. Perfom measurement on the first qubit (in computational basis) and identify all possible results of measurement and all post-measurement states of this system?

13) How does CNOT gate transforms basis vectors of the two-qubit system $|q_1q_2>$ if q_2 is the control qubit and q_1 is the target qubit?

14) Propose a quantum circuit which generates the Bell state $|\beta_{00}> = (|00>+|11>)/2^{-1/2}$ from an initial fiducial state $|00>$.

15) Write down the operator $(I\otimes P_0 + Z\otimes P_1)$ in matrix representation using the standard computational basis. $P_k = |k><k|$ for k = 0,1. How would you call this operator?

16) Prove $tr(UAU^+) = tr(A)$ where U is a unitary operation.

17) Calculate the Bloch vector for the pure state $|\psi> = (|0>+i|1>)/2^{-1/2}$.

18) Let the Hamiltonian be given as $H=h\nu Z/2$ where h is the Planck constant, ν is frequency. Write down the evolution operator U(t) for time t in matrix representation in standard computational basis.

19) To distinguish the Bell states using projective measurement in standard computational basis one needs to rotate the Bell basis to the standard computational basis. Propose a circuit to carry out this rotation.

20) Write down a density matrix for the following ensemble $\{(p_\psi=1/4, |\psi\rangle = (|0\rangle+i|1\rangle)/2^{-1/2}), (p_\phi=3/4, |\phi\rangle = (|0\rangle+|1\rangle)/2^{-1/2})\}$.

21) Calculate the reduced density matrix for the qubit A of the two-qubit system AB described by the following density matrix
$\rho = ½ (|01\rangle\langle 01| - i |01\rangle\langle 10| + i |10\rangle\langle 01| + |10\rangle\langle 10|)$. (Hint: partial trace over B)

22) Calculate the Schmidt decomposition for the state $|\phi\rangle = (|00\rangle+i|11\rangle)/2^{-1/2}$
(Hint: recall the matrix a).

23) Write down the following two qubit density operator in a matrix representation

$$\rho = ½ (|01\rangle\langle 01| - i/2 |01\rangle\langle 10| + i/2 |10\rangle\langle 01| + |10\rangle\langle 10|).$$

24) The density matrix σ below does not describe a normalized quantum state. Calculate the norm and normalize the density matrix

$$\sigma = \begin{pmatrix} 3/2 & -i/8 \\ i/8 & 1/2 \end{pmatrix}$$

25) Calculate the Bloch vector of the state given by the following density matrix

$$\rho = \begin{pmatrix} 3/4 & 1/16 \\ 1/16 & 1/4 \end{pmatrix}$$

26) Does the density matrix in the question 3 represent a pure state or mixed state?
(Hint: calculate the purity)

27) Calculate the density matrix $\mathcal{E}(\rho)$ for the qubit which has passed through the bit-flip error channel described by the Kraus operators $E_0 = (1-p)^{1/2}$ I and $E_1 = p^{1/2}$ X, and was initially in the state

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

28) The depolarizing error channel is the error process which with the probability p replaces a quantum state given by the density matrix ρ by the completely mixed state i.e. whose density matrix is given as I/2 (where I is the identity matrix). Write down the quantum circuit that simulates this error process. (Hint: Use conditional swap operation).

29) Describe three-qubit repetition code for bit flip quantum error correction, i.e. encoding, error syndrome measurement and recovery procedure.