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GRAPHENE ELECTRONICS

and

QFT IN 2+1 DIMENSIONS

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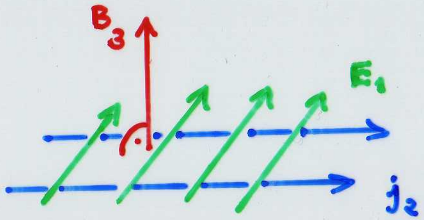
6 JANUARY 2007

OVERVIEW

- INTEGER QHE
- GRAPHENE ELECTRONICS
- THE RELATIVISTIC QHE
- SPONTANEOUS EDGE CURRENTS [with M. Gruber, W. Nahm]

INTEGER QUANTUM HALL EFFECT

B_3 breaks time reversal symmetry



$$\boxed{j_2 = \sigma_{21}\, E_1}$$ (OHM-HALL)

HALL CONDUCTIVITY $\left[\frac{e^2}{h}\right]$

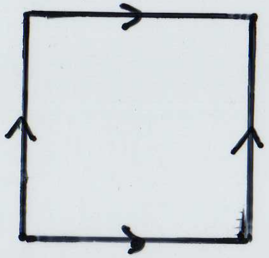
H Hamiltonian, $\quad H|0\rangle = E_0|0\rangle$

$$\sigma_{21} = \frac{\delta \langle 0 | j^2 | 0 \rangle}{\delta E_1}$$

By ordinary 1st order perturb. $\boxed{\delta H = e x_1 \delta E_1}$

$$= 2\, \mathrm{Im} \sum_{n \neq 0} \langle 0 | j^2 (E_n - E_0)^{-2} j^1 | 0 \rangle$$

KUBO - FORMULA



$T: \mathbb{R}^2 / \Lambda$ where $\Lambda \cong \mathbb{Z}^2$

$$L^2(\mathbb{R}^2) \cong \int_{T^*}^{\oplus} L^2(T)\, d^2k$$

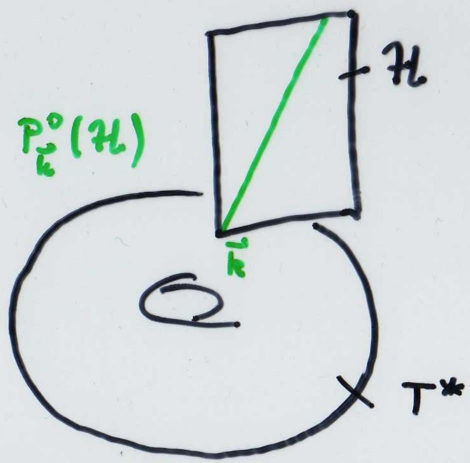
$$H \longmapsto H(\vec{k})$$

$$\partial_j \longmapsto \partial_j - i k_j \qquad (j = 1, 2)$$

$$\sigma_{21}^0(\vec{k}) = i e^2 \, \mathrm{Tr}_{L^2(T)} \{ P^0 [\partial_{k_1} P^0, \partial_{k_2} P^0] \} \qquad P^0 \equiv P^0_{\vec{k}} := |0_{\vec{k}}\rangle\langle 0_{\vec{k}}|$$

$\sigma_{21}^0(\vec{k})\, e^{-2}\, dk_1 \wedge dk_2$ curvature 2-form of $\boxed{P^0_{\vec{k}} \circ \nabla_{\vec{k}}}$ (adiabatic connection)

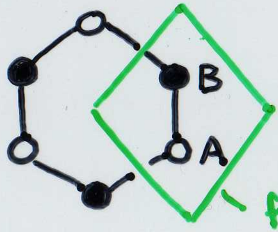
$$\boxed{\sigma_{21}^0 = \frac{1}{2\pi} \int_{T^*} \sigma_{21}^0(\vec{k})\, d^2k}$$



CHERN NUMBER $C_1(\mathcal{E}) \in \mathbb{Z}$

[TKNN 1982]

Graphene $:=$ a single atomic layer of carbon $\boxed{D = 2}$



A, B inequivalent carbon atoms

IV a)

6 C carbon

$1s^2$

$2s^2\ 2p^2$

Hartree $\quad \Psi = \bigwedge_{\text{electrons}} |1\text{-electron}\rangle \in \mathcal{H}$

Classify ψ by
- $[H, T] = 0 \qquad \underbrace{T_{\vec R}\, \psi_{\vec k}(\vec x)}_{\equiv\, \psi_{\vec k}(\vec x + \vec R)} = e^{i\vec k \vec R}\, \psi_{\vec k}(\vec x)$
- spin $\uparrow, \downarrow$

$\Rightarrow$ ansatz: $\quad \Psi = \bigwedge_{\vec k} \psi_{\vec k}^{(\uparrow)} \wedge \bigwedge_{\vec k} \psi_{\vec k}^{(\downarrow)}$

where $\quad \psi_{\vec k} = \psi_{\vec k}^{1s} \wedge \psi_{\vec k}^{\sigma} \wedge \psi_{\vec k}^{\pi} \qquad$ for $\uparrow, \downarrow$

$\rightsquigarrow$ subspaces $\qquad \mathcal{H}_{1s} \qquad \mathcal{H}_{\sigma} \qquad \mathcal{H}_{\pi}$

Classification
w.r.t. similar energy E
symmetry P

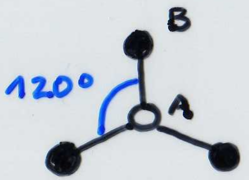
$\underbrace{\text{bound} \quad (2s, 2p_x, 2p_y) \quad \text{also bound}}_{\text{invariant under } z \mapsto -z}$ $\quad 2p_z \Rightarrow$ nuclei A, B

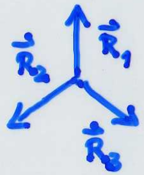
$\psi^{\pi} \mapsto -\psi^{\pi}$

$$\boxed{E \overset{!}{=} \min_{\mathcal{H}_\pi \subset \mathcal{H}}}$$

TIGHT BINDING APPROX.

one electron state: $\Psi_{\vec{k}}(\vec{x}) = \sum_{\vec{R}} e^{i\vec{k}\vec{R}} \underbrace{\varphi_j(\vec{x}-\vec{R})}_{\text{state around nucleus}}$, $j = A, B$





nearest neighbour only

$\varphi_j = \sum_k c_{jk} w_k$ $\{w_k\}$ basis of $\mathcal{H}_\pi$

Want $H\varphi_i \sim E_i \varphi_i$

$$\langle w_j | H | \varphi_i \rangle = E_i \langle w_j | \varphi_i \rangle$$

$$\sum_k c_{ik} \underbrace{\langle w_j | H | w_k \rangle}_{H_{jk}} \qquad \sum_k E_i c_{ik} \underbrace{\langle w_j | w_k \rangle}_{S_{jk}}$$

overlap matrix

$$H = \begin{pmatrix} \epsilon_{2p} & t\cdot \sum_i e^{i\vec{k}\vec{R}_i} \\ t\cdot \sum_i e^{-i\vec{k}\vec{R}_i} & \epsilon_{2p} \end{pmatrix} \qquad S = \begin{pmatrix} 1 & s\cdot \sum_i e^{i\vec{k}\vec{R}_i} \\ s\cdot \sum_i e^{-i\vec{k}\vec{R}_i} & 1 \end{pmatrix}$$

$$\frac{\partial E_j(\vec{k})}{\partial c_{ij}^*(\vec{k})} \overset{!}{=} 0 \quad \Rightarrow \quad \det\,(H - E_j S) \overset{!}{=} 0$$ (secular equ.)

$$E_j(\vec{k}) = \begin{cases} \dfrac{\epsilon_{2p} - t\,|\sum e^{i\vec{k}\vec{R}_i}|}{1 - s\,|\sum e^{i\vec{k}\vec{R}_i}|} & \pi^* \text{ antibonding} \\ \text{same with "+"} & \pi \text{ bonding} \end{cases}$$

energy band

Brillouin zone

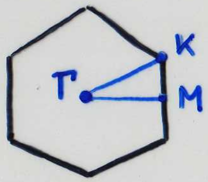


Fig. 2.3 (b)

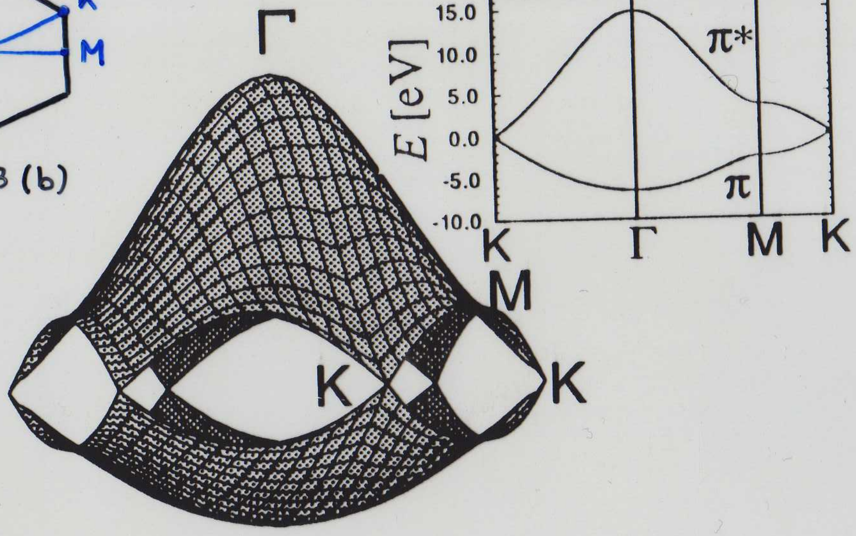
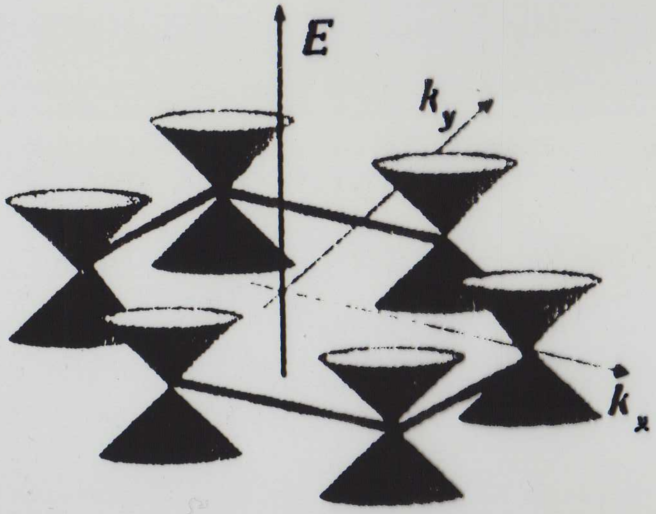


Fig. 2.4: The energy dispersion relations for 2D graphite are shown throughout the whole region of the Brillouin zone. The inset shows the energy dispersion along the high symmetry directions of the triangle $\Gamma M K$ shown in Fig. 2.3(b) (see text).

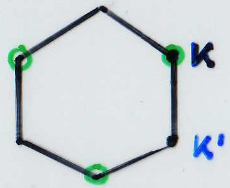


Approximation of the low energy band structure of graphene

<u>Tight binding :</u> a very simple and crude approximation.

Why is it relevant ?

For $\vec{K}$ invariant under a lattice symmetry group G



the eigenvectors of H-ES form a representation of G.

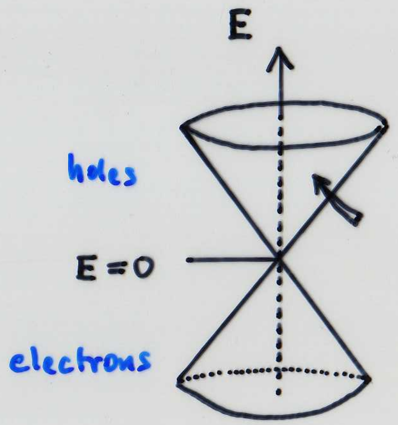
Under small symmetry preserving deformations

this representation does not change.

For non-abelian G this may yield degeneracies

(irreducible representations of dim > 1)

In our case, G = dihedral group of triangle.



"pseudospin"

charge carriers treated as relativistic massless particles

with an analog of velocity of light $\boxed{c_* \approx 10^6 \, \frac{m}{s}}$

$\Rightarrow$ low dimensional relativistic QFT !

nature Vol. 438, 10 Nov:
(Novoselov, Geim et al.)

$\boxed{\text{1st experimental study of a relativistic QHE}}$

Experiment (graphene): $\sigma_H = 2, 6, 10 \left[\frac{e^2}{h}\right]$

Theoretically,

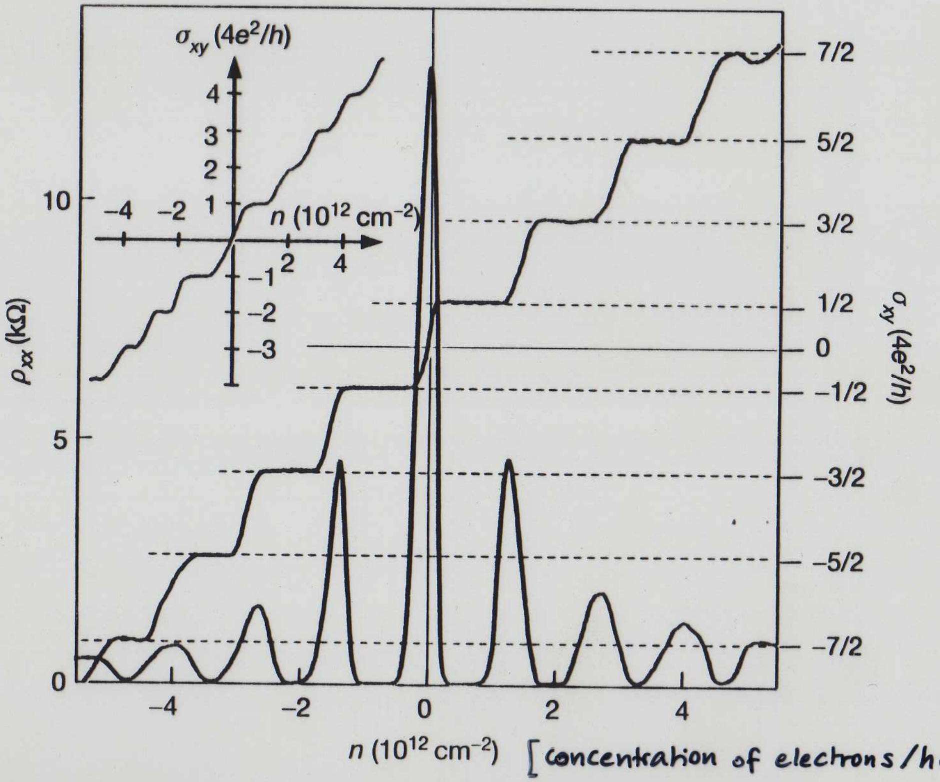
Gusynin, Sharapov : UNCONVENTIONAL INTEGER QHE IN GRAPHENE
(PRL 95, Sept. 2005)

$$\boxed{\sigma_H = \left(4\,\frac{e^2}{h}\right)\left(N+\frac{1}{2}\right)} \qquad (N \geq 0)$$

degeneracy factor:

2 = # C-atoms in fund. cell (fermion doubling)

2 = # spins



[concentration of electrons/holes]

Figure 4 | QHE for massless Dirac fermions. Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of graphene as a function of their concentration at $B = 14$ T and $T = 4$ K. $\sigma_{xy} \equiv (4e^2/h)\nu$ is calculated from the measured dependences of $\rho_{xy}(V_g)$ and $\rho_{xx}(V_g)$ as $\sigma_{xy} = \rho_{xy}/(\rho_{xy}^2 + \rho_{xx}^2)$. The behaviour of $1/\rho_{xy}$ is similar but exhibits a discontinuity at $V_g \approx 0$, which is avoided by plotting σ_{xy}. Inset: σ_{xy} in 'two-layer graphene' where the quantization sequence is normal and occurs at integer ν. The latter shows that the half-integer QHE is exclusive to 'ideal' graphene.

nature, Vol. 438, Nov. 2005

A nonrelativistic treatment yields

$$\boxed{\sigma_H = 4\,\frac{e^2}{h}\cdot N} \qquad (N \geq 0)$$

in accordance with the CONVENTIONAL INTEGER QHE for ordinary semiconductors

Experiment:

graphite film (double-Layer graphene)

Explanation of the $\frac{1}{2}$ shift from the conventional QHE

Landau - level (LL) formation for electrons in graphene (under perpend. magn. field)

$$|E_N^{\pm}| = \sqrt{2 e \hbar c_*^2 B \left(N + \tfrac{1}{2} \pm \tfrac{1}{2}\right)} \quad , \quad N \geq 0$$

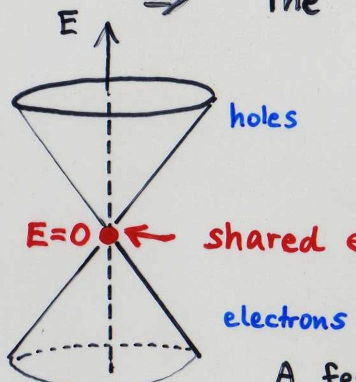
pseudospin
(electrons, holes)

$$\Rightarrow \quad |E_{N+1}^{-}| = |E_N^{+}|$$

All levels $\boxed{N > 0}$ occupied by fermions with <u>both</u> ($\pm$) pseudospins.

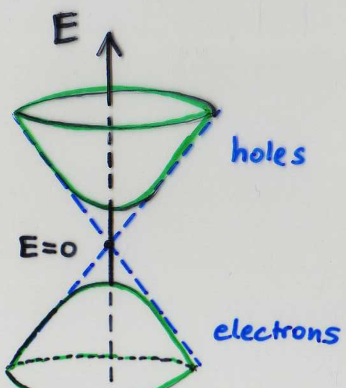
BUT $\boxed{E_0^{-} = 0}$ has no E^{+} counterpart (i.e., halved degeneracy)

$\Rightarrow$ The 1st Hall plateau appears at half the normal filling.



shared equally by electrons and holes

A <u>fermion mass</u> would open a gap around $E=0$:



Therefore,

" We attribute this qualitative transition between graphene and its two-layer counterpart to the fact that fermions in the latter exhibit a finite mass near $n \approx 0$ and can no longer be described as massless Dirac particles. " (nature)

THE RELATIVISTIC QUANTUM HALL EFFECT

$$\boxed{H = -i\vec\sigma\vec\nabla + m\sigma_3} \longrightarrow H(\vec k) = \vec\sigma\vec k + m\sigma_3$$

m breaks time reversal symmetry

$$\sigma^{(-)}_{21}(\vec k) = ie^2 \sum_{\vec\kappa\in\Lambda^*} \mathrm{Tr}_{\mathbb{C}^2}\left\{P^{(-)}_{\vec k+\vec\kappa}\left[\partial_1 P^{(-)}_{\vec k+\vec\kappa}, \partial_2 P^{(-)}_{\vec k+\vec\kappa}\right]\right\} \qquad \text{where} \quad P_{\vec k} = P_{|0_{\vec k}\rangle}$$

$$= ie^2\, \mathrm{Tr}_{\Lambda\mathbb{C}^2}\left\{P^{(0)}_{mult(\vec k)}\left[\partial_1 P^{(0)}_{mult(\vec k)}, \partial_2 P^{(0)}_{mult(\vec k)}\right]\right\} \qquad \text{where} \quad P_{mult(\vec k)} = P_{\bigwedge_{\vec\kappa\in\Lambda^*}|0_{\vec k+\vec\kappa}\rangle}$$

$$\left.\begin{array}{l} = \dfrac{e^2}{2}\displaystyle\sum_{\vec\kappa\in\Lambda^*}\frac{m}{\left[(\vec k+\vec\kappa)^2+m^2\right]^{3/2}} \\[2em] \dfrac{1}{2\pi}\displaystyle\int_{T^*}\sigma^{(-)}_{21}(\vec k)\, dk_1\wedge dk_2 = \ ? \end{array}\right\} \quad \frac{1}{2\pi}\,\frac{e^2}{2}\int_{\mathbb{R}^2}\frac{m}{\left[\vec k^2+m^2\right]^{3/2}}\, d^2k$$

THE RELATIVISTIC QUANTUM HALL EFFECT [M.L. cond-mat/0505428]

$$\boxed{H = -i\vec{\sigma}\vec{\nabla} + m\sigma_3} \longrightarrow H(\vec{k}) = \vec{\sigma}\vec{k} + m\sigma_3 \equiv (\underline{\sigma}\cdot\underline{k})\Big|_{F_m := \{\underline{k}\in\mathbb{R}^3 \mid k_3 = m\}}$$

m breaks time reversal symmetry

$$\sigma_{21}^{(-)}(\vec{k}) = ie^2 \sum_{\vec{\kappa}\in\Lambda^*} \mathrm{Tr}_{\mathbb{C}^2}\left\{P^{(-)}_{\vec{k}+\vec{\kappa}}\left[\partial_1 P^{(-)}_{\vec{k}+\vec{\kappa}}, \partial_2 P^{(-)}_{\vec{k}+\vec{\kappa}}\right]\right\} \quad \text{where} \quad P_{\vec{k}} = P_{|0_{\vec{k}}\rangle}$$

$$= ie^2\,\mathrm{Tr}_{\wedge\mathbb{C}^2}\left\{P^{(0)}_{\mathrm{mult}(\vec{k})}\left[\partial_1 P^{(0)}_{\mathrm{mult}(\vec{k})}, \partial_2 P^{(0)}_{\mathrm{mult}(\vec{k})}\right]\right\} \quad \text{where} \quad P_{\mathrm{mult}(\vec{k})} = P_{\bigwedge_{\vec{\kappa}\in\Lambda^*}|0_{\vec{k}+\vec{\kappa}}\rangle}$$

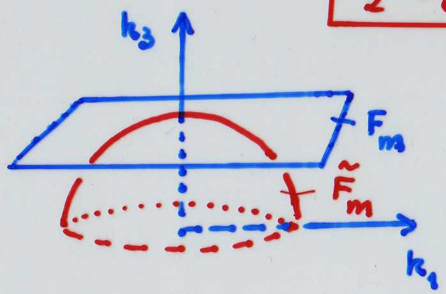
$$= \frac{e^2}{2}\sum_{\vec{\kappa}\in\Lambda^*}\frac{m}{[(\vec{k}+\vec{\kappa})^2+m^2]^{3/2}}$$

$$\frac{1}{2\pi}\int_{T^*}\sigma_{21}^{(-)}(\vec{k})\, dk_1\wedge dk_2 = \cancel{?}$$

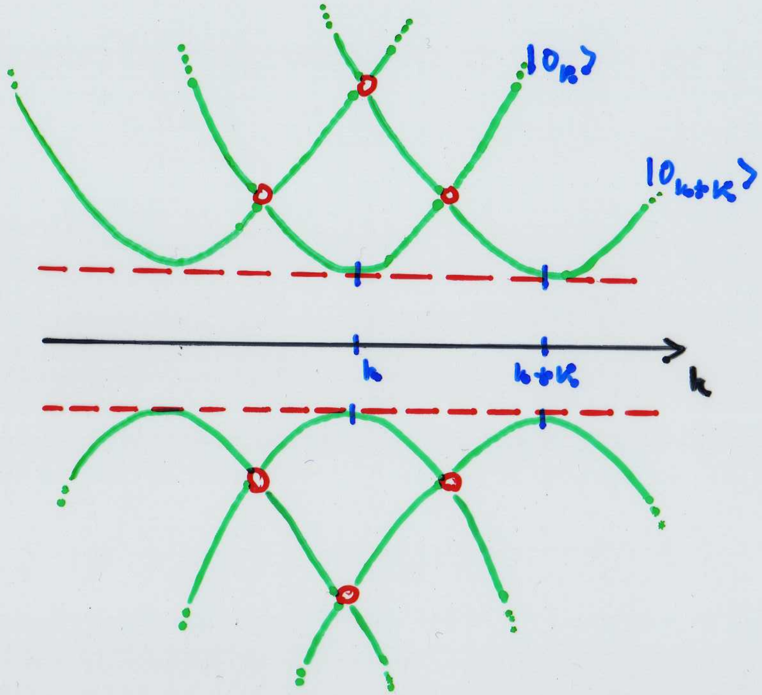
$$\frac{1}{2\pi}\,\frac{e^2}{2}\int_{\mathbb{R}^2}\frac{m}{[\vec{k}^2+m^2]^{3/2}}\, d^2k =$$

$$\boxed{\tfrac{1}{2}\,\mathrm{sgn}(m)} = \frac{e^2}{4\pi}\cdot\frac{1}{2}\,\varepsilon^{abc}\int_{F_m\subset\mathbb{R}^3}\frac{k_a\, dk_b\wedge dk_c}{|\underline{k}|^3} = \sigma_{21}^{(-)}$$

$m>0$:



area of upper halfsphere $[\tilde{F}_m \subset S^2]$



1-particle : energy degeneracies

jumping dim. of eigenspaces

multi particle : non-degeneracy (Pauli exclusion)

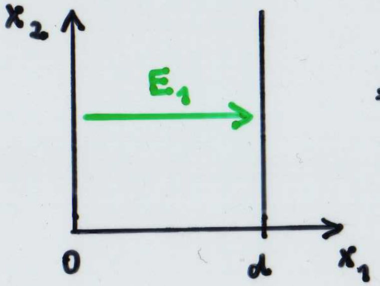
transition fct. : sign ambiguity

(∞ number of interchanges of factors in $\bigwedge_{k\in\Lambda^*} |0_{k+K}\rangle$)

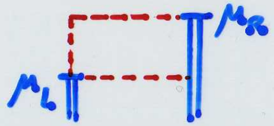
no global Hilbert space

distance of k to local minima determines order of wedge factors

SPONTANEOUS EDGE CURRENTS [M. Gruber, M. L. LMP (2006)]



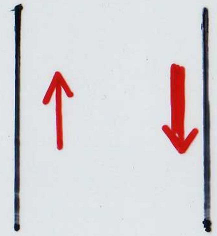
$\Rightarrow$



$|\Delta| = |q E_1 d|$

chemical potentials on boundary

$\Rightarrow$



$$J^e_d(\Delta) = \int_0^d j_\Delta \, dx_1$$

EDGE CURRENT

Then $d \to \infty$: Δ = occupied energy interval in gap

$$\boxed{J^e_{(\Delta)} = \sigma^e_{(\Delta)} \frac{|\Delta|}{q}}$$

↑ EDGE CONDUCTIVITY

Hermitian extension to half plane

$$H_\gamma(k_2) = -i\sigma_1 \partial_1 + k_2 \sigma_2 + m\sigma_3$$

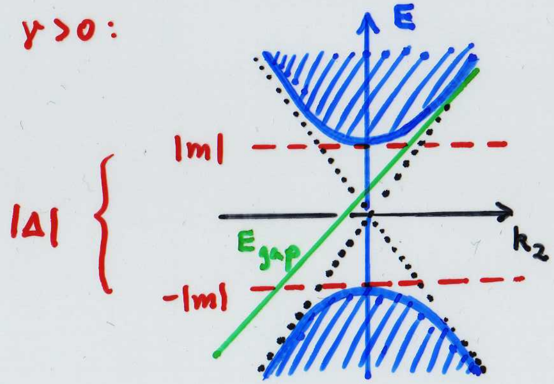
For k_2 fixed,

$$\begin{pmatrix} \psi_1(x_1) \\ \psi_2(x_1) \end{pmatrix} \quad \text{with} \quad \psi_2(0) = i\gamma\, \psi_1(0)$$

$$\gamma \in \mathbb{R} \cup \{\infty\}$$

spectrum:

$\gamma > 0$:



$$E(k_2) \geq \sqrt{k_2^2 + m^2} \qquad \text{(bulk)}$$

$$E_{gap}(k_2) = \frac{2\gamma}{\gamma^2+1} k_2 + \frac{1-\gamma^2}{1+\gamma^2} m \quad \text{if} \quad \boxed{k_2(\gamma^2 - 1) > -2m\gamma}$$

(normalisation condition)

E_{gap} goes through Δ iff $\boxed{m\gamma > 0}$

CALCULATION OF EDGE CONDUCTIVITY

$$j^e(k_2) = \langle \psi_{k_2} | e\sigma_2 | \psi_{k_2} \rangle_{L^2(\mathbb{R}_+)} = e \cdot \frac{dE_{gap}}{dk_2} \equiv j^e$$

$$\sigma^e = \frac{e}{|\Delta|} \, j^e$$

$$= \frac{e}{|\Delta|} \int_{E_{gap}(k_2) \in \Delta} j^e \, \frac{dk^2}{2\pi}$$

$$= \frac{e^2}{2\pi} \underbrace{\left(\frac{1}{|\Delta|} \int_{E_{gap} \in \Delta} dk_2 \right)}_{= \left| \frac{1}{\text{slope of } E_{gap}} \right|} \frac{dE_{gap}}{dk_2} = \begin{cases} \operatorname{sgn}(m) \, \frac{e^2}{h} & \text{if } m\gamma > 0 \\ 0 & \text{otherwise} \end{cases}$$

j^e spontaneous edge current

Note: magn. Schrödinger with random potential: $\sigma^{Kubo} = \sigma^e$

constant Dirac with mass: $\sigma^{Kubo} = \frac{0 + \operatorname{sgn}(m)}{2} = \frac{1}{2} \operatorname{sgn}(m)$

Application ?

Close to $\gamma = 0$,

the spontaneous current depends very sensitively on γ.

Can this be used like a transistor ?

PROBLEM : $\gamma \neq 0, \infty$ may be impossible to realise since the bulk excitations develop divergent currents close to the edge, even in free QFT.

$$j^\mu(x) = \int_0^\infty \int_{-\infty}^\infty \frac{dl\, dk}{|k^0|} \, \overline{\psi}_{lk}(x) \, \gamma^\mu \, \psi_{lk}(x)$$

$l := |k^1|$

$k := k^2$

$k^0 = -\sqrt{k^2 + l^2 + m^2}$

↗ solutions of the Dirac eq.

$$\psi_{lk}(x) = \left\{ c \begin{pmatrix} i\rho \\ 1 \end{pmatrix} e^{ilx_1} + d \begin{pmatrix} i\rho^* \\ 1 \end{pmatrix} e^{-ilx_1} \right\} e^{ikx_2} \,, \qquad \rho := \frac{k + il}{m - k^0}$$

and $\langle \psi_{lk} | \psi_{\tilde{l}\tilde{k}} \rangle = |k^0| \, \delta(k - \tilde{k}) \, \delta(l - \tilde{l})$

This yields

$$j^2(x_1) = \frac{\pi\gamma}{\gamma^2 - 1} \left(\frac{1}{x_1^2} + \frac{2m}{x_1} \right) e^{-2m x_1} .$$