Coupled Cavity QED

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(Some) recent papers

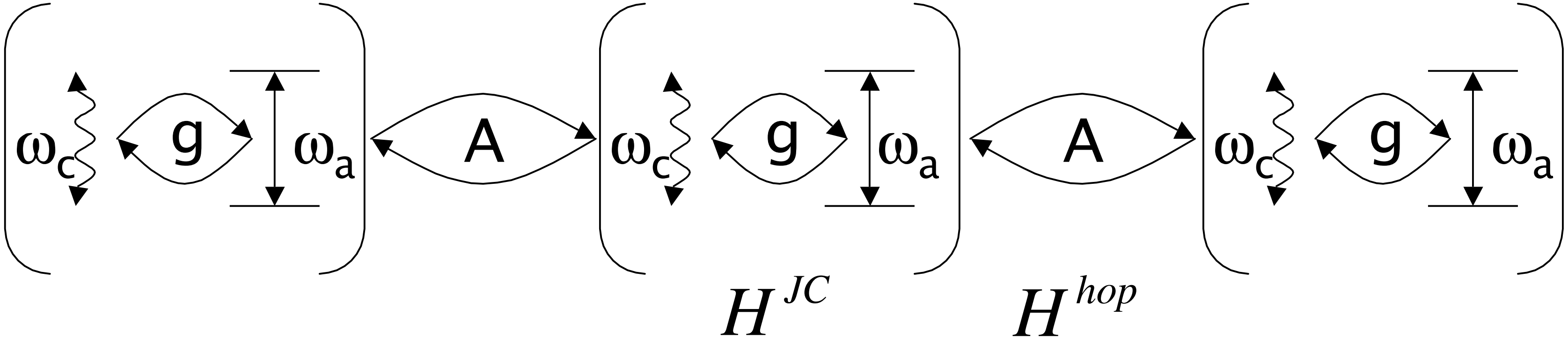
- "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays," Angelakis, Santos and Bose, quant-ph/0606159 (2006).
- "Generation and verification of high-dimensional entanglement from coupled-cavity arrays," Angelakis and Bose, J. Opt. Soc. Am. B 24, 266 (2007).
- "Cluster state quantum computation in coupled cavity arrays," Angelakis and Kay, quant-ph/0702133 (2007).
- "Strongly interacting polaritons in coupled arrays of cavities," Hartmann, Brandao, and Plenio, Nature Phys. **2**, 849 (2006).
- "Quantum phase transitions of light," Greentree, Tahan, Cole and Hollenberg, Nature Phys. 2, 856 (2006).
- "Atomic Entanglement vs Photonic Visibility for Quantum Criticality of Hybrid System," Huo, Li, Song, and Sun, quant-ph/0702078 (2007).
- "Coherent output of photons from coupled superconducting transmission line resonators controlled by charge qubits," Zhou, Gao, Song, and Sun, cond-mat/0608577 (2006).

Motivations

- Quantum phase transitions with photons
- Simulation of other systems, e.g. spin chains
- Phase transitions in highly addressable and tunable systems
- Quantum information processing: entanglement creation, single photon generation, cluster-state computing
- Phase transitions similar to Bose-Hubbard model but possessing some unique features

Coupled cavity model

- 1-D chain of N cavities
- Each contains a two-level atom
- Photons can hop between adjacent cavities



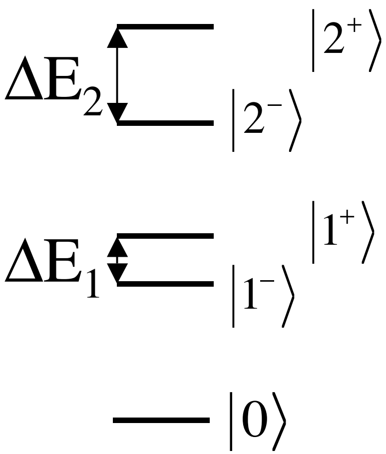
$$H^{JC} = \sum_{i=1}^{N}\left[\omega_c \hat{a}_i^\dagger \hat{a}_i + \omega_a |e_i\rangle\langle e_i| + g\left(\hat{a}_i^\dagger |g_i\rangle\langle e_i| + \hat{a}|e_i\rangle\langle g_i|\right)\right]$$

$$H^{hop} = A\sum_{i=1}^{N-1}\left(\hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_{i+1}^\dagger \hat{a}_i\right)$$

Polaritons

Eigenstates of the Jaynes–Cummings Hamiltonian

$$H^{JC} = \sum_{i=1}^{N} \left[\omega_c \hat{a}_i^\dagger \hat{a}_i + \omega_a |e_i\rangle\langle e_i| + g\left(\hat{a}_i^\dagger |g_i\rangle\langle e_i| + \hat{a}|e_i\rangle\langle g_i| \right) \right]$$



$$|0\rangle = |g,0\rangle$$

$$|n^-\rangle = \sin(\theta(n,g,\Delta)/2)|e,n-1\rangle + \cos(\theta(n,g,\Delta)/2)|g,n\rangle$$

$$|n^+\rangle = \cos(\theta(n,g,\Delta)/2)|e,n-1\rangle - \sin(\theta(n,g,\Delta)/2)|g,n\rangle$$

$$\Delta = \omega_a - \omega_c$$

$$\Delta E_n = \sqrt{\Delta^2 + 4ng^2}$$

Two excitations

Same cavity

$|2_i^+\rangle$ $|2_i^-\rangle$

Different cavities

$|1_i^-\rangle \otimes |1_j^-\rangle$ $|1_i^-\rangle \otimes |1_j^+\rangle$ $|1_i^+\rangle \otimes |1_j^+\rangle$

Energy spectrum for A=0 (no hopping):

$|1_i^+\rangle \otimes |1_j^+\rangle$ —— $2E_1^+$

$|2_i^+\rangle$ —— E_2^+

$|1_i^-\rangle \otimes |1_j^+\rangle$ —— $E_1^- + E_1^+$

$|2_i^-\rangle$ —— E_2^-

$|1_i^-\rangle \otimes |1_j^-\rangle$ —— $2E_1^-$

- Order is the same for all values of g and Δ
- Level splittings can change significantly, which alters the eigenstates

Coupling

$$H^{hop} = A\sum_{i=1}^{N-1}\left(\hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_{i+1}^\dagger \hat{a}_i\right)$$

- Conserves total excitation number

$$\mathcal{N} = \sum_{i=1}^{N}\left(\hat{a}_i^\dagger \hat{a}_i + |e_i\rangle\langle e_i|\right)$$

- Photons hop, not polaritons:

$$|2_i^-\rangle \otimes |0_j\rangle \rightarrow \begin{cases} |1_i^-\rangle \otimes |1_j^-\rangle \\ |1_i^-\rangle \otimes |1_j^+\rangle \\ |1_i^+\rangle \otimes |1_j^-\rangle \\ |1_i^+\rangle \otimes |1_j^+\rangle \end{cases}$$

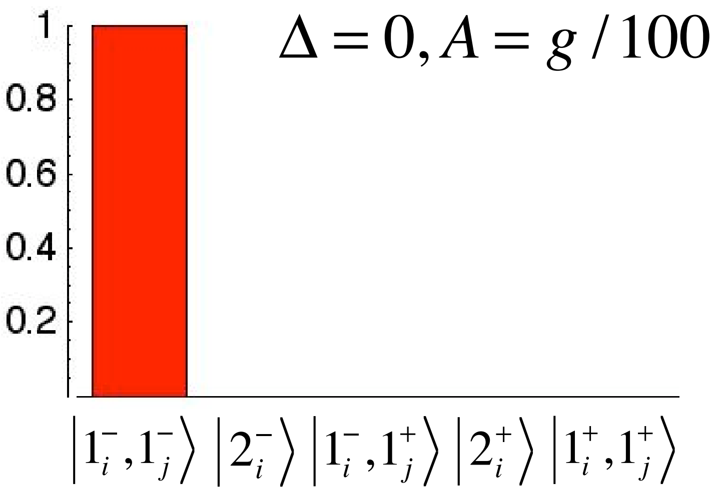
The probability of each transition depends on the balance between Δ, g, and A

Photon blockade

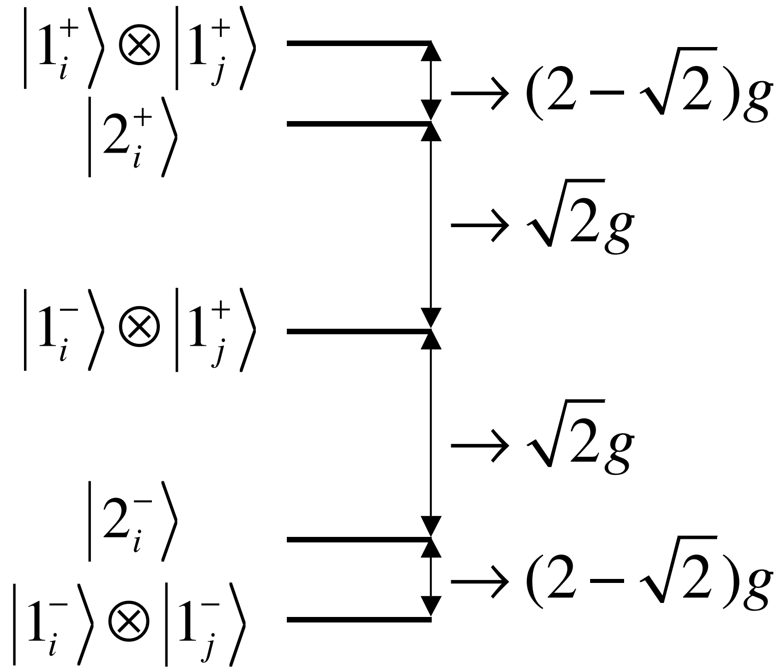
$$\Delta = 0, A \ll g$$

States are polaritonic:

$$\left|n^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|e, n-1\right\rangle \pm \left|g, n\right\rangle\right)$$



3 cavities, 2 excitations



Ground state:

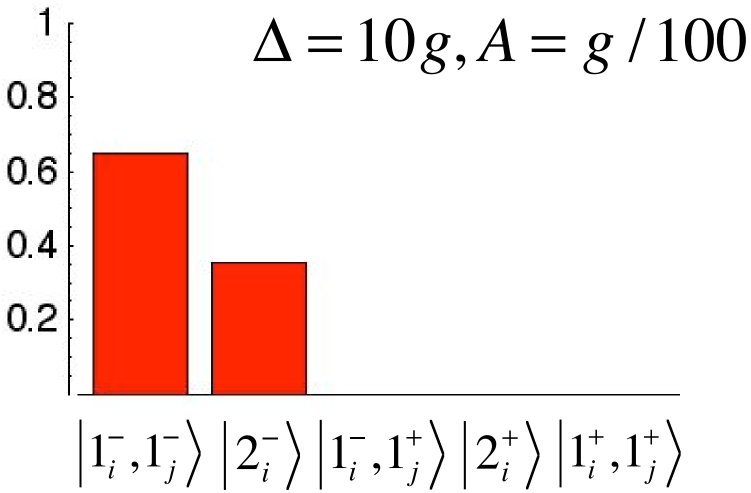
polaritonic Mott insulator

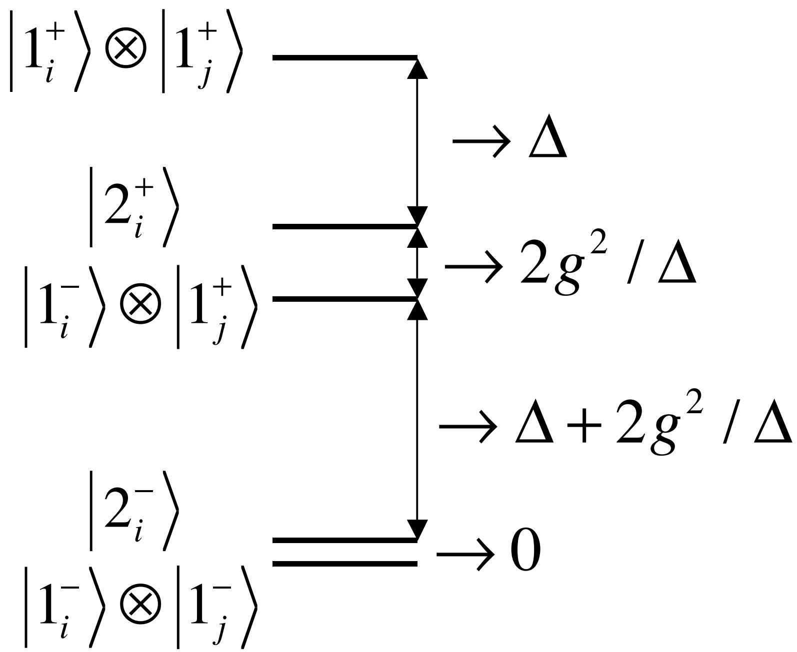
Large positive detuning

$$\Delta \gg g \gg A, \Delta > 0$$

Lowest two states are photon-like:

$$|n^-\rangle \rightarrow |g,n\rangle, |n^+\rangle \rightarrow |e,n-1\rangle$$





Ground state:

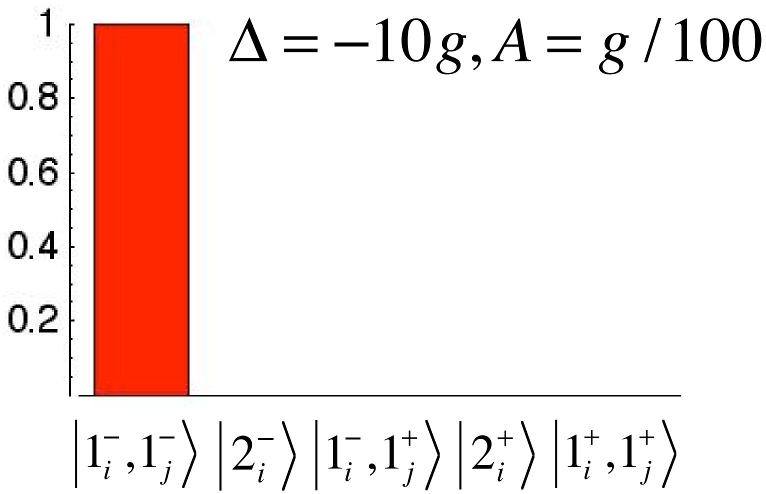
photonic superfluid

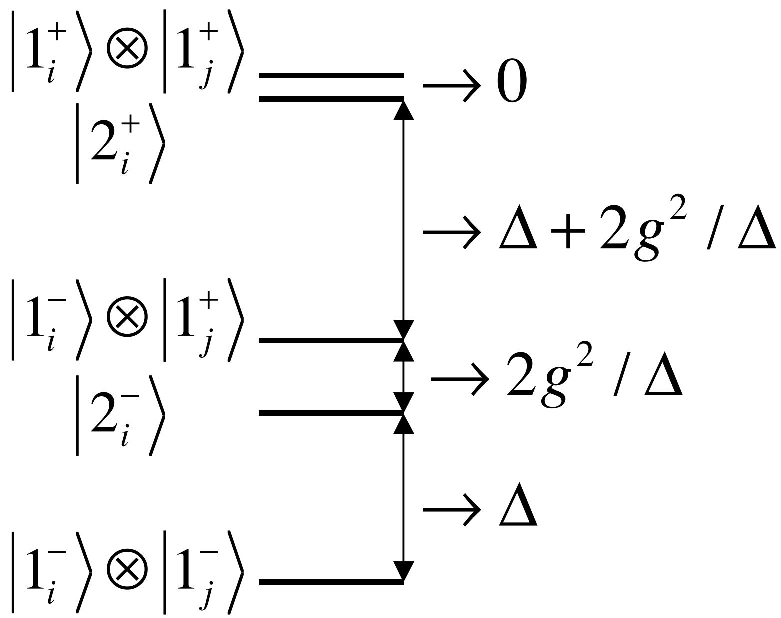
Large negative detuning

$$|\Delta| \gg g \gg A, \Delta < 0$$

Lowest state is atom-like:

$$|n^-\rangle \to |e, n-1\rangle, |n^+\rangle \to |g, n\rangle$$



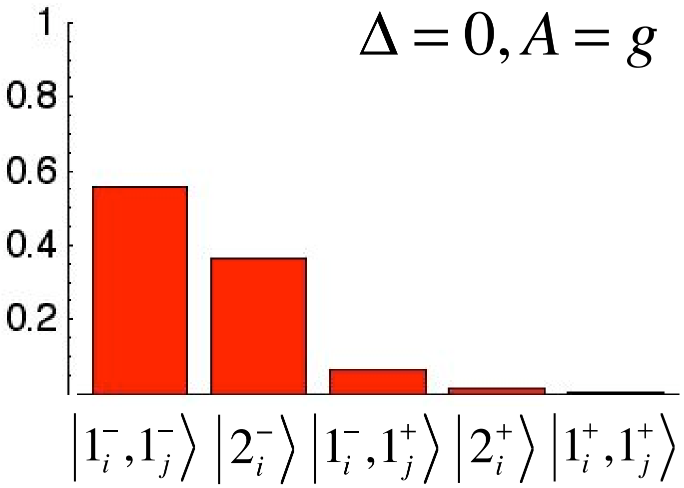


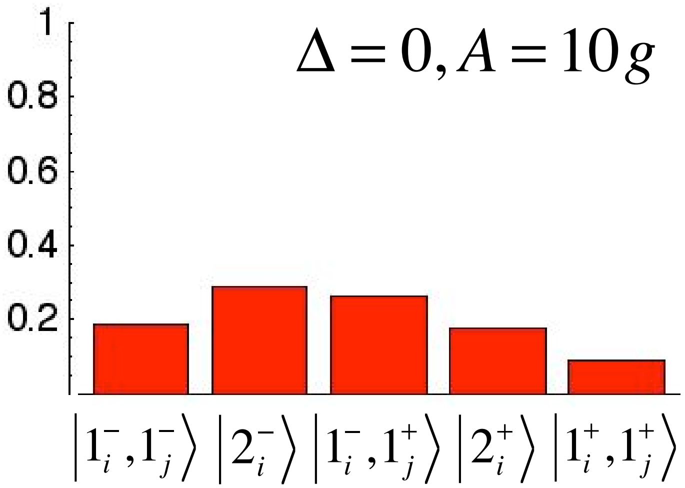
Ground state:

atomic Mott insulator

Increased photon hopping

$$\Delta = 0, A \geq g$$





Ground state:

polaritonic superfluid

Both polariton species are involved!

Differences from Bose-Hubbard model

- Effective repulsion caused by photon blockade
- Hopping is among photons rather than polaritons
- Detuning-controlled phase transition involves a change in the nature of the particles
- Hopping-controlled transition involves two different species of polaritons
- Limited number of cavities and definite number of excitations (as opposed to thermodynamic limit)

Further work

- Identifying the consequences of having polaritons rather than bosons
- Characterisation of superfluid and insulator phases in this model
- Investigating the polaritonic superfluid phase
- Applications?