

# Dynamic properties of dark resonance systems

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## Introduction

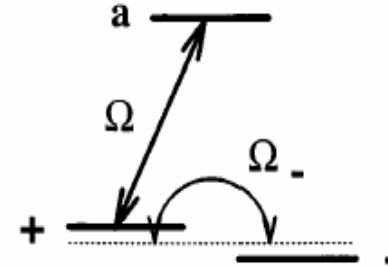
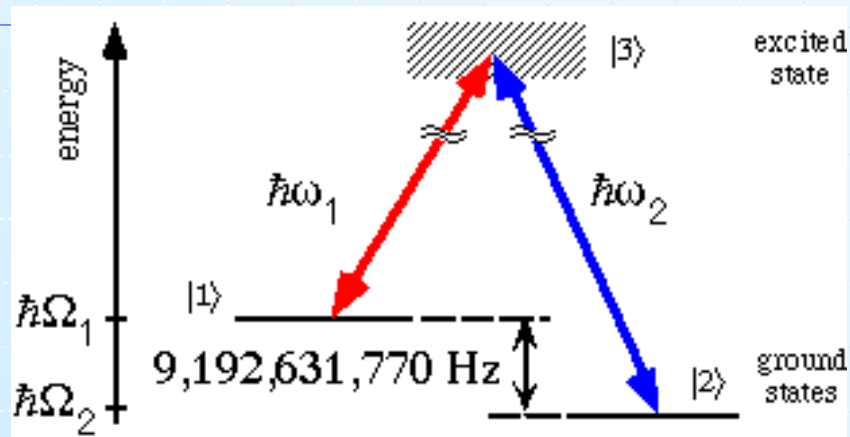
Atoms or molecules prepared in specified quantum states or coherent superposition states have led to many applications in different areas such as

- ◆ Chemical-reaction dynamics;
- ◆ Atom optics;
- ◆ Cavity quantum electrodynamics;
- ◆ Quantum information and computation;

Several methods can be used to prepare atoms or molecules in desired target states

- ◆  $\Pi$  -pulse technique;
- ◆ (F-)Stimulated Raman adiabatic passage technique;
- ◆ Stark-chirped rapid adiabatic passage technique.

## Dark state/resonance

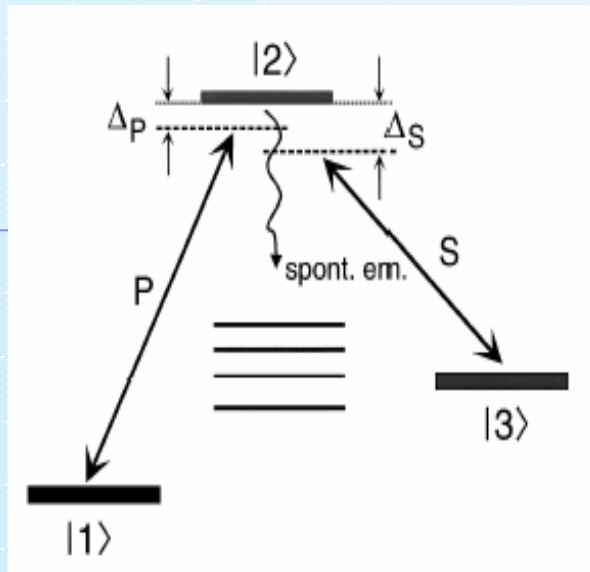


$$|D\rangle = \cos\Theta |1\rangle - \sin\Theta |2\rangle$$

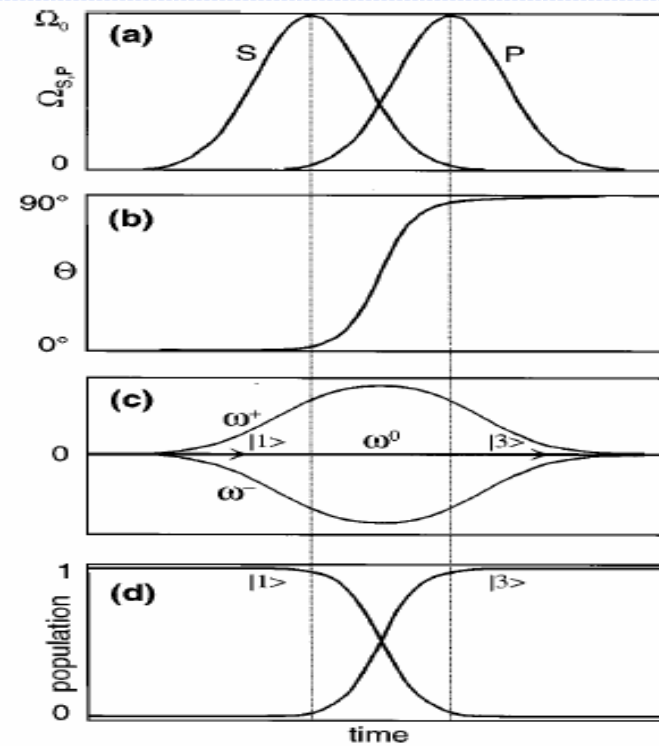
All the populations are trapped in the superposition state of  $|1\rangle$  and  $|2\rangle$ . The atom will not absorb any photon and therefore it is termed as “dark state”.

## Review of some previous work

### Three-state stimulated Raman adiabatic passage (STIRAP)



three-level excitation scheme



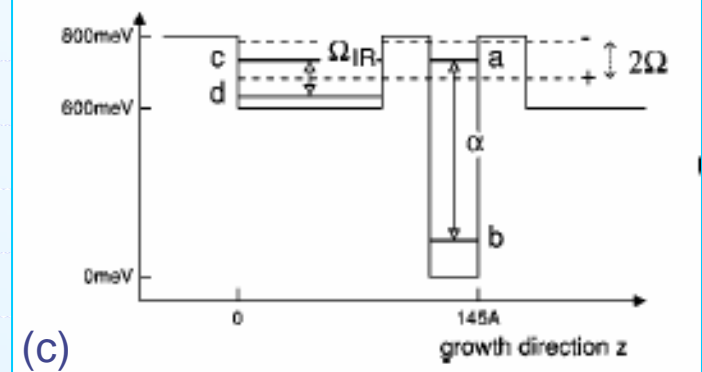
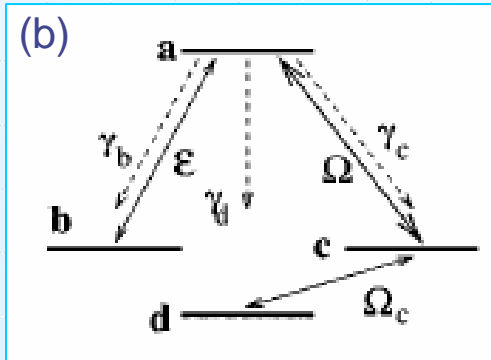
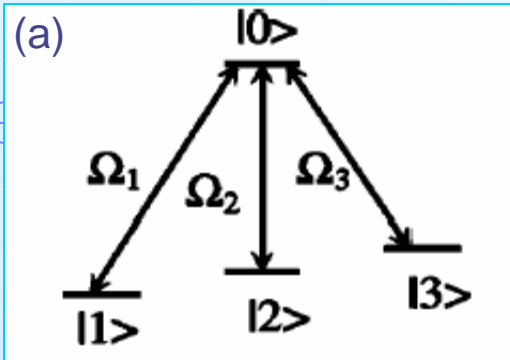
under the condition of two-photon resonance

(from Rev. Mod. Phys., 70, 1003(1998), by Bergmann *et al.*)

## Some applications:

**Laser controlled chemical-reaction;  
Electromagnetically induced transparency;  
Atomic frequency standard;  
Adiabatic population transfer;  
Preparation of desired superposition states;  
Slow group velocity;  
Quantum information storing and retrieving;  
.....**

# Double-dark state/resonance



With the application of another field, dark state system evolves to double-dark state case.

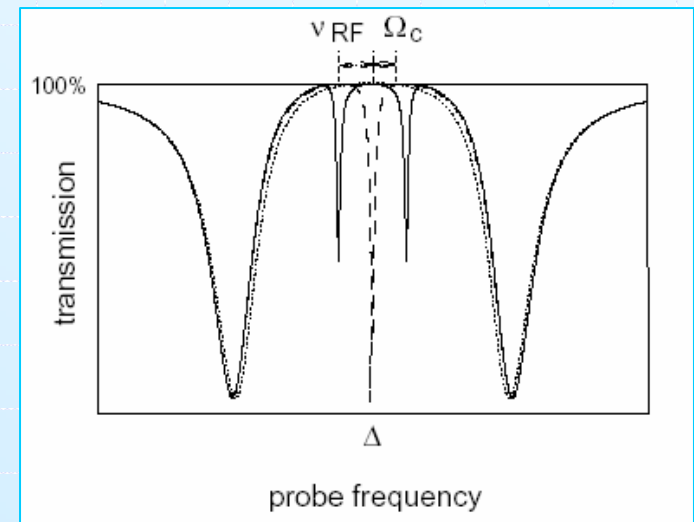
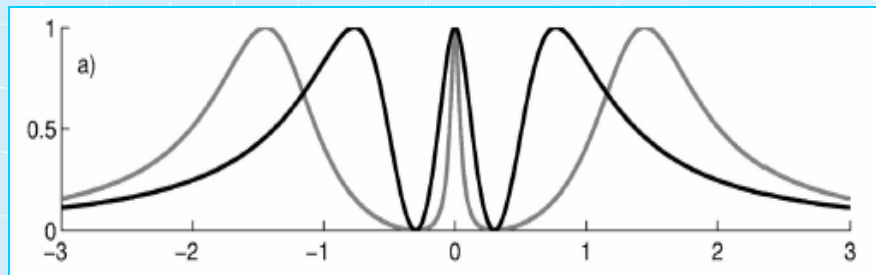
Relative applications:

**Sub-Doppler and subnatural narrowing of an absorption line;**

**Extremely sharp resonances;**

**Nonlinear generation of a comb of sidebands;**

...

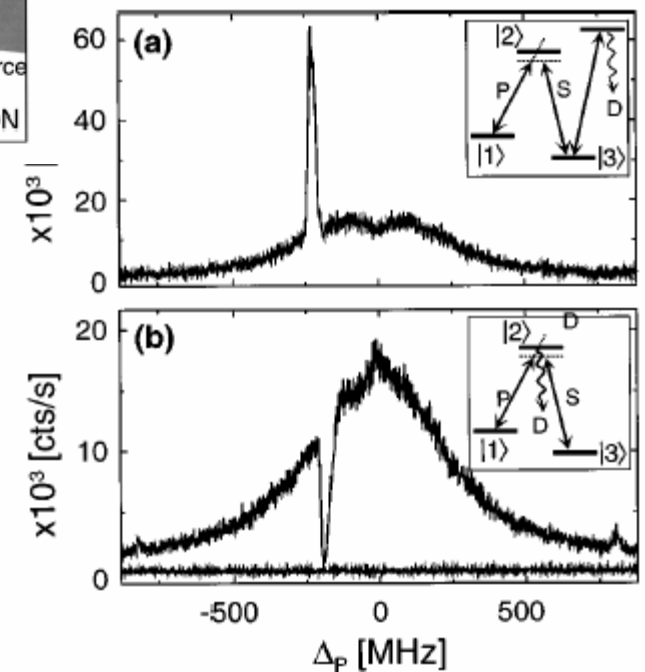
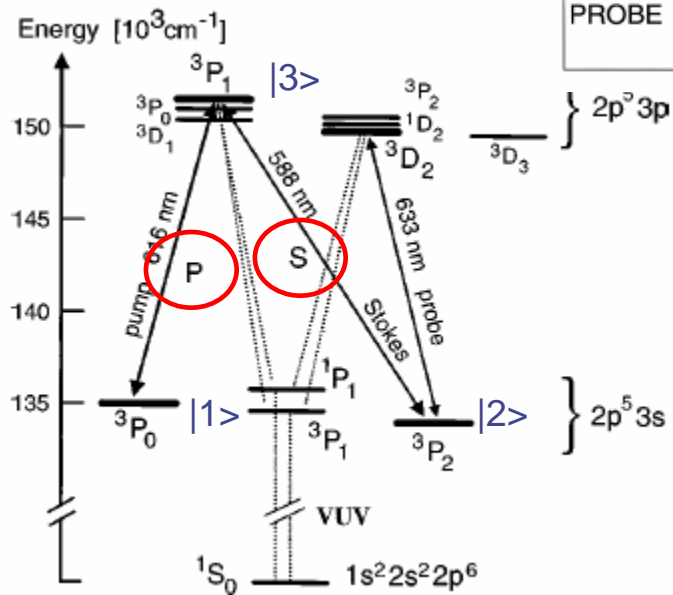
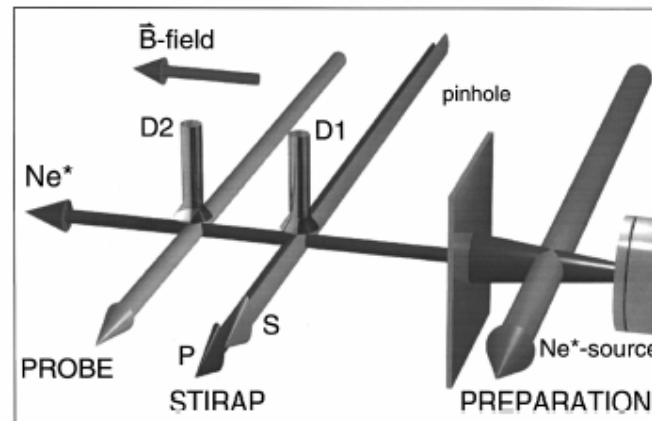


(from PRA, 68, 063801; PRA, 66, 013803; PRA, 69, 063802)

# Creation of atomic coherent superposition states without multi-photon resonance



## Introduction:



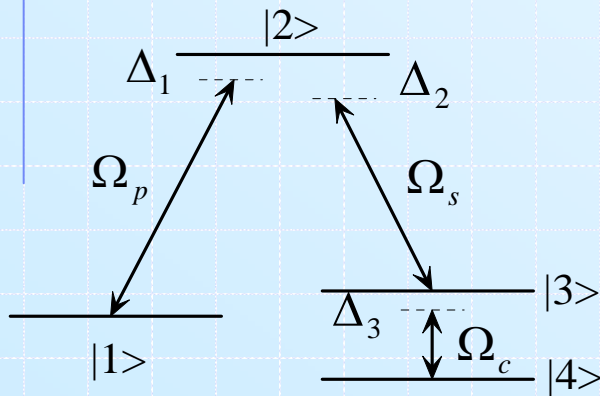
It is known that in STIRAP (stimulated Raman adiabatic passage) technique, besides the adiabatic passage, **the condition of two-photon resonance must be satisfied !** Otherwise, dark resonance does not exist and one can not get the desired results.

(Bergmann k, et al, Rev. Mod. Phys., 2, 1008 (1998))



# Creation of atomic coherent superposition states without multi-photon resonance

## 1. The proposed model



$$\Delta_{\pm} = \frac{\Delta_3 \pm \sqrt{\Delta_3^2 + 4\Omega_c^2}}{2}$$

Double dark states exist, they are

$$|\psi\rangle_+ = \cos\theta|1\rangle - \sin\theta(\cos\phi|3\rangle + \sin\phi|4\rangle)$$

$$|\psi\rangle_- = \cos\theta|1\rangle - \sin\theta(\cos\phi|3\rangle - \sin\phi|4\rangle)$$

A constant control field, which may be a microwave or quasi-static field, is employed between the twofold of the final state.

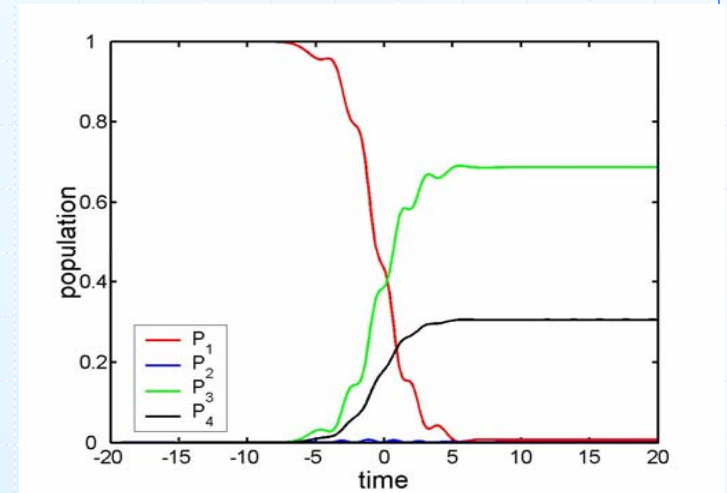
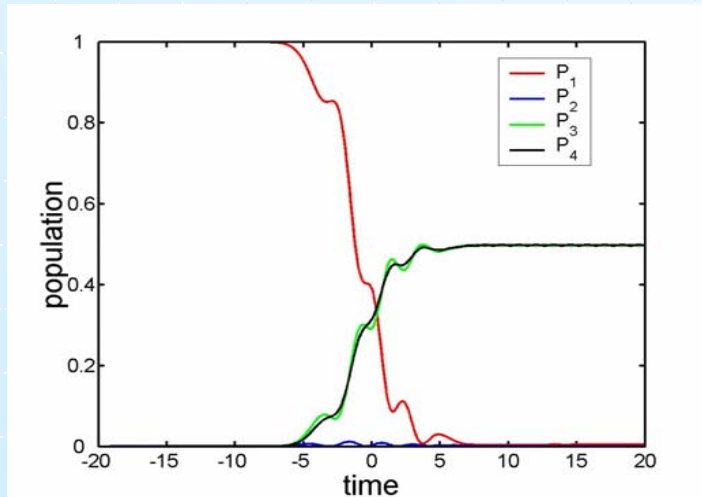
After the interaction of the pump and the Stokes pulses, all the population resides in the ground state  $|1\rangle$  is transferred to the superposition state:

$$\cos\phi|3\rangle + \sin\phi|4\rangle$$

$$\cos\phi|3\rangle - \sin\phi|4\rangle$$

These two superposition states have equal amplitude but inverse phases, which has some applications in quantum information.

## 2. Realization of coherent superposition states



When the control field is on resonance, the ultimate superposition state has the form of

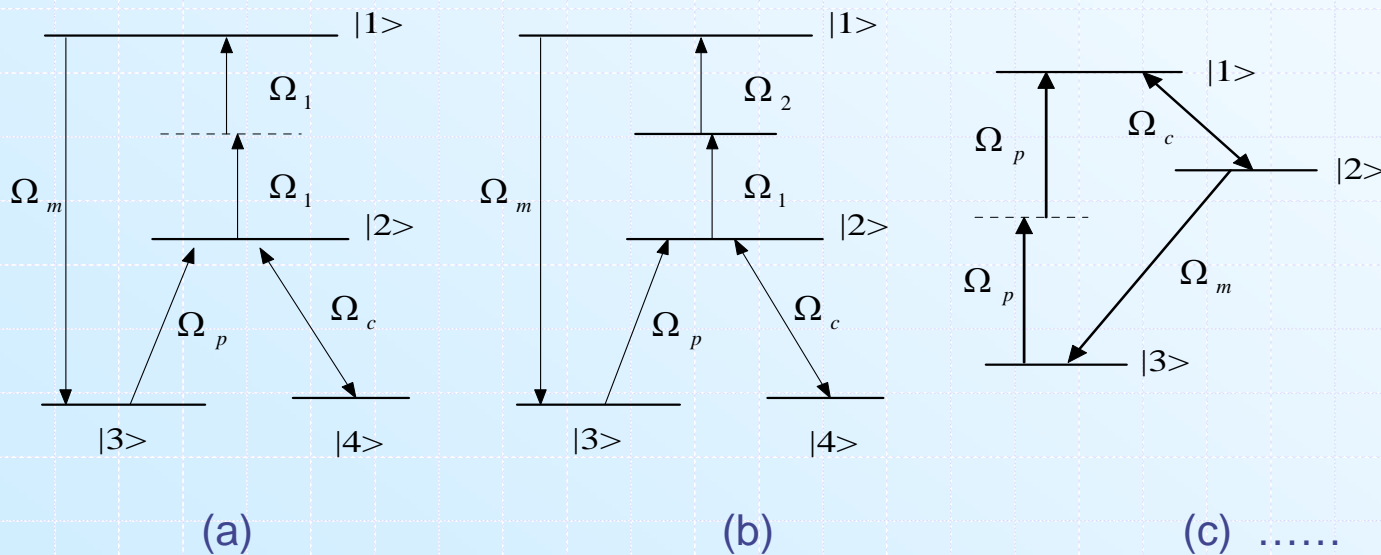
$$|\psi\rangle_+ = (|3\rangle + |4\rangle)/\sqrt{2} \quad |\psi\rangle_- = (|3\rangle - |4\rangle)/\sqrt{2}$$

corresponding to inverse two-photon detuning. They are the maximal superposition states of the twofold state and are orthogonal indeed.

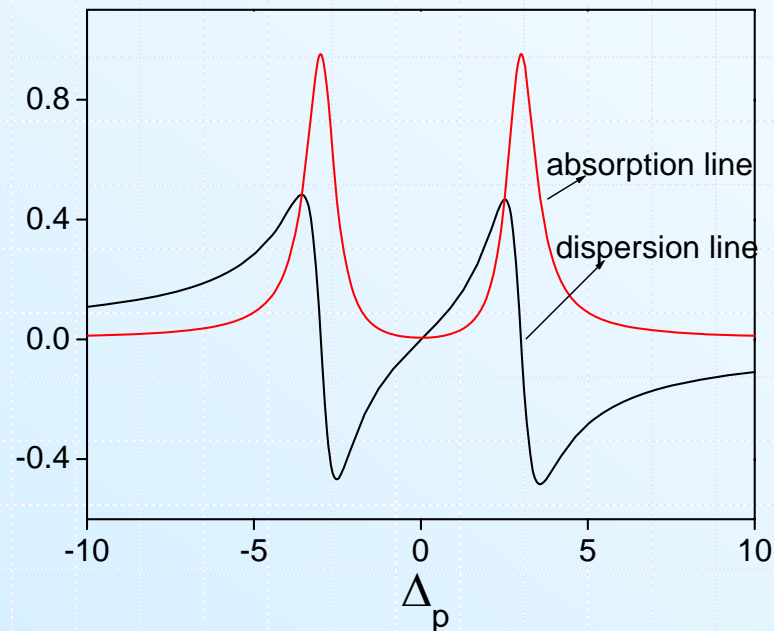
# Controllable enhancement of FWM induced by double dark resonances

## 1. Introduction

Recently, several schemes for enhancement of four-wave mixing based on electromagnetically induced transparency (EIT) have been investigated:



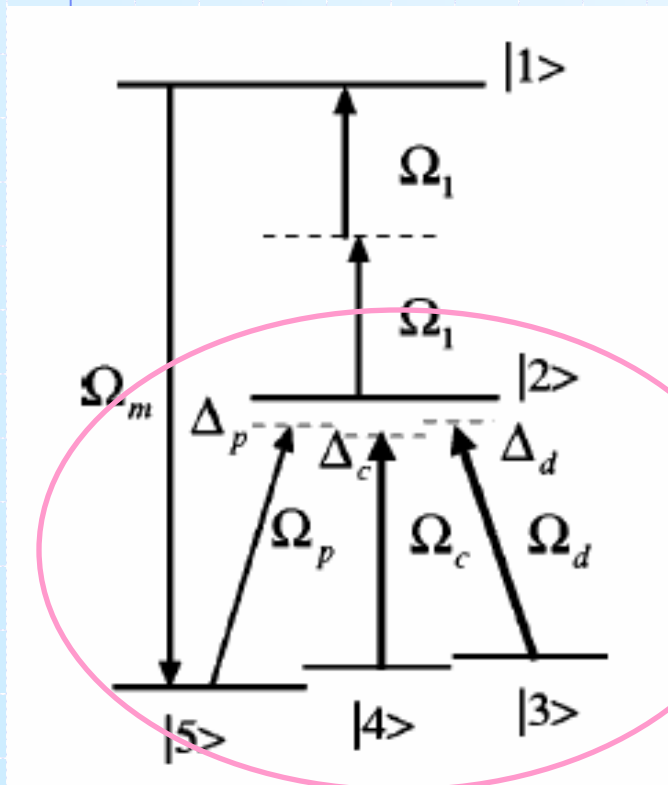
It has been shown that an EIT for a pump wave can open an efficient FWM channel that is otherwise prohibited by strong absorption of the pump wave.



The electromagnetically induced transparency (EIT) effect can lead to suppressed linear absorption and slow group velocity, just can be seen from the absorption and dispersion lines.

This property can be used to enhance the frequency conversion efficiency.

## 2. The scheme



1 ) FWM can be generated between levels  $|1\rangle$  and  $|5\rangle$  :  $\omega_m = \omega_p + 2\omega_1$

The lower four states compose a tripod configuration and can display double dark states.

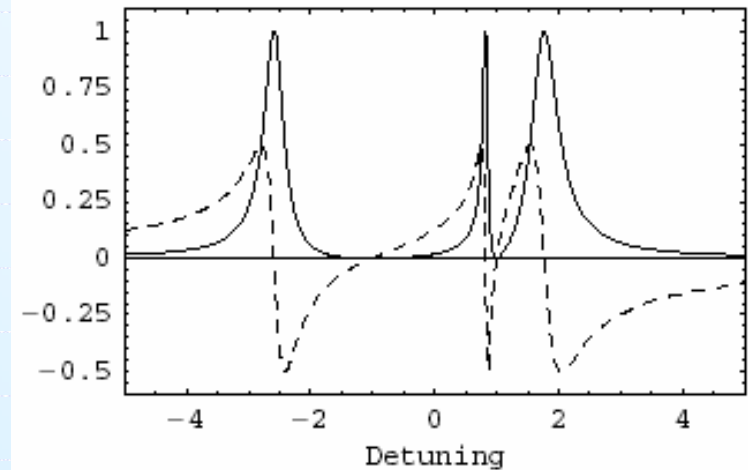
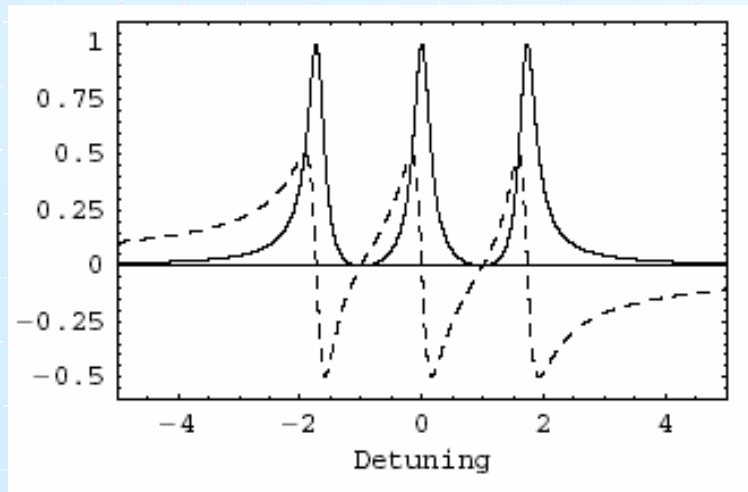
2 ) Two transparencies can be produced among levels  $|2\rangle$ ,  $|3\rangle$ ,  $|4\rangle$  and  $|5\rangle$  :

$$\Delta_p = \Delta_c \quad \text{or} \quad \Delta_p = \Delta_d$$

The two channels both own the property of the reduced linear absorption and slow group velocity.



### 3. The absorption and dispersion properties of the double-dark resonance system



By adjusting of the two control fields  $\Omega_c$  and  $\Omega_d$ , the absorption and dispersion properties of the signal field  $\Omega_p$  may be controlled. Accordingly, the intensity of the generated FWM field also can be manipulated.

(from J.Opt.B 4, S372(2002), by E. Paspalakis *et al.*)

## 4. Calculation of the FWM signal

The equations of atomic motion:

$$\begin{cases} \frac{\partial}{\partial t} C_1 = -i(\Delta_1 C_1 - \Omega_1 C_2 - \Omega_m C_4) - \gamma_1 C_1 \\ \frac{\partial}{\partial t} C_2 = -i(C_2 - \Omega_1^* C_1 - \Omega_c C_3) - \gamma_2 C_2 \\ \frac{\partial}{\partial t} C_3 = -i(C_3 - \Omega_c^* C_2 - \Omega_p C_4) - \gamma_3 C_3 \\ \frac{\partial}{\partial t} C_4 = -i(-\Omega_m^* C_1 - \Omega_p^* C_3) \end{cases}$$

By Fourier transformation, we obtain:

$$\begin{cases} \alpha_1 = \tau W_m D_m - \tau \Omega_1 \tau \Omega_c \tau W_p D_m D_p / \delta_2, \\ \alpha_3 = \tau W_p D_p. \end{cases}$$

The propagation of the probe and generated FWM fields are:

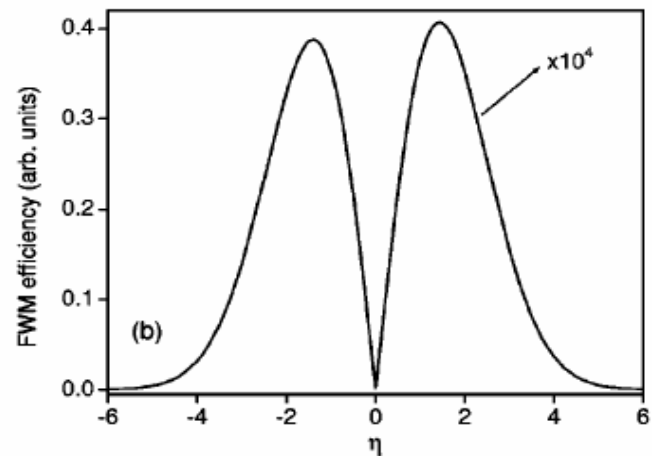
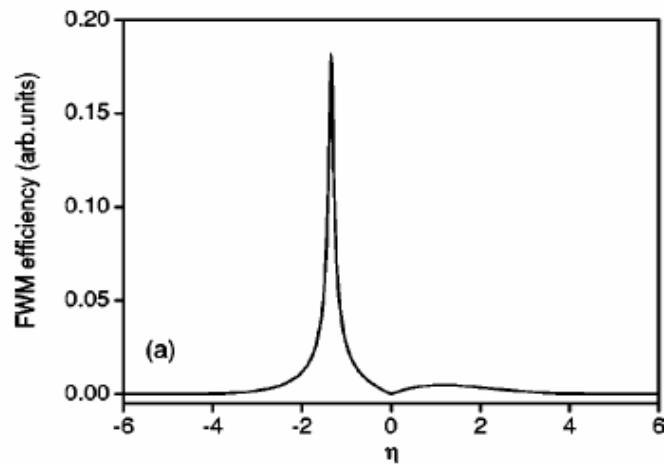
$$\begin{cases} \frac{\partial}{\partial z} W_p - i \frac{\eta}{c} W_p = i \kappa_p \alpha_3 \alpha_4^* \\ \frac{\partial}{\partial z} W_m - i \frac{\eta}{c} W_m = i \kappa_m \alpha_1 \alpha_4^* \end{cases}$$

$W_{m,p}(z, \eta)$  is the Fourier transform of  $\Omega_{m,p}(z, t)$

The resulted FWM signal:

$$W_m(z, \eta) = (\kappa_m c \tau^2)(\Omega_1 \tau)(\Omega_c \tau) D_p D_m \frac{e^{i\eta z/c\tau} (e^{iD_m(\kappa_m v \tau^2)z/c\tau} - e^{iD_p(\kappa_p v \tau^2)z/c\tau})}{\delta_2 [D_p(\kappa_p c \tau^2) - D_m(\kappa_m c \tau^2)]} W_p(0, \eta)$$

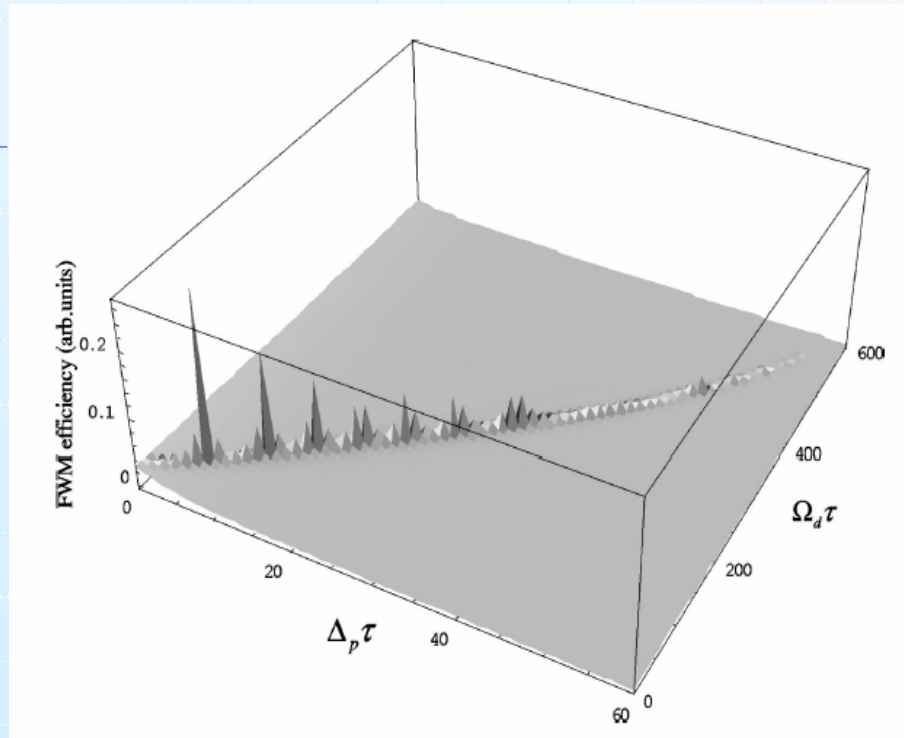
## 5. Simulated results



when  $\Omega_c > \Omega_d$ ,

← FWM signal produced at  $\Delta_p = \Delta_d$  channel;

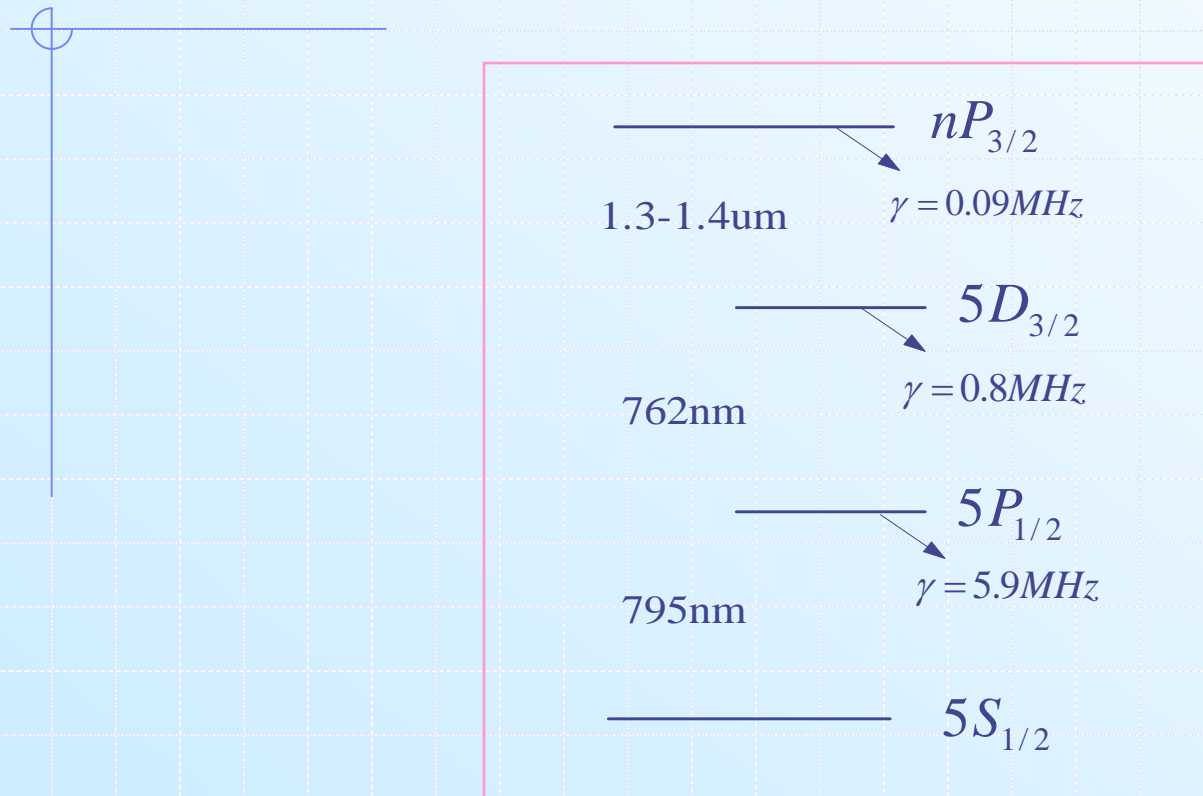
← FWM signal produced at  $\Delta_p = \Delta_c$  channel.



This simulated result shows that:

- 1) If  $\Omega_c$  is set and larger than  $\Omega_d$ , the stronger FWM signal is always produced at the channel of  $\Delta_p = \Delta_d$  ;
- 2) By properly tuning the relative intensities of the two control fields, controllable enhanced FWM can be realized.

## 6. Realistic energy levels



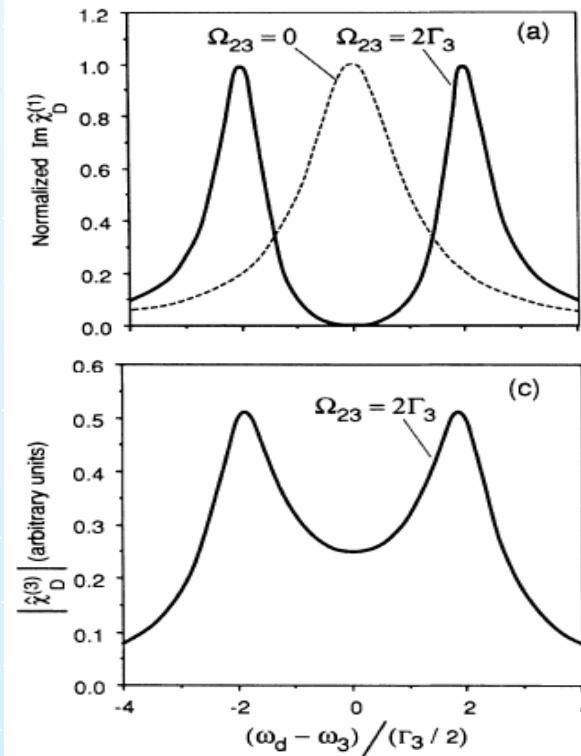
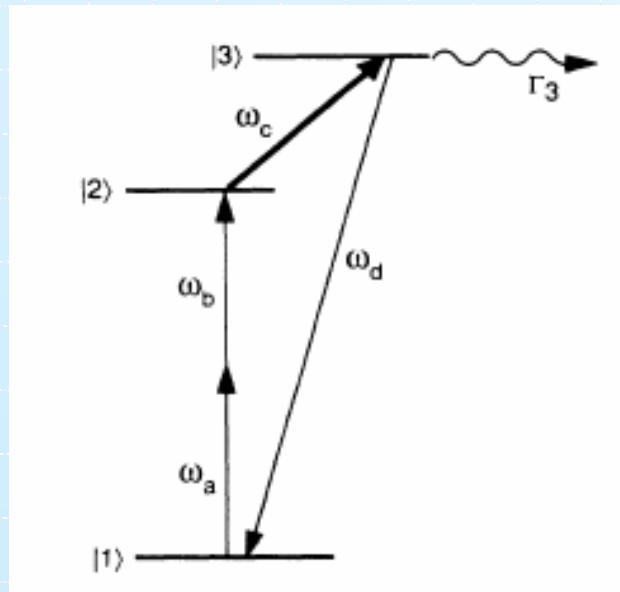
The relative energy level of  $^{85}\text{Rb}$ , and the produced FWM signal wavelength is about 300 nm.

# Giant Kerr nonlinearity induced by interacting dark resonances

# Introduction

## 1) Enhancement of optical nonlinearities

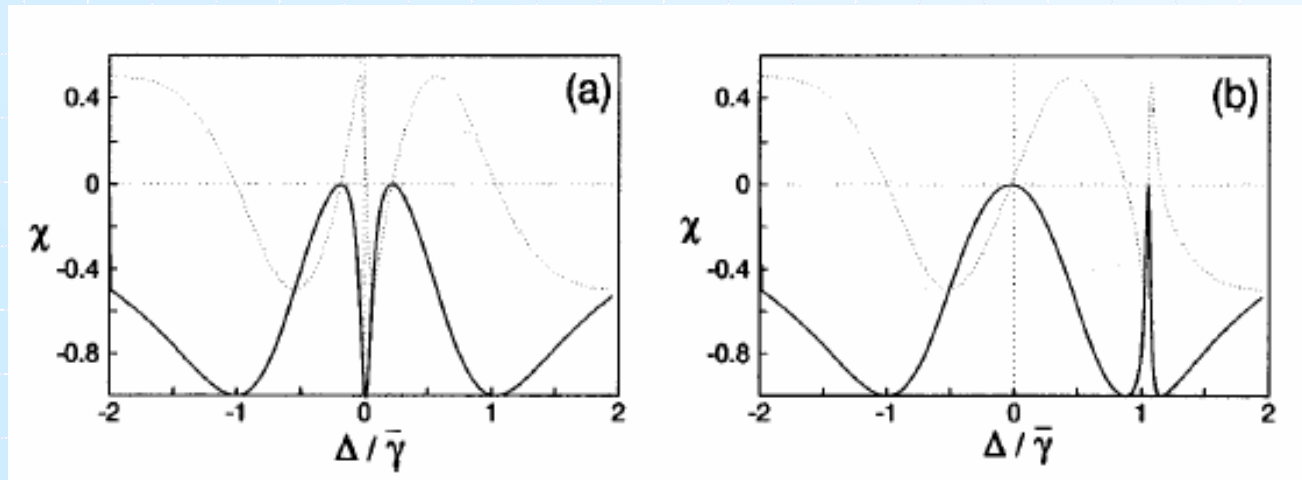
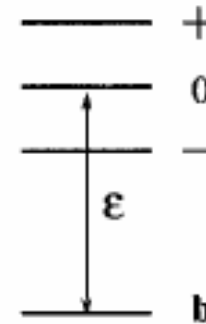
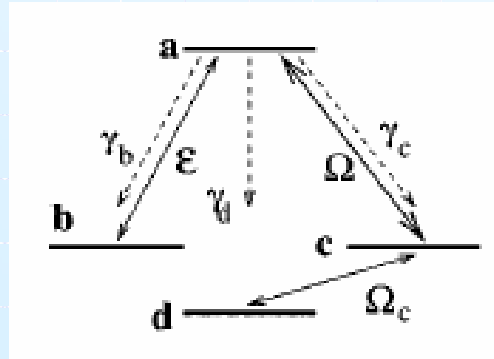
Since the optical nonlinear susceptibilities play important roles in areas of frequency conversion, generation of optical solitons, polarization phase gate, etc., it is desirable for people to have large nonlinear susceptibilities under conditions of low light powers.



(from Phys.Rev.Lett., 64, 1107(1990), by Harris *et al.*)



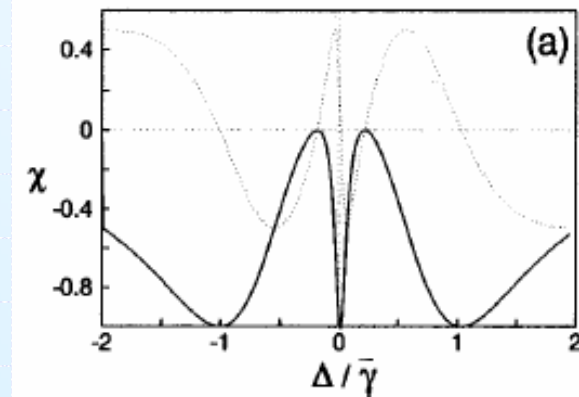
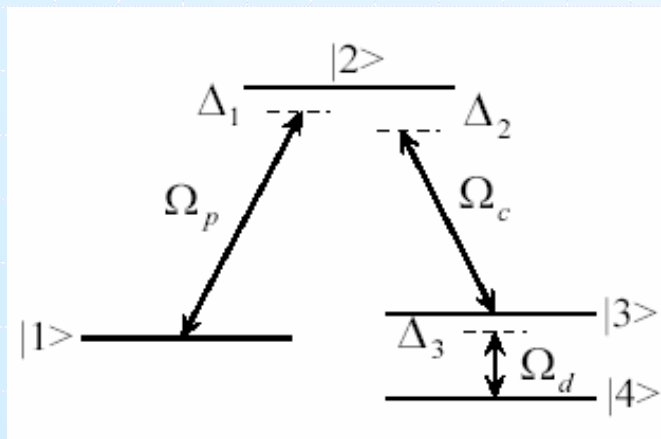
## 2) The properties of the interacting double-dark resonance



(from Phys.Rev.A 60, 3225(1999), by M.D Lukin *et al.*)

# Giant Kerr nonlinearity induced by interacting dark resonances

## 1. The scheme



It is the coherent control field  $\Omega_d$  that cause the occurrence of two distinct dark resonances, the interaction of which gave rise to the emergence of a strong absorption. By proper tuning of the coherent control field, the position and width of the absorption line can be engineered.

Therefore, question rises that whether this property can be utilized for engineering the third-order susceptibility?

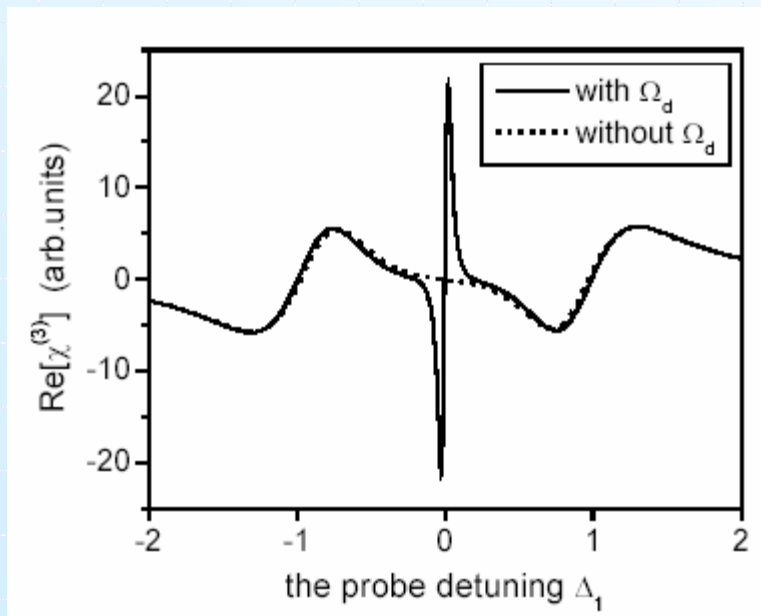
## 2. The analytical expression of the third-order susceptibility

$$\chi^{(3)} = \frac{-2N |\mu_{12}|^4}{3\varepsilon_0 \hbar^3} \frac{1}{((-ir + \Delta_1)(\Delta_1^2 - \Delta_1\Delta_3 - \Omega_d^2) - (\Delta_1 - \Delta_3)\Omega_c^2)} \left[ \frac{2\Omega_d^2\Omega_c^2(-\Delta_1\Delta_3 + \Delta_3^2 + \Omega_d^2)}{(-i(r - i\Delta_1)(\Delta_1^2 - \Delta_1\Delta_3 - \Omega_d^2) + (\Delta_1 - \Delta_3)\Omega_c^2)^2} - \frac{(\Delta_1^2 - \Delta_1\Delta_3 - \Omega_d^2)^3(\Delta_3^2 + 2(r^2 + \Omega_d^2))}{((ir + \Delta_1)(\Delta_1^2 - \Delta_1\Delta_3 - \Omega_d^2)\Omega_c - (\Delta_1 - \Delta_3)\Omega_c^3)^2} + \frac{(\Delta_1(-\Delta_1 + \Delta_3) + \Omega_d^2)^2(-i(r - 13i\Delta_1)(\Delta_1 - \Delta_3) + 12\Omega_d^2) - 2\Delta_1(\Delta_1 - \Delta_3)^3\Omega_c^2}{((ir + \Delta_1)(\Delta_1^2 - \Delta_1\Delta_3 - \Omega_d^2) - (\Delta_1 - \Delta_3)\Omega_c^2)^2} \right].$$

It is obtained by solving the density matrix elements equations with a perturbative iterative method up to third-order.

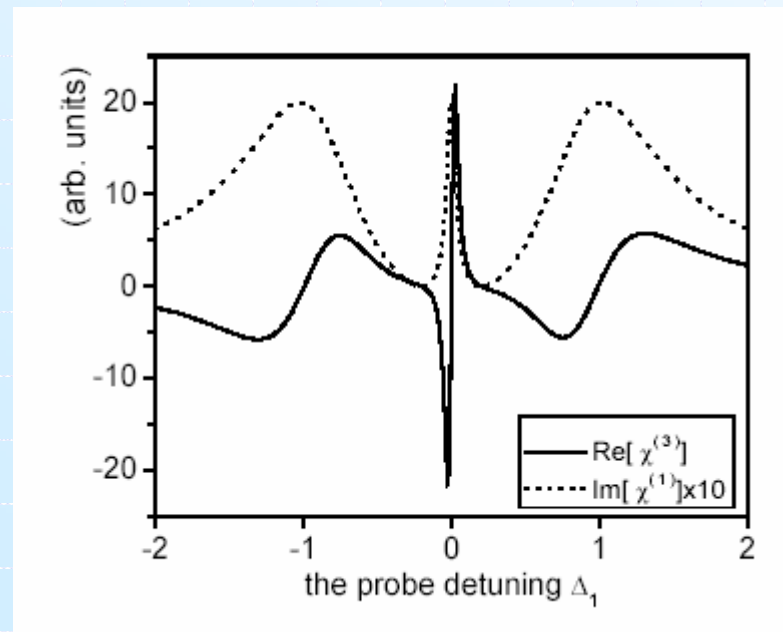
### 3. Numerical simulations

First consider the case that the coherent control field is on-resonance, i.e.,  $\Delta_3 = 0$

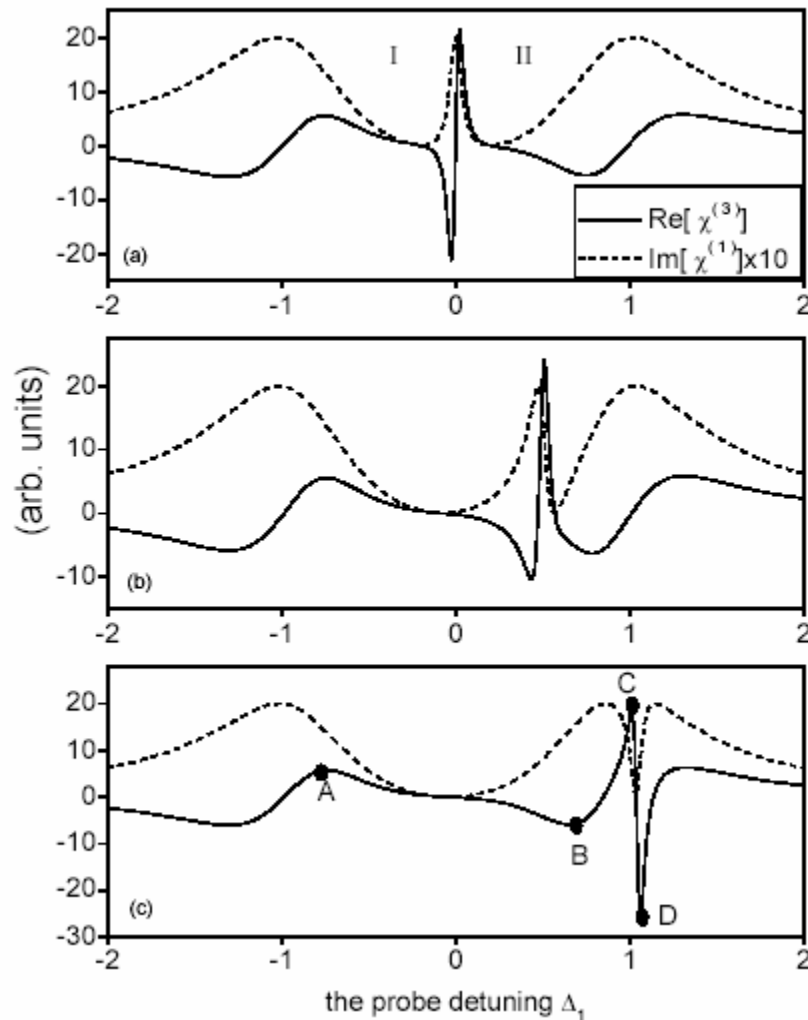


The Kerr nonlinearity  $\text{Re}[\chi^{(3)}]$ , i.e., the refractive part of the third-order susceptibility, is now enhanced by about **two orders of magnitude in the vicinity of the resonance**, compared with the conventional EIT system.

Nevertheless, the enhanced Kerr nonlinearity and the strong absorption are now superposed, as shown below. That is to say, although the Kerr nonlinearity is enhanced dramatically, it is accompanied by a strong linear absorption. This is not desired for applications of low-level intensity nonlinear optics.



Fortunately, the case varies with the tuning of the control field, which makes the enhanced Kerr nonlinearity with vanishing absorption possible.



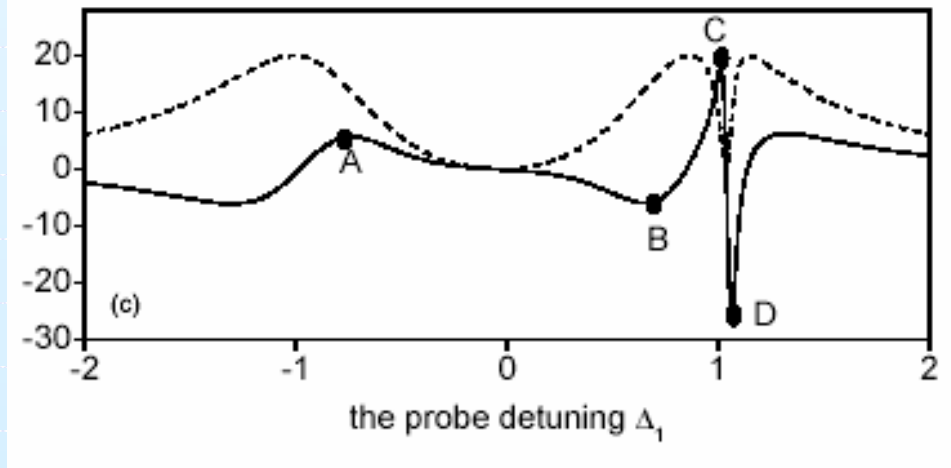
Figures (a), (b) and (c) corresponds to  $\Delta_3 = 0$ ,  $\Delta_3 = 0.5$  and  $\Delta_3 = 1.0$  respectively.

With the increasing of the detuning, the two distinct EIT windows become different: one broadens (region I) while the other narrows (region II).

When  $\Delta_3$  changes from 0 to 1.0, it is surprising to see that the enhanced Kerr nonlinearity gradually enters into the narrower EIT window.



From figure(c), it is clear that the Kerr nonlinearity is dramatically enhanced with suppressed linear absorption (region II).



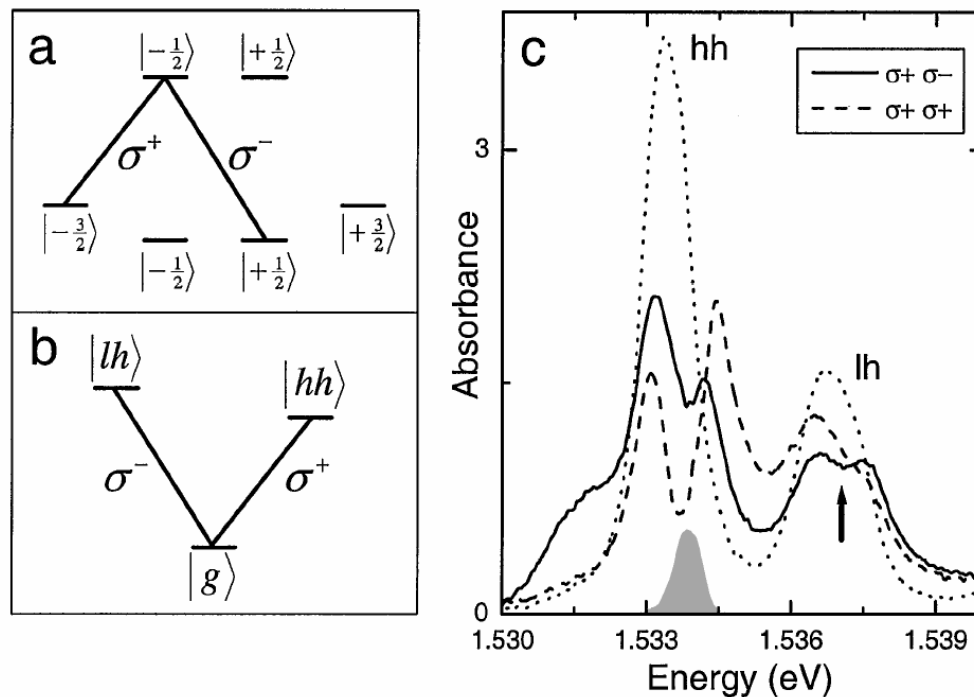
Since  $\text{Im}[\chi^{(1)}]$  and  $\text{Re}[\chi^{(3)}]$  of the conventional  $\Lambda$ -type EIT system are just the same to those shown in region I, a comparison between the present scheme and the single dark resonance system could be drawn by making an analysis of the two regions. In region I, for certain probe detunings (dot A and B), the enhanced  $\text{Re}[\chi^{(3)}]$  is accompanied by fractional linear absorption. In region II, however, the giant enhanced  $\text{Re}[\chi^{(3)}]$  (dot C and D) corresponds to vanishing absorption. This striking contrast implies the principal result of the present paper. Compared with the single dark resonance case, the interacting double dark resonances cause giant enhancement of the Kerr nonlinearity with vanishing linear absorption.



## Enhancing Kerr nonlinearity in an asymmetric double quantum well via Fano interference

# Introduction

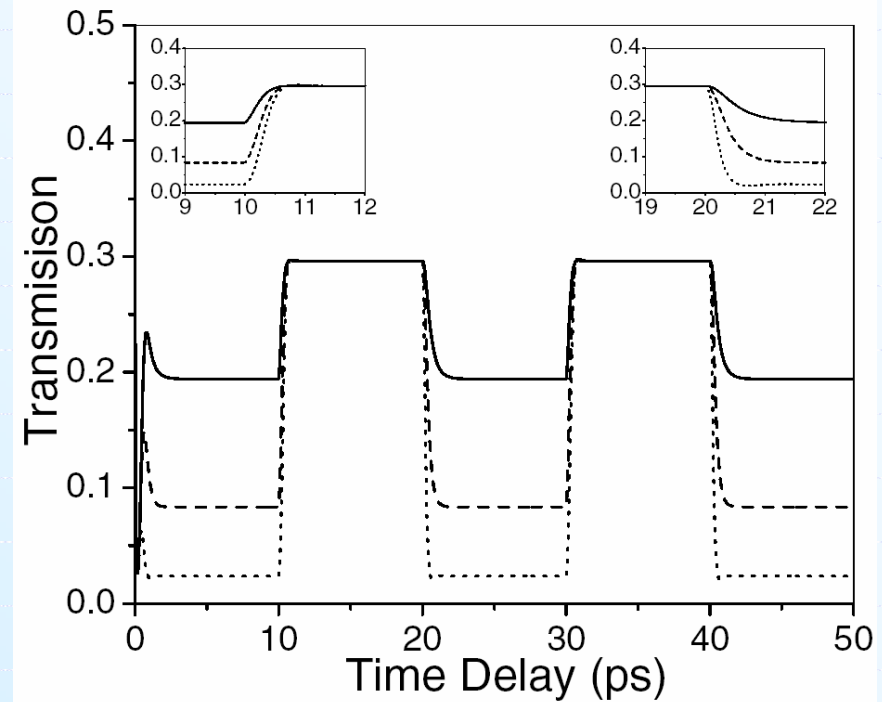
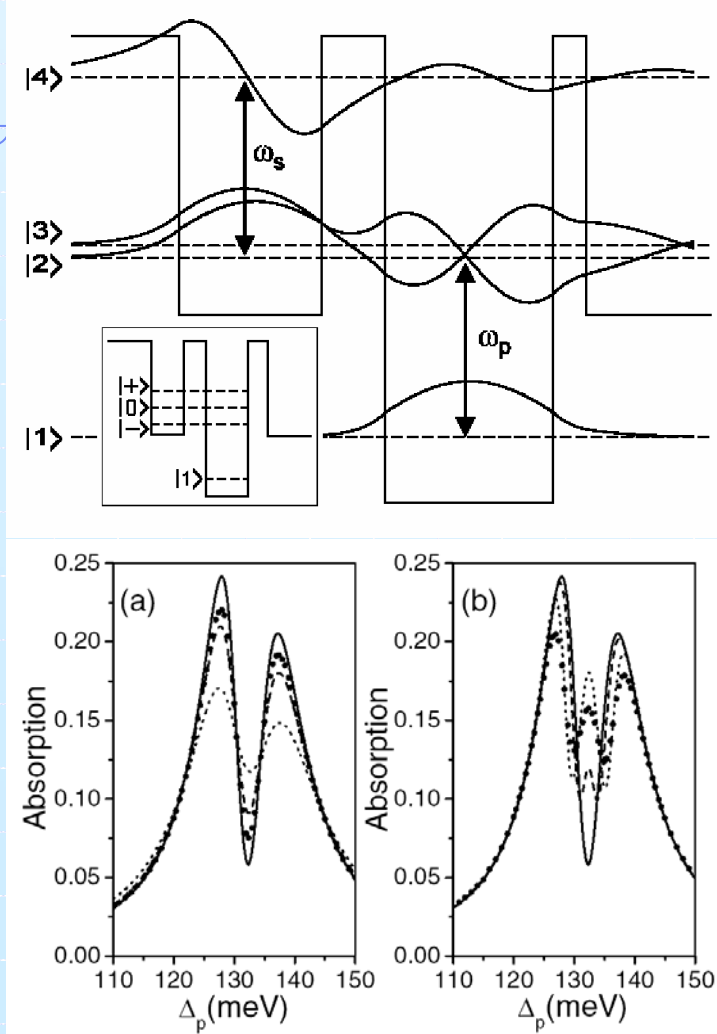
## 1) EIT in a GaAs quantum well



By using the nonradiative coherence between the heavy-hole and the light-hole valence bands, electromagnetically induced transparency in the transient optical response in a GaAs quantum well was observed experimentally.

(from Opt.Lett., 28, 831 (2003), by Phillips *et al.*)

## 2) Ultrafast all optical switching via tunable Fano interference

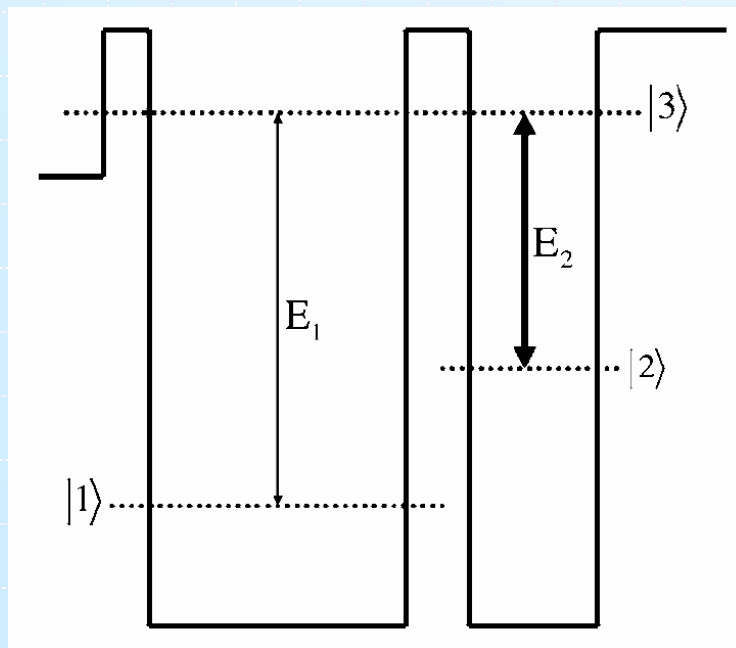


The transmitted probe either in the “open” or “close” position is less than one picosecond.

(from Phys. Rev. Lett., 95, 057401 (2005), by Jinhui Wu *et al.*)

# Enhancing Kerr nonlinearity in an asymmetric double quantum well via Fano interference

## 1. The model



The presence of the continuum for the decay of the excited state  $|3\rangle$  by tunneling gives rise to asymmetric line shapes due to Fano interference.

## 2. The basic equations

$$\dot{\sigma}_{11} = -\gamma_1 \sigma_{11} + 2 \operatorname{Im}(\tilde{\Omega}_1 \sigma_{31}) + \lambda_2 \sigma_{22} + \lambda_3 \sigma_{33} + \lambda_c \sigma_{cc}, \quad (1)$$

$$\dot{\sigma}_{22} = -(\gamma_2 + \lambda_2) \sigma_{22} + 2 \operatorname{Im}(\tilde{\Omega}_2 \sigma_{32}), \quad (2)$$

$$\dot{\sigma}_{33} = -(\Gamma_3 + \lambda_3) \sigma_{33} - 2 \operatorname{Im}(\tilde{\Omega}_1^* \sigma_{31}) - 2 \operatorname{Im}(\tilde{\Omega}_2^* \sigma_{32}), \quad (3)$$

$$\dot{\sigma}_{21} = -[i(\Delta_1 - \Delta_2) + (\gamma_{21}^p + \gamma_{21}^{e-e})] \sigma_{21} + i\tilde{\Omega}_1 \sigma_{23} - i\tilde{\Omega}_2 \sigma_{31}, \quad (4)$$

$$\dot{\sigma}_{31} = -[i\Delta_1 + (\gamma_{31}^p + \gamma_{31}^{e-e})] \sigma_{31} - i\tilde{\Omega}_2 \sigma_{21} - i\tilde{\Omega}_1 \sigma_{11} + i\tilde{\Omega}_1^* \sigma_{33}, \quad (5)$$

$$\dot{\sigma}_{32} = -(i\Delta_2 + \gamma_{32}^p) \sigma_{32} + i\tilde{\Omega}_2^* \sigma_{33} - i\tilde{\Omega}_2 \sigma_{22} - i\tilde{\Omega}_1 \sigma_{12}, \quad (6)$$

Rabi frequencies

$$\tilde{\Omega}_1 = \Omega_1 \left(1 - \frac{i}{q_1}\right), \quad \tilde{\Omega}_2 = \Omega_2 \left(1 - \frac{i}{q_2}\right),$$

Asymmetric parameters

$$q_1 = \frac{2\Omega_1}{\sqrt{\gamma_1 \Gamma_3}}, \quad q_2 = \frac{2\Omega_2}{\sqrt{\gamma_2 \Gamma_3}},$$

### 3. The expressions of the linear and nonlinear susceptibilities

$$\text{Im}(\chi^{(1)}) = -\frac{N\mu_{13}^2}{\hbar\epsilon_0} \left( \frac{2}{\Gamma_3} + \frac{\mathcal{A}\mathcal{D} - \mathcal{B}\mathcal{C}}{\mathcal{C}^2 + \mathcal{D}^2} \right),$$

$$\chi^{(3)}\mathcal{E}_1^3 = \frac{N}{3\epsilon_0} \left[ \left( s'_1 - \frac{i}{2}\gamma'_1 \right) \mathcal{E}_1 \sigma_{11}^{(2)} + \mu_{13} \left( 1 - \frac{i}{q_1} \right) \sigma_{31}^{(3)} \right],$$

$$\mathcal{A} = \frac{2}{q_1} (\gamma_{21}^p + \gamma_{21}^{e-e}) - \left( 1 - \frac{1}{q_1^2} \right),$$

$$\mathcal{B} = \left( 1 - \frac{1}{q_1^2} \right) (\gamma_{21}^p + \gamma_{21}^{e-e}) + \frac{2}{q_1} (\Delta_1 - \Delta_2),$$

$$s'_1 = P \sum_c \frac{|\mu_{1c}|^2}{\hbar^2(\nu_1 - \omega_{c1})},$$

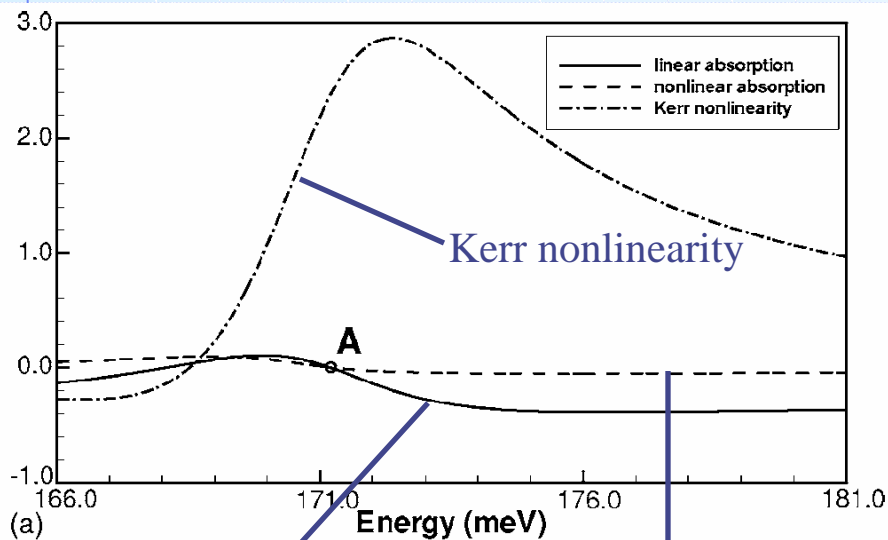
$$\gamma'_1 = \frac{4\mu_{13}^2}{q_1^2 \Gamma_3},$$

$$\mathcal{C} = \Delta_1(\Delta_1 - \Delta_2) - (\gamma_{21}^p + \gamma_{21}^{e-e})(\gamma_{31}^p + \gamma_{31}^{e-e}) - \Omega_2^2 \left( 1 - \frac{1}{q_2^2} \right),$$

$$\mathcal{D} = \frac{2\Omega_2^2}{q_2} - [\Delta_1(\gamma_{21}^p + \gamma_{21}^{e-e}) + (\Delta_1 - \Delta_2)(\gamma_{31}^p + \gamma_{31}^{e-e})].$$

### 3. Results

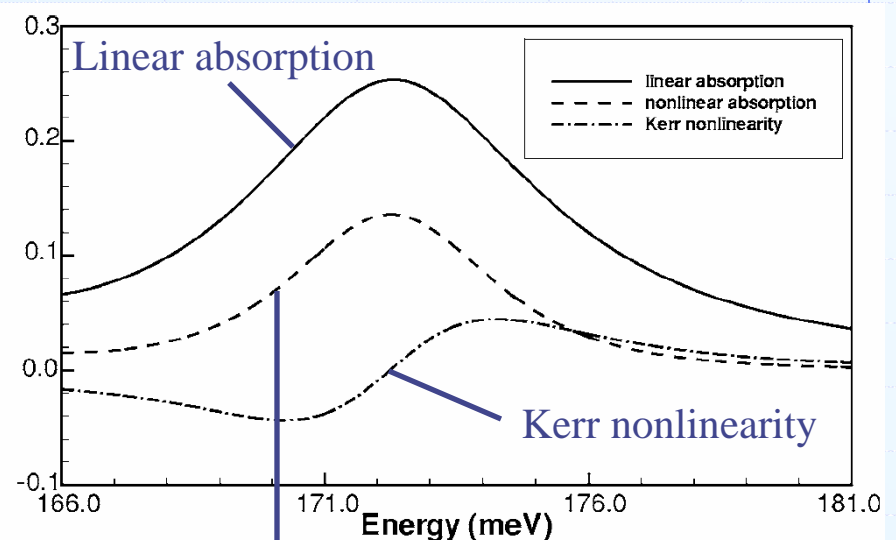
with Fano interference



Linear absorption

Nonlinear absorption

without Fano interference



Nonlinear absorption

The enhancement of Kerr nonlinearity with vanishing linear and nonlinear absorptions is obtained due to Fano inference.



## Summary

- We realized creation of coherent superposition state without multi-photon resonance condition (**PRA69,023805(2004)**);
- We proposed a new scheme for enhancement of FWM using the property of the double-dark resonance (**PRA71,043819(2005)**);
- Giant Kerr nonlinearity induced by interacting dark resonances (**Optics Lett.30, 3371(2005)**);
- The Kerr nonlinearity can be enhanced in an asymmetric double quantum well via Fano interference (**PRB74,155314(2006)**).
- It shows that the double-dark resonance property gives rise to various interesting phenomena and enlarge the domain of dark-state-based physics.

**Thank you**