Implementations of Topological Order with Atomic, Molecular and Optical Systems

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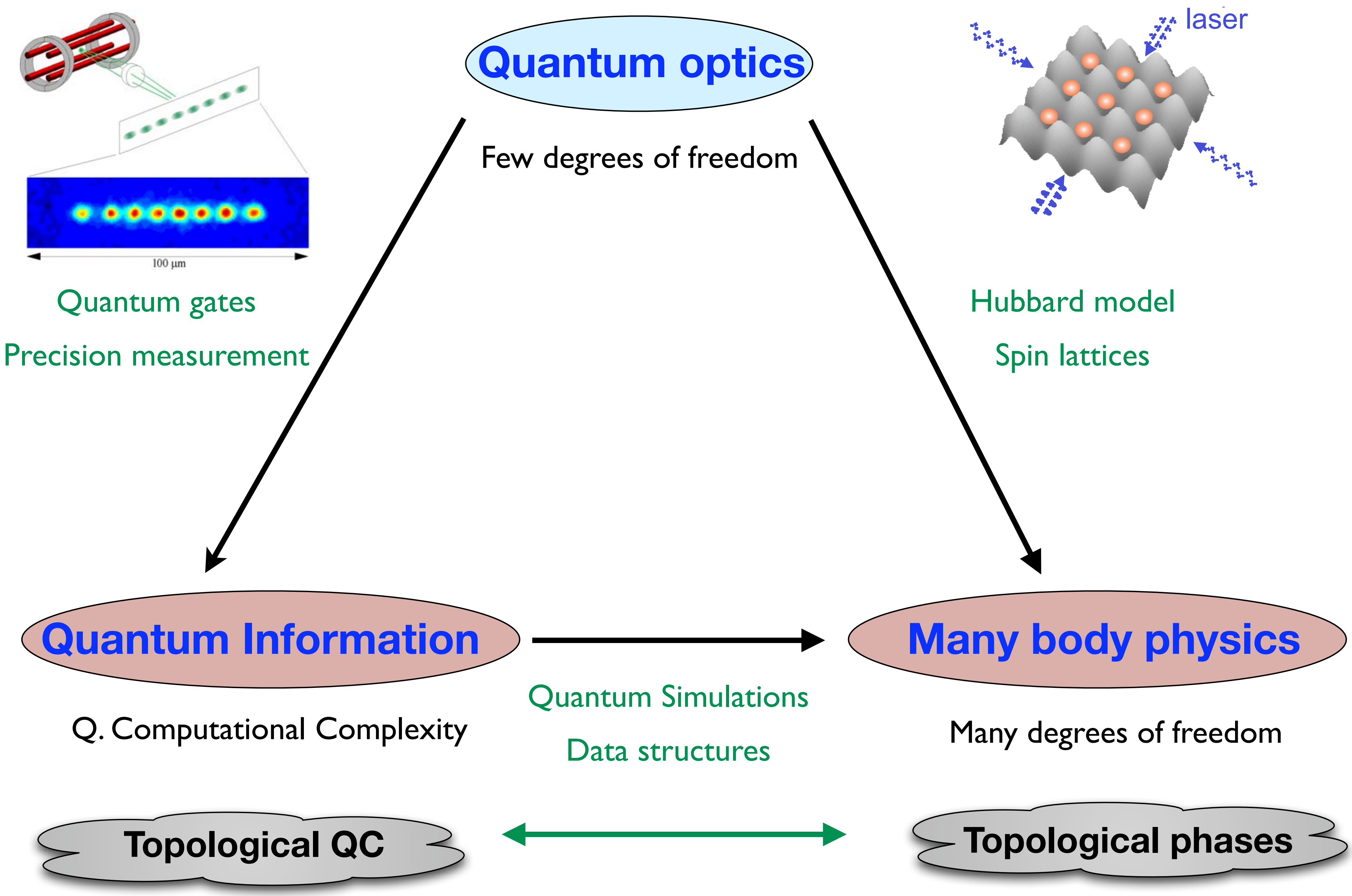
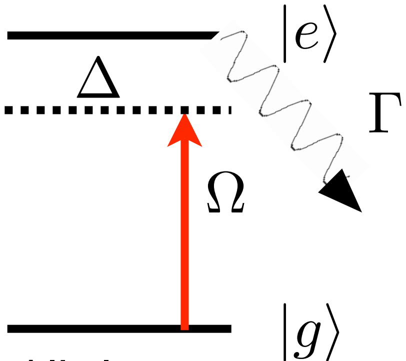

Quantum optics
Few degrees of freedom
100 μm
Quantum gates
Precision measurement
laser
Hubbard model
Spin lattices
Quantum Information
Many body physics
Quantum Simulations
Data structures
Q. Computational Complexity
Many degrees of freedom
Topological QC
Topological phases

Outline

- Trapped atoms in optical lattices: the short road to q. simulations
- Topological order in spin lattices
- Implementations
  - Spin-1/2 models -----> Kitaev honeycomb model
  - Spin>1/2
- Summary & Outlook
- Open questions

Optical lattices



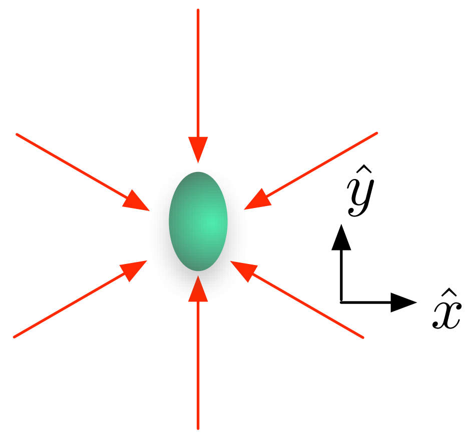
Alkali atom

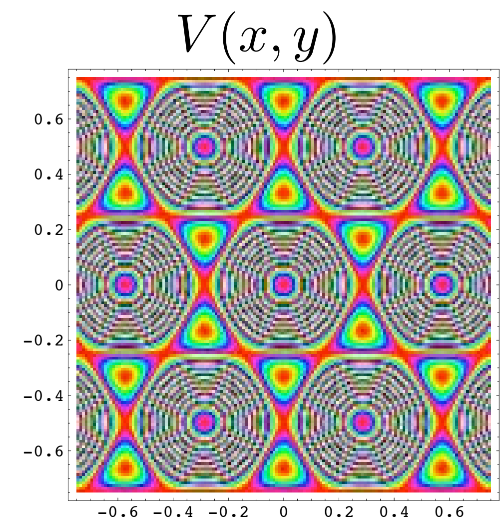
Coherent for large intensity and detuning

Can build 1D, 2D, 3D lattices with adjustable topography by tuning intensity, polarization, & detuning

$$V(\mathbf{x}) = -\langle \mathbf{E}(\mathbf{x},t) \cdot \overleftrightarrow{\alpha} \cdot \mathbf{E}^{*}(\mathbf{x},t)\rangle$$

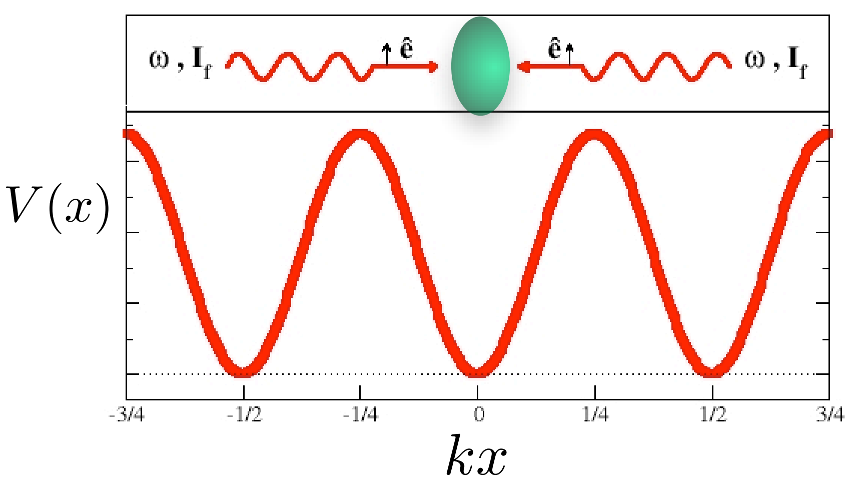
$$V_0 \simeq \frac{\Omega^2}{\Delta} \qquad \gamma \simeq \Gamma\left(\frac{\Omega}{\Delta}\right)^2$$

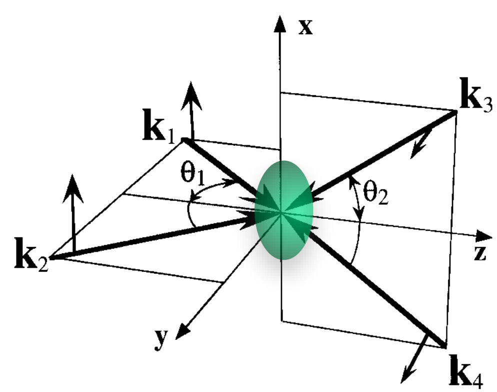


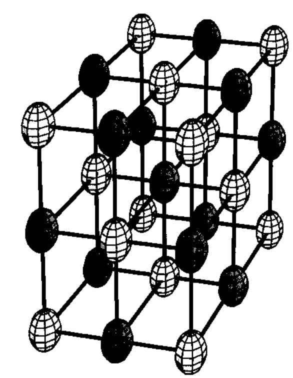


Trapping with counter-propagating lasers

$$V(x) = V_0 \cos^2(kx)$$

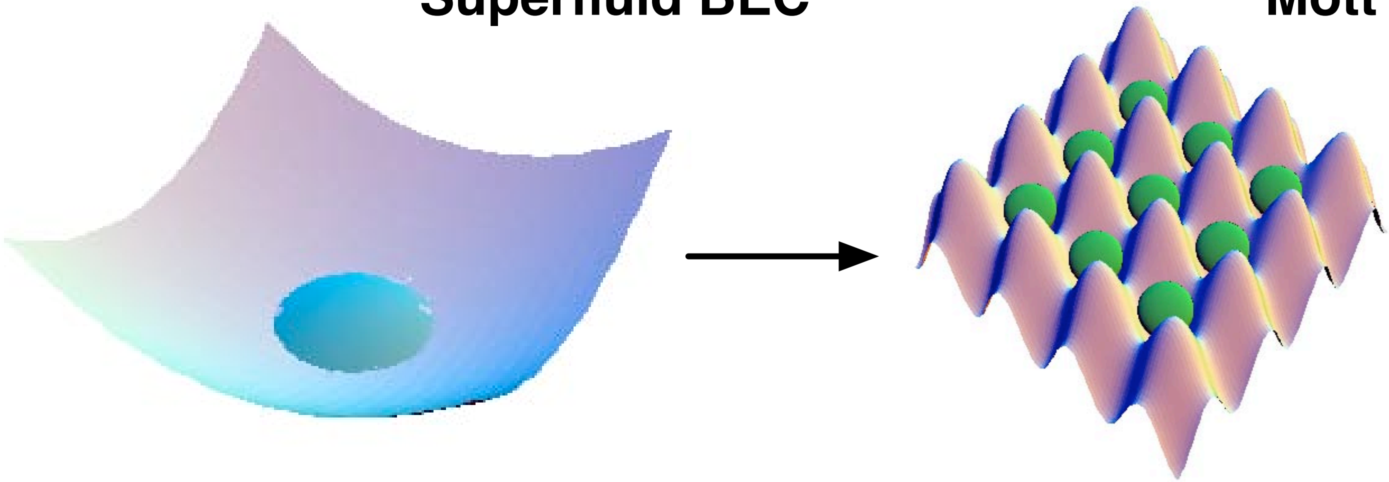






- State preparation via q. phase transition

Superfluid BEC **Mott Insulator**

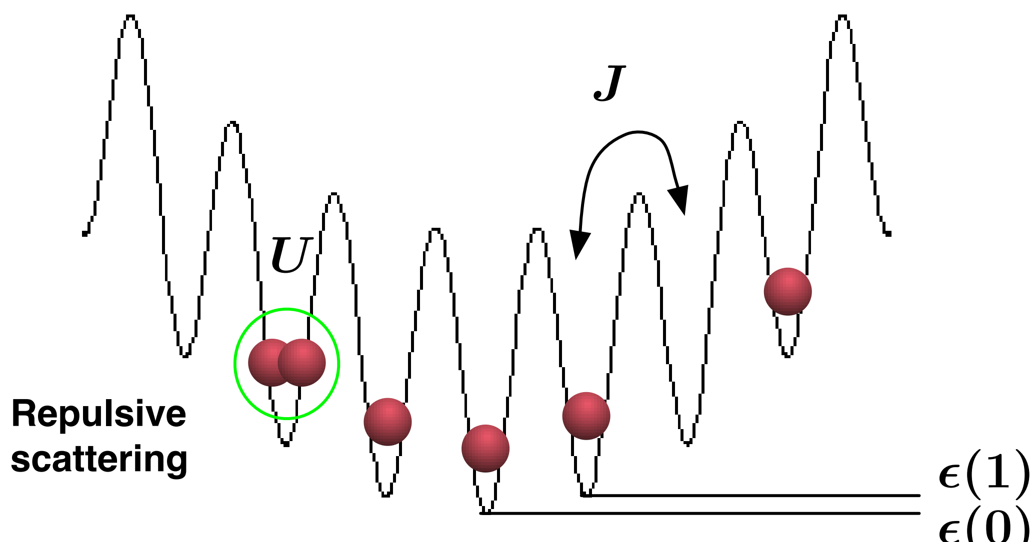


Theory: Jaksch *et al.* PRL 81, 3108 (1998)

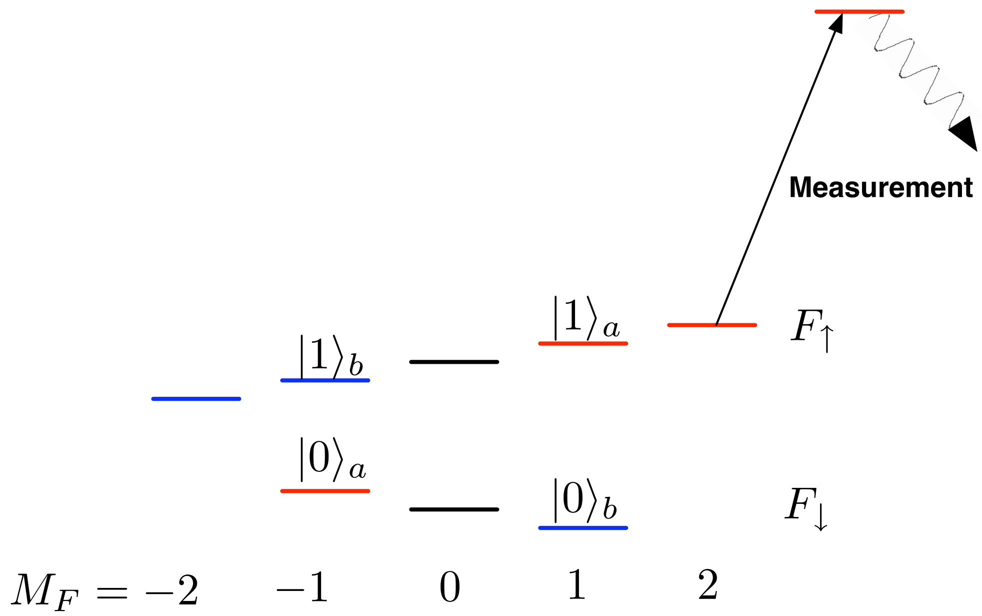
Exp: M. Greiner *et al.* Nature 415, 39 (2003)

- Bose-Hubbard dynamics

$$H_{BH} = -\sum_{<j,k>} J(a_j^\dagger a_k + a_k^\dagger a_j) + \sum_j \frac{U}{2} n_j(n_j - 1) + \epsilon(j) n_j$$

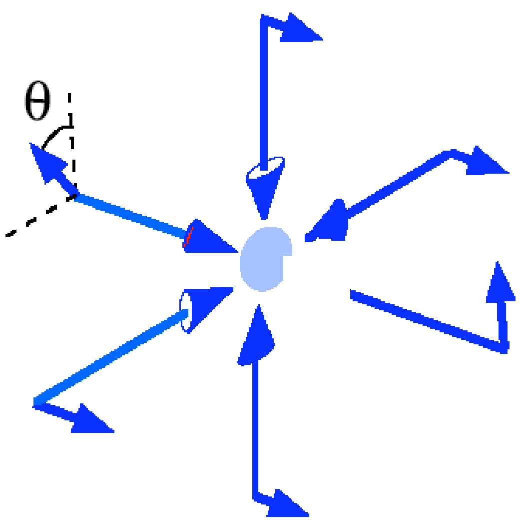


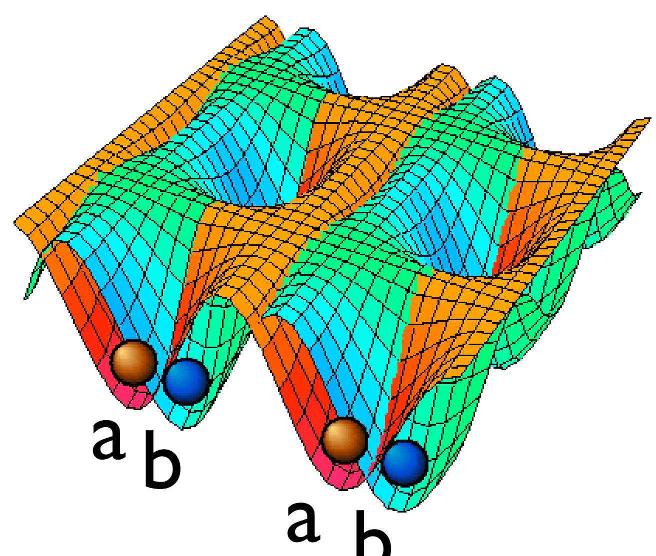
- Quantum gates
  - Information encoded in hyperfine levels.



Decoherence free subspace wrt magnetic field fluctuations

  - Collisional interactions. Tunable using state dependent lattices





Theory: GKB *et al.*, PRL, 82, 1060 (1999); Jaksch *et al.* 82, 1975 (1999)

Exp: O. Mandel, *et al.* Nature 425, 937 (2003)

Spin lattice models with TO

- **A Hamiltonian on spins represented as edges on a surface cellulation**

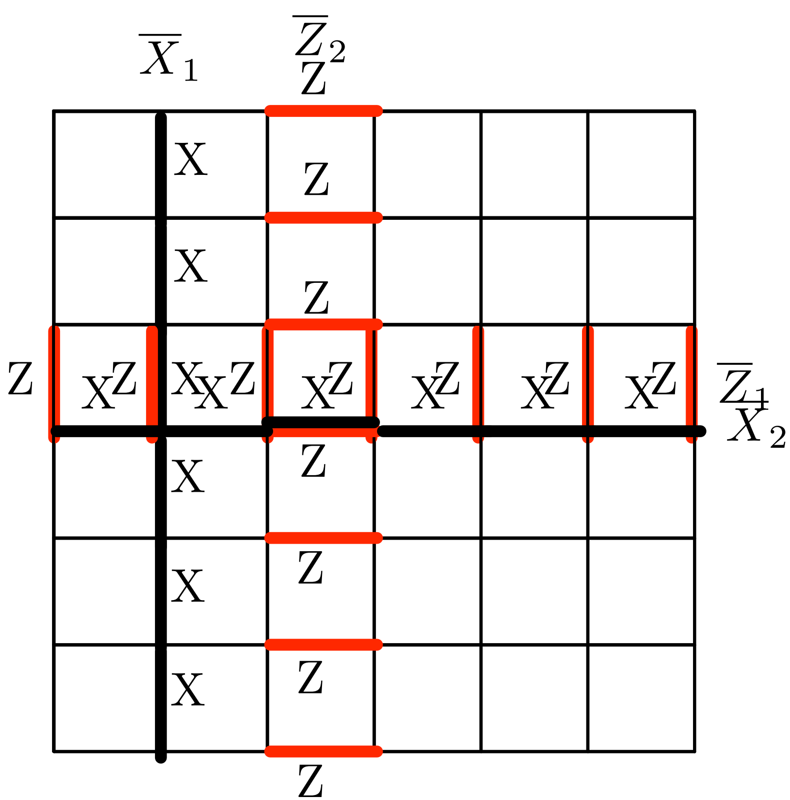
$$H = -U\Big(\sum_{v \in \mathcal{V}} g_v + \sum_{f \in \mathcal{F}} g_f\Big) \qquad g_v = \prod_{e \in \{[*,v],[v,*]\}} Z_e \qquad g_f = \prod_{e \in \partial f} X_e$$

$$[g_v, g_{v'}] = [g_f, g_{f'}] = [g_v, g_f] = 0$$

  - **Sum of generators of the stabilizer group G** $\quad G = \langle \{g_v, g_f\} \rangle$
  - **Ground states of H are eigenstates of G with eigenvalue +1**

- **Ex: Qubits on a torus**

$$H = -U\Big(\sum_{+} Z_{e_1} Z_{e_2} Z_{e_3} Z_{e_4} + \sum_{\square} X_{e_1} X_{e_2} X_{e_3} X_{e_4}\Big)$$



$$\dim \mathcal{H}_{\rm gr} = \mathrm{Trace}\Big[\frac{1}{\#G} \sum_{g \in G} g\Big] = \mathrm{Trace}\Big[\frac{1}{2^{n-2}} \sum_{g \in G} g\Big] = 4$$

Generically

$$\dim \mathcal{H}_{\rm gr} = \# H_1(\Gamma, \mathbb{F}_2) = 2^{2g}$$

genus of surface

*A.Yu. Kitaev, Annals of Physics, **303**, 2 (2003); quant-ph/9707021
M Freedman and D. Meyer, Found. Comp. Math. 1, 325 (2001).

Generalized surface codes

- **Represent state space of each spin on a lattice by a qudit (d levels)**

- **Single spin operator basis**

$$
\begin{aligned}
X\,|j\rangle &= |j+1 \bmod d\rangle \\
Z\,|j\rangle &= \xi^j\,|j\rangle\,, \qquad \text{for } \xi = \exp(2\pi i/d)
\end{aligned}
$$

Chain — **Computational basis state**

$$
\omega = \sum_{e\in\mathcal{E}} n_e e \quad \leftrightarrow \quad |\omega\rangle
$$

- **Hamiltonian (d-prime)**

$$
H = U\sum_v H_v + h\sum_f H_f
$$

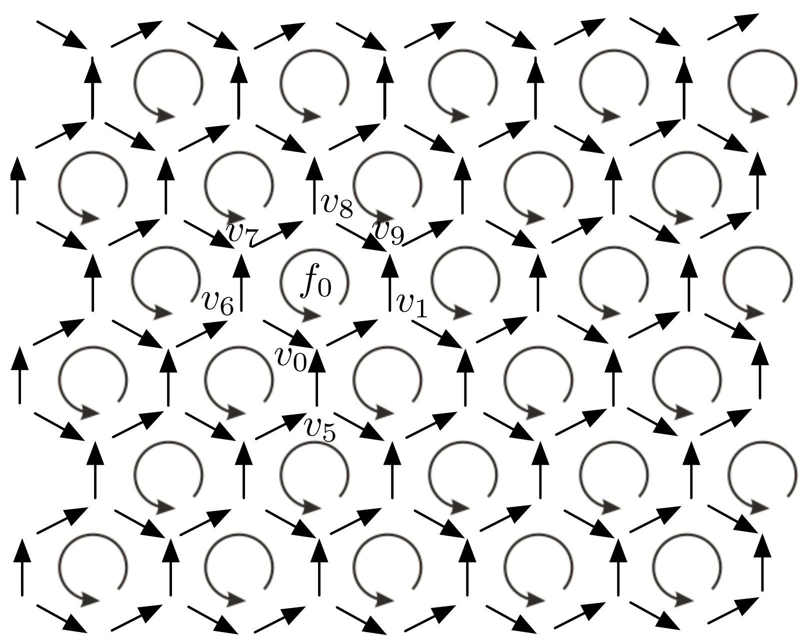
$$
\begin{aligned}
g_v &= \prod_{e=[*,v]} Z_e \prod_{e=[v,*]} Z_e^{-1} \\
H_v &= -(g_v + g_v^\dagger)
\end{aligned}
$$

$$
\begin{aligned}
g_f &= X_{e_1}^{o_1} X_{e_2}^{o_2} X_{e_3}^{o_3} \ldots X_{e_p}^{o_p} \\
H_f &= -(g_f + g_f^\dagger)
\end{aligned}
$$

$$
\dim\mathcal{H}_{\mathrm{gr}} = \#H_1(\Gamma, \mathbb{F}_d) \qquad \mathcal{H}_{\mathrm{gr}} \cong (\mathbb{C}^d)^{2g}
$$

Example:

$$
\begin{aligned}
g_{v_0} &= Z_{[v_6,v_0]} Z_{[v_5,v_0]} Z^{-1}_{[v_0,v_1]} \\
g_{f_0} &= X_{[v_0,v_1]} X_{[v_1,v_9]} X^{-1}_{[v_8,v_9]} X^{-1}_{[v_7,v_8]} X^{-1}_{[v_6,v_7]} X_{[v_6,v_0]}
\end{aligned}
$$

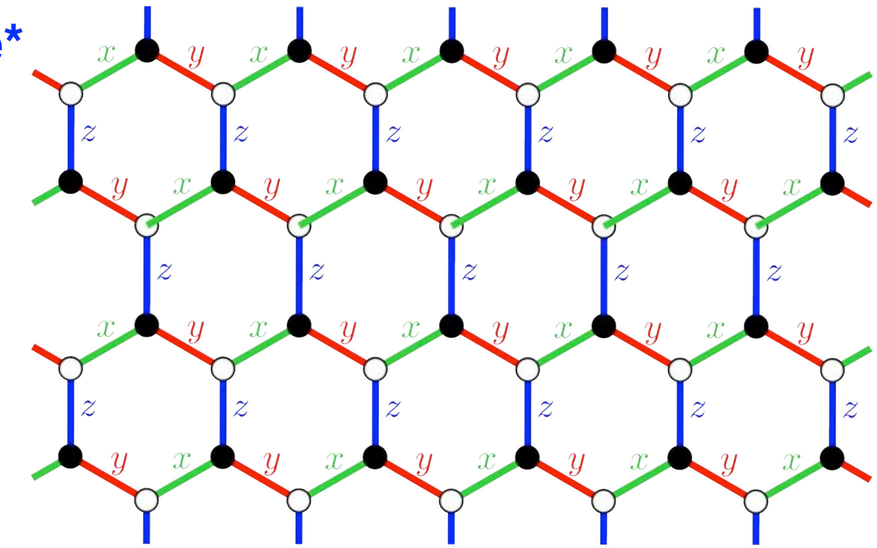


For efficient homological qudit codes see H. Bombin and MA Martin-Delgado, quant-ph/0605094

*SS Bullock and GKB, J. Phys. A submitted, quant-ph/0609070

From k-local to 2-local

- **Spin-1/2 particles on a honeycomb lattice***



$$H = J_\perp \sum_{x-\text{links}} \sigma_j^x \sigma_k^x + J_\perp \sum_{y-\text{links}} \sigma_j^y \sigma_k^y + J_z \sum_{z-\text{links}} \sigma_j^z \sigma_k^z.$$

  - **Exactly solvable**

*A.Yu. Kitaev, Annals of Physics, 321,2 (2006)

- **In the limit, $|J_z| \gg |J_\perp|$, pairs of spins along z-links are mapped to a qubit**
  - **New spin operators on each z-link:**

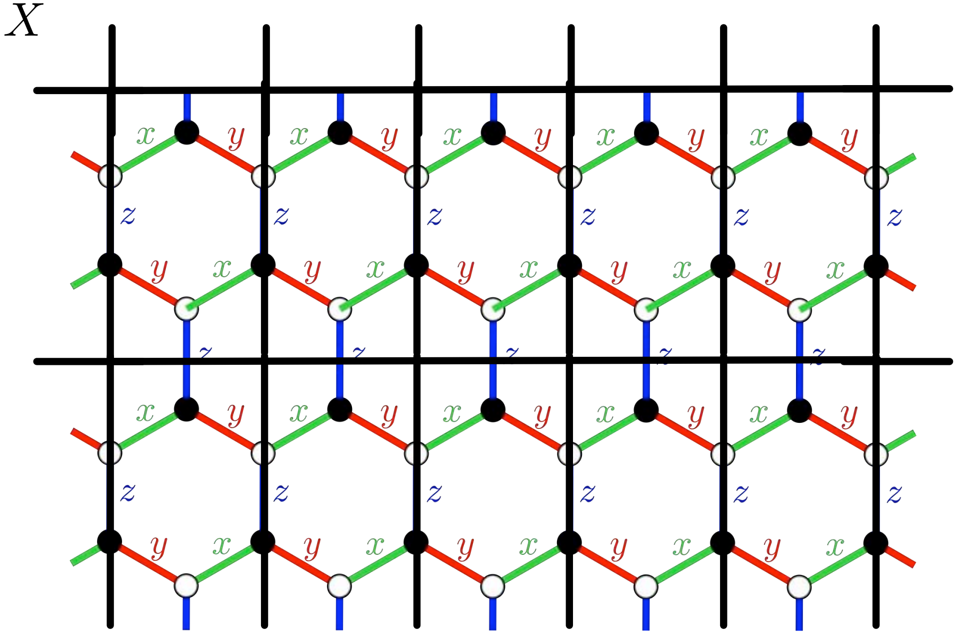
$$\mathbf{1}_{2(1)} \otimes \sigma_2^z \to Z \qquad \sigma_1^y \otimes \sigma_2^x \to Y \qquad \sigma_1^x \otimes \sigma_2^x \to X$$

$$H_{\text{eff}} = -J_{\text{eff}} \sum_{\diamond} Y_{\text{left}} Z_{\text{up}} Y_{\text{right}} Z_{\text{down}}$$

Unitary transformation: $\prod_{j \ni \text{white}} e^{iX_j \pi/4}$

$$H_{\text{eff}} = -J_{\text{eff}} \left(\sum_{+} Z_{e_1} Z_{e_2} Z_{e_3} Z_{e_4} + \sum_{\square} X_{e_1} X_{e_2} X_{e_3} X_{e_4} \right)$$

  - **Protected q. memory** $\quad J_{\text{eff}} = \dfrac{J_\perp^4 |J_z|}{16 J_z^4}$



String net condensed states

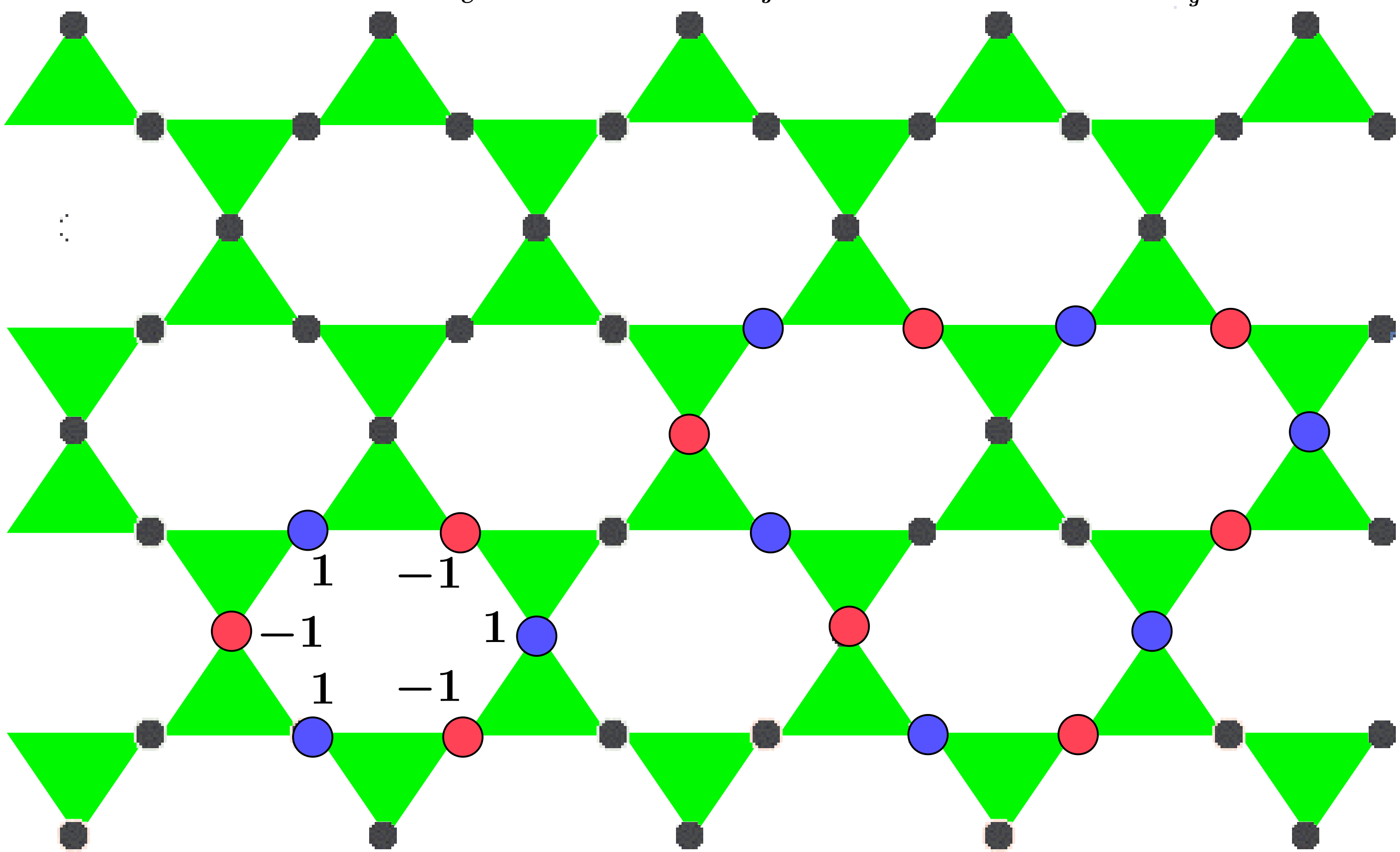
Spin-1

$$H = \underbrace{-U \sum_{\triangle} \Big(\sum_{j=1}^{3} S_j^z \Big)^2}_{H_U} + \underbrace{J \sum_j (S_j^z)^2}_{H_J} - t \sum_{<i,j>} (S_i^+ S_j^- + S_i^- S_j^+)$$

$U \gg t \gg J$

$$H_{\text{eff}} = \underbrace{-U \sum_{\triangle} \Big(\sum_{j=1}^{3} S_j^z \Big)^2}_{H_U} + \underbrace{J \sum_j (S_j^z)^2}_{H_J} \underbrace{- g \sum_{<i,j>} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.)}_{H_g}$$

$$g = \frac{3t^3}{U^2}$$



X.G. Wen, Phys. Rev. B, 68, 115413 (2003)

Properties

- **Emergent (local) U(1) Gauge invariance, i.e. wavefunction invariant under the transformation**

$$U(\phi_j) = e^{\left(i \sum_{\triangle} \phi_{\triangle} \sum_{k=1}^{3} S_k^z\right)}$$

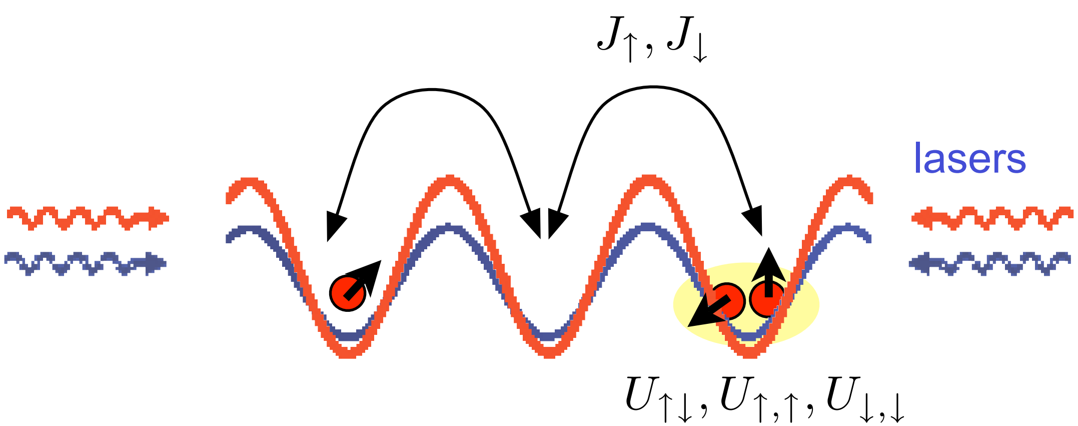
- **Artificial light polarization defined in terms of ordering of strings: +-+-+-... and -+-+-+**

- **Robust to perturbations. Energy 2U to break a cycle**

- **By adding a string tension term $J \sum_j (S_j^z)^2$ the system acquires two distinct phases in the ground state: a confined phase characterized by small closed loops, and a deconfined phase with large fluctuating loops**

Implementions with atoms

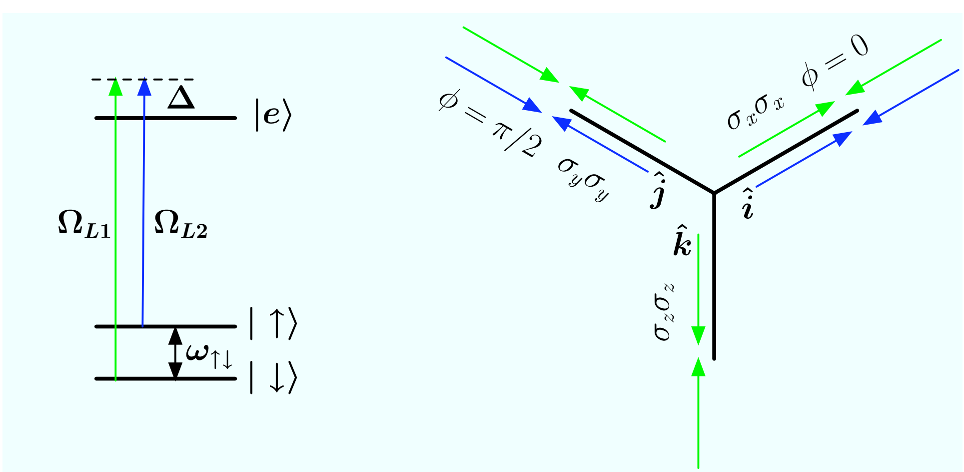
- **Hubbard model with atoms**
  - **State dependent collisions. Restrict to subspace with one particle per well**

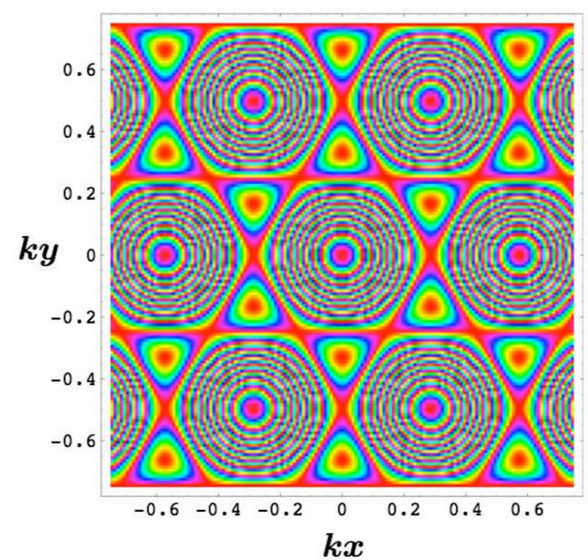


$$J_\perp = -\frac{J_\uparrow J_\downarrow}{U_{\uparrow\downarrow}}$$

$$J_z = \frac{J_\uparrow^2 + J_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{J_\uparrow^2}{U_\uparrow} - \frac{J_\downarrow^2}{U_\downarrow}$$

$$\begin{aligned} \prod_j P_{N_j=1}(H_{BH}+V_B)\prod_j P_{N_j=1} &= \sum_j J_\perp(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y) + J_z\sigma_j^z\sigma_{j+1}^z \\ &= H_{XXZ}, \end{aligned}$$

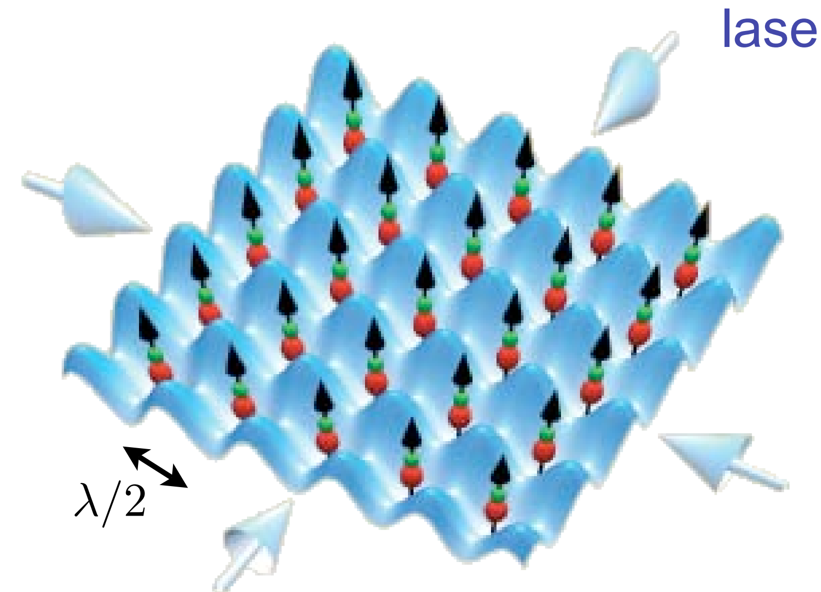


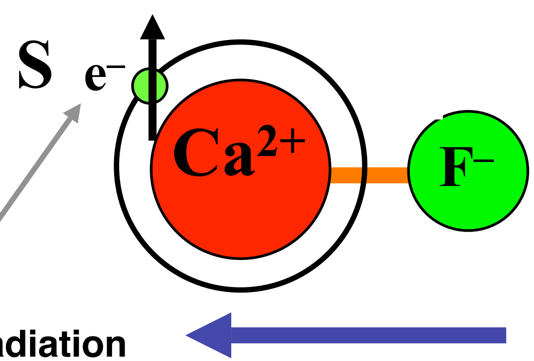


L.M. Duan, E. Demler, M.D. Lukin, Phys. Rev. Lett. 91, 09402 (2003)

Implemations with polar molecules

- System: $^2\Sigma_{1/2}$ hetero-nuclear molecules in electronic-vibrational ground-states
  - Alkaline-earth monohalides (CaF,CaCl,MgCl...)
  - single electron in outer shell
- Electric dipole moment in superposition
- of rotational states





talks to optical radiation

dipole moment

talks to microwave radiation
... as rotations on ~20 GHz

$T \sim 500\text{nK}$ **Energy scales:**

$\gamma/\hbar \sim 100\ \text{MHz}$	**Spin-rotational coupling**
$B/\hbar \sim 10\ \text{GHz}$	**Rotational constant**
$\omega_{osc} \sim 100\ \text{kHz} - 1\text{MHz}$	**Lattice trap spacing**
$\Gamma/\hbar \sim 10^{-3}\ \text{Hz}$	**Black-body scattering rate**
$\Gamma_{\text{scat}}/\hbar \sim 10^{-1}\text{Hz}$	**Spontaneous emission**

A. Micheli, GKB, and P. Zoller, Nature Phys., May, 2006

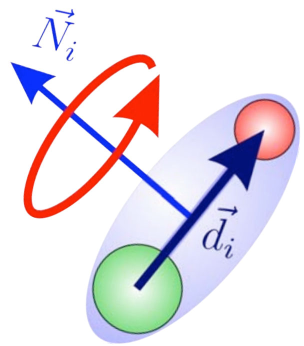
Rotational spectra of a single molecule

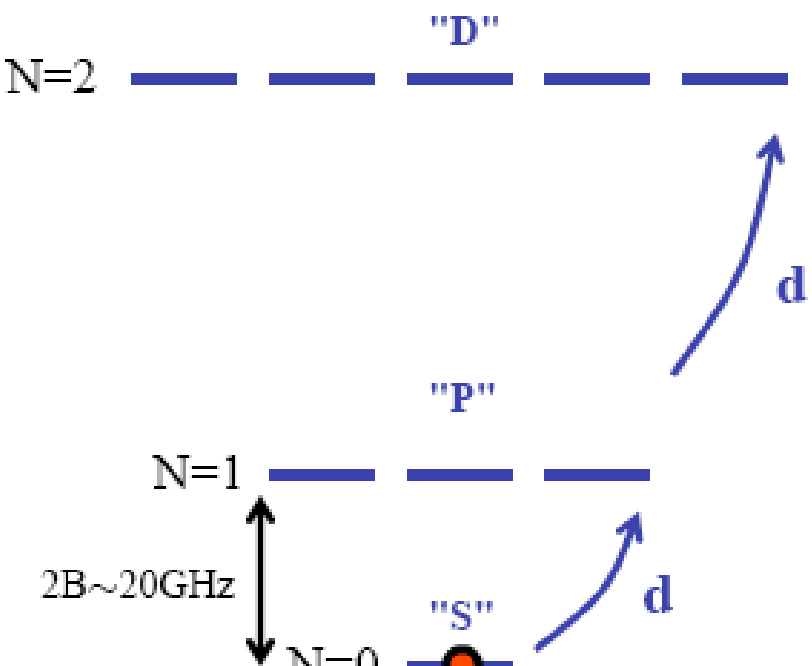
- rigid rotor

$$H = B\,\mathbf{N}^2$$

$$|N, M_N\rangle$$

$$E_N = B\,N(N+1)$$





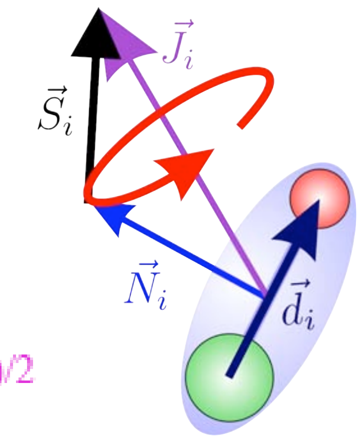
rotational ground state ...

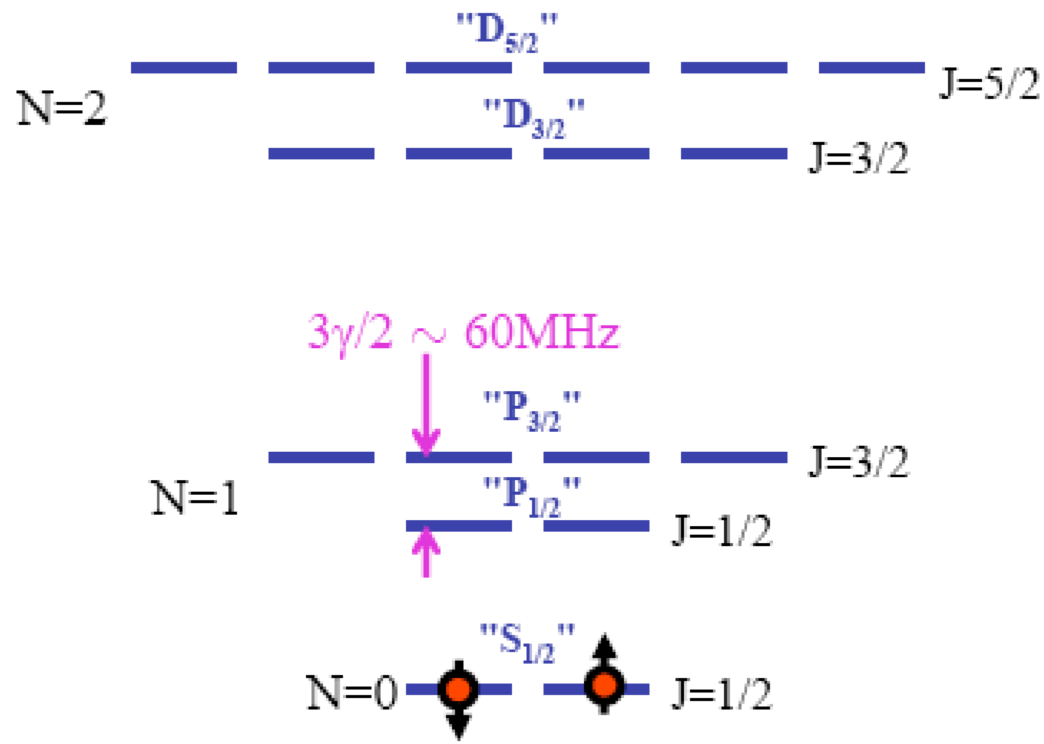
- add spin-rotation coupling

$$H = B\,\mathbf{N}^2 + \gamma\,\mathbf{N}\cdot\mathbf{S}$$

$$|N, J, M_J\rangle \quad (J = |N \pm 1/2|)$$

$$E_{N,J=N\pm 1/2} = B\,N(N+1) + \begin{cases} +\gamma N/2 \\ -\gamma (N+1)/2 \end{cases}$$

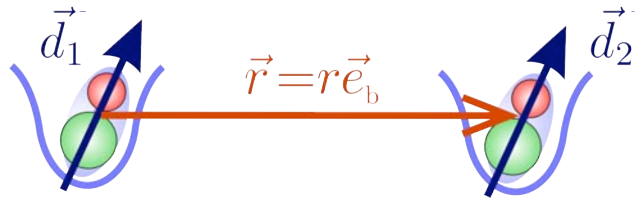




... as spin-1/2-system

Two polar molecules: dipole-dipole interactions

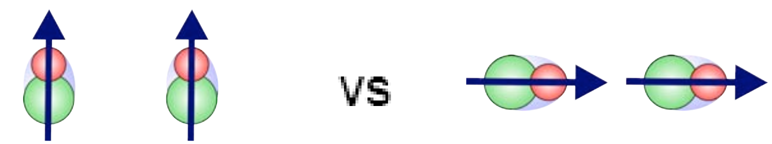
- interactions of two polar molecules



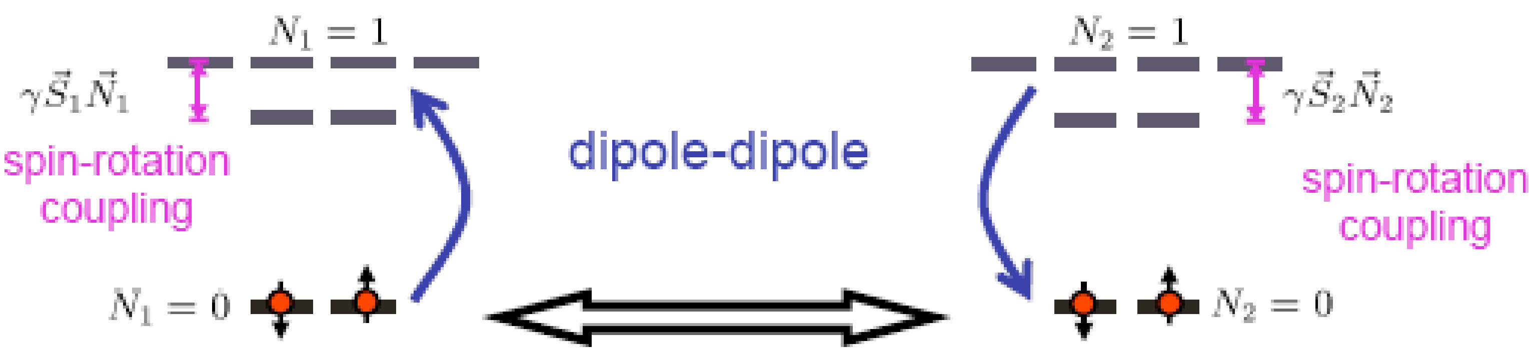
$$V_{dd} = \frac{\vec{d}_1 \cdot \vec{d}_2 - 3(\vec{d}_1 \cdot \vec{e}_b)(\vec{e}_b \cdot \vec{d}_2)}{r^3}$$

features of dipole-dipole interaction:

- long range $\sim 1/r^3$
- angular dependence (anisotropic)



- include spin-rotation coupling in adiabatic potentials for molecular dimers

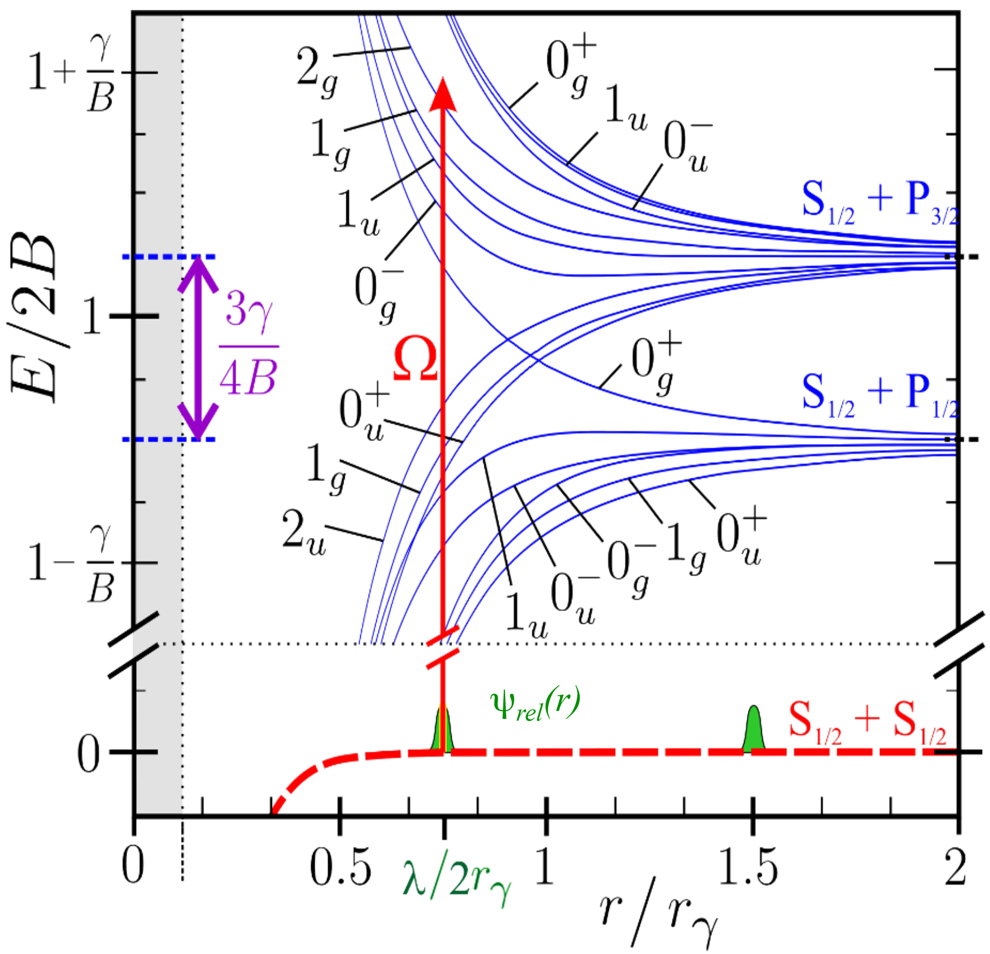


- At typical optical lattice spacing : $\lambda/2 \sim r_\gamma = (2d^2/\gamma)^{1/3}$
  - rotation of dimers strongly coupled to spins
  - Hunds case (c) excited states, $\{|Y|_{g,u}^{\pm}(r)\}$ $(Y = \Sigma_{i=1,2} M_{N,i} + M_{S,i})$
  - solvable in closed form due to symmetries

Tunable spin patterns

- Adiabatic mixing with dipole-dipole coupled states by microwave fields

$$H_{\text{eff}}(r) = \sum_{i,f} \sum_{\lambda(r)} \frac{\langle g_f | H_{\text{mf}} | \lambda(r) \rangle \langle \lambda(r) | H_{\text{mf}} | g_i \rangle}{\hbar\omega_F - E(\lambda(r))} |g_f\rangle\langle g_i| \qquad H_{\text{spin}} = \langle H_{\text{eff}}(r) \rangle_{\text{rel}}$$



Feature 1:
By tuning close to a given resonance one can select a *specific spin pattern*:

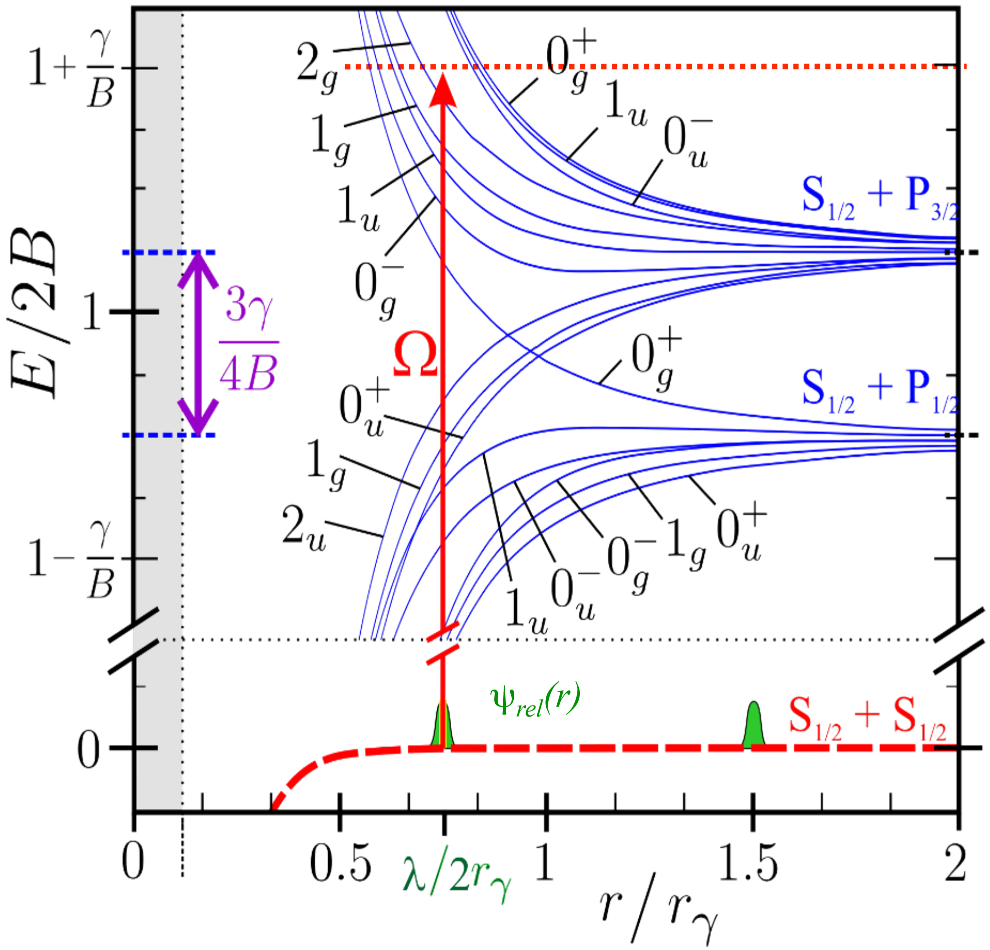
Polarization	Resonance	Spin pattern
$\hat{x}$	2_g	$\sigma^z\sigma^z$
$\hat{z}$	0_u^+	$\vec{\sigma}\cdot\vec{\sigma}$
$\hat{z}$	0_g^-	$\sigma^x\sigma^x + \sigma^y\sigma^y - \sigma^z\sigma^z$
$\hat{y}$	0_g^-	$\sigma^x\sigma^x - \sigma^y\sigma^y + \sigma^z\sigma^z$
$\hat{y}$	0_g^+	$-\sigma^x\sigma^x + \sigma^y\sigma^y + \sigma^z\sigma^z$
$(\hat{y}-\hat{x})/\sqrt{2}$	0_g^+	$-\sigma^x\sigma^y - \sigma^y\sigma^x + \sigma^z\sigma^z$

polarization rel. to body axis, here set $\vec{e}_b = \hat{z}$

Tunable spin patterns

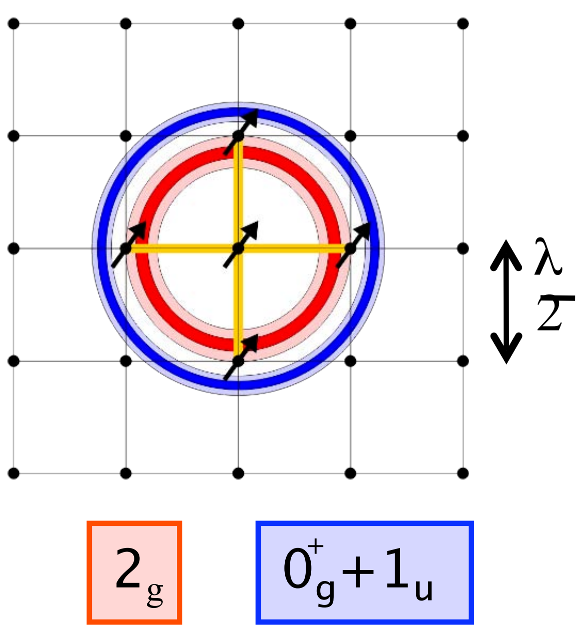
- Adiabatic mixing with dipole-dipole coupled states by microwave fields

$$H_{\rm eff}(r) = \sum_{i,f}\sum_{\lambda(r)} \frac{\langle g_f|H_{\rm mf}|\lambda(r)\rangle\langle\lambda(r)|H_{\rm mf}|g_i\rangle}{\hbar\omega_F - E(\lambda(r))}|g_f\rangle\langle g_i| \qquad H_{\rm spin} = \langle H_{\rm eff}(r)\rangle_{\rm rel}$$



Feature 2:

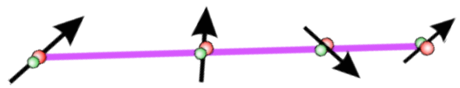
Can choose the *range* of the interaction for a given spin texture



Multiple fields

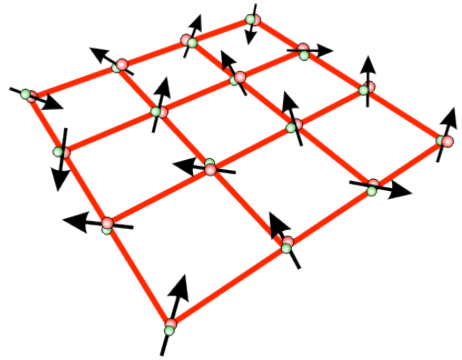
Feature 3: for a *multifrequency* field spin textures are *additive* ⇨ toolbox.

- 1D XYZ model



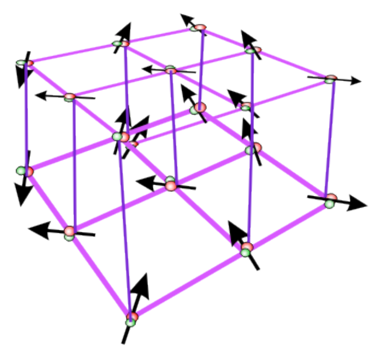
$$H = \sum_{<i,j>} J_x \sigma_i^x \sigma_j^x + J_y \sigma_i^y \sigma_j^y + J_z \sigma_i^z \sigma_j^z$$

- 2D Ising model



$$H = \sum_{<i,j>} J \sigma_i^z \sigma_j^z$$

- 3D Heisenberg model



$$H = \sum_{<i,j>} J \vec{\sigma_i} \cdot \vec{\sigma_j}$$

- Typical coupling strengths:

$$J \sim 10 - 100\mathrm{kHz}$$

Polarization	Resonance
$\hat{z}$	0_u^+
$\hat{y}$	0_g^-
$\hat{y}$	0_g^+
$\hat{x}$	2_g
$\hat{x}$	0_u^+
$\hat{z}$	0_g^-
$\hat{z}$	0_u^+
$\hat{x}$	1_u

sign adjustable by tuning above or below given resonance

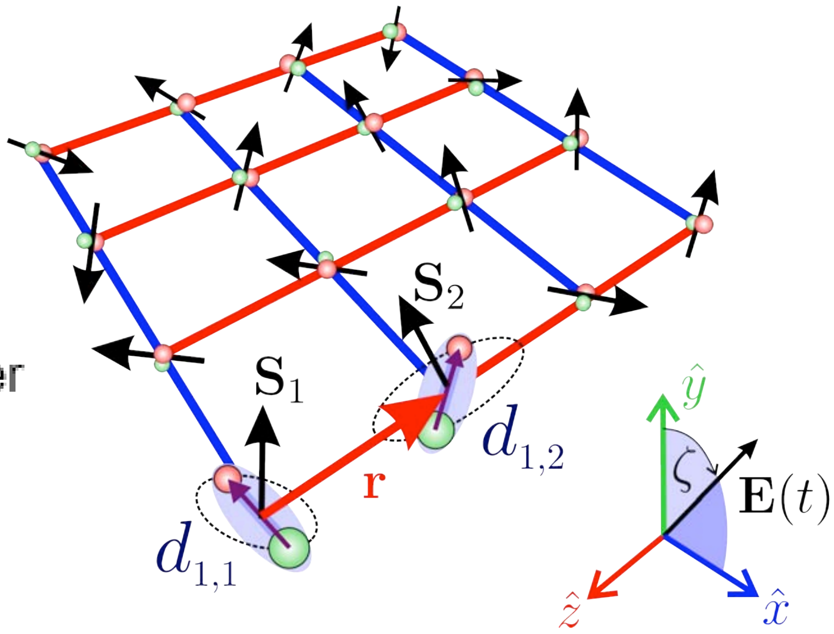
Model I: Error protected ground states

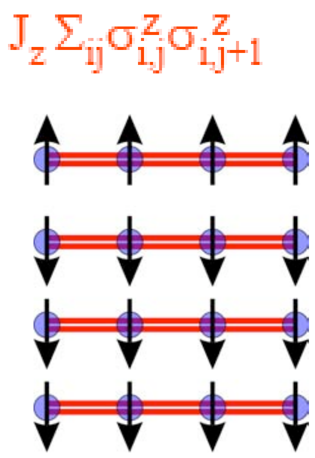
* B. Douçot, M.V. Feigel'man, L.B. Ioffe, A.S. Ioselevich, Phys. Rev. B **71**, 024505 (2005).

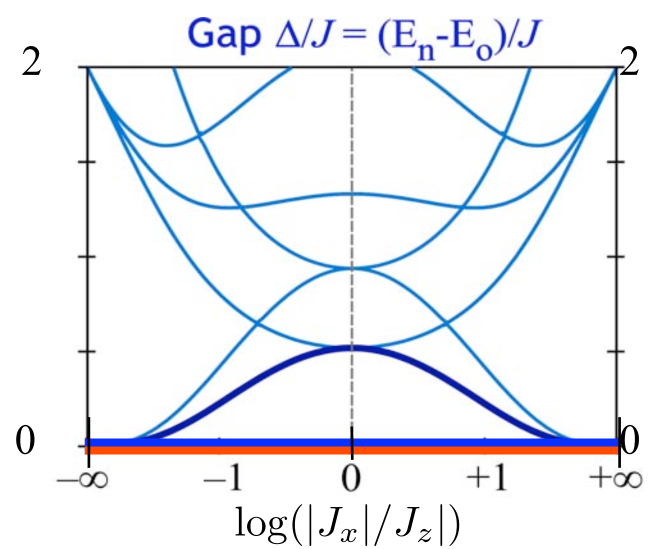
- Model on 2D square lattice*

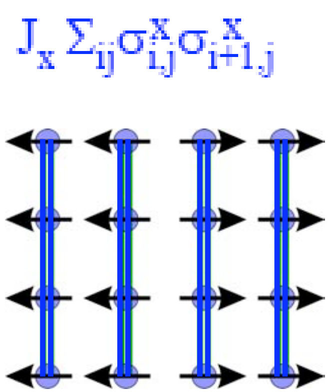
$$H^{(1)}_{\text{spin}} = \sum_{i=1}^{\ell-1}\sum_{j=1}^{\ell-1} J(\sigma^z_{i,j}\sigma^z_{i,j+1} + \cos\zeta\,\sigma^x_{i,j}\sigma^x_{i+1,j})$$

  - gapped spectrum with 2-fold degenerate ground-state (for $\zeta \neq \pm\pi/2$) $\rightarrow |0\rangle_L, |1\rangle_L$
  - ground-states robust to local errors up to ℓ-*th* order



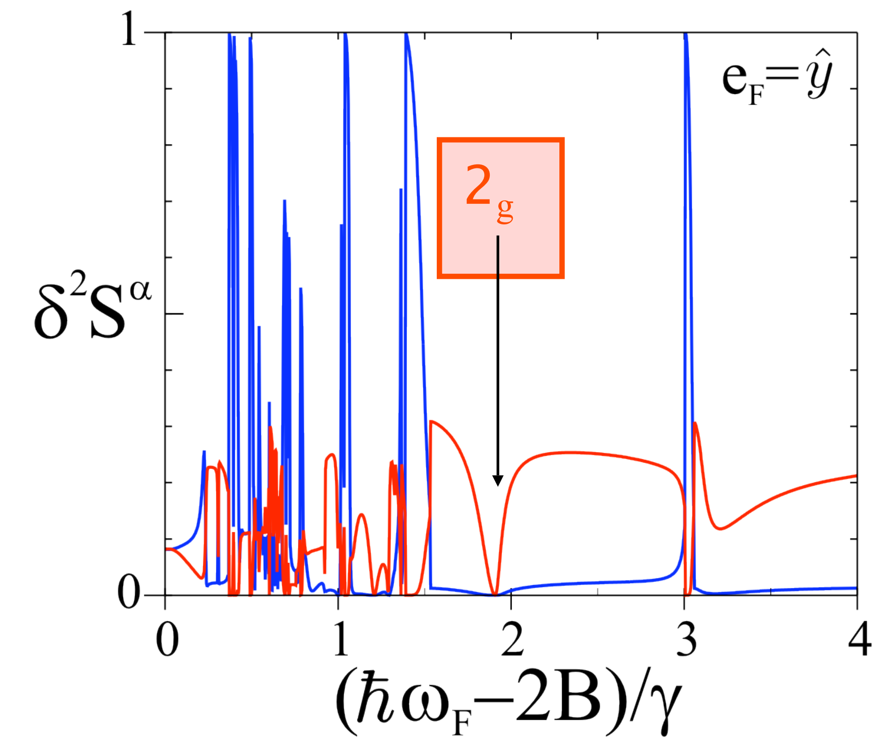




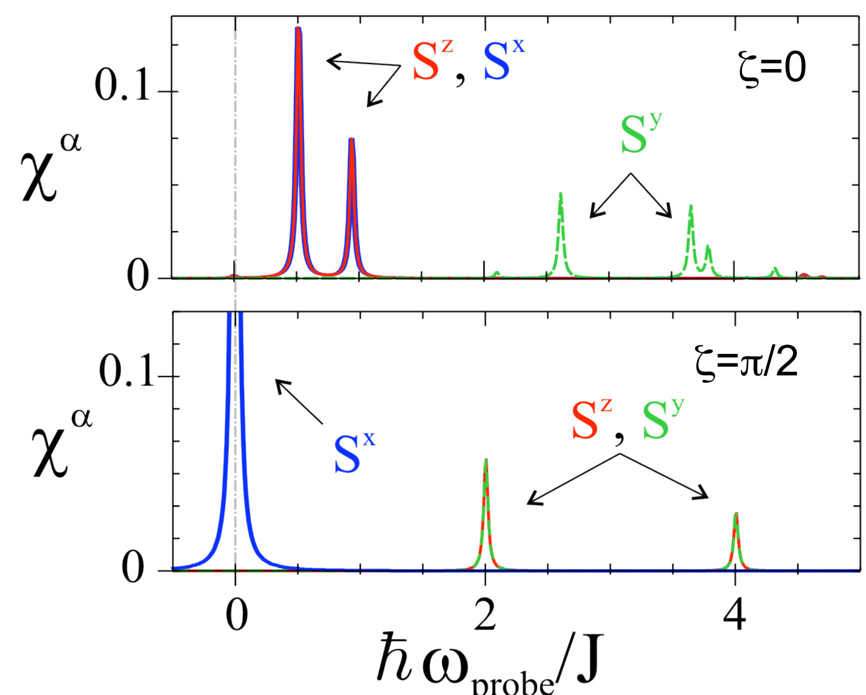


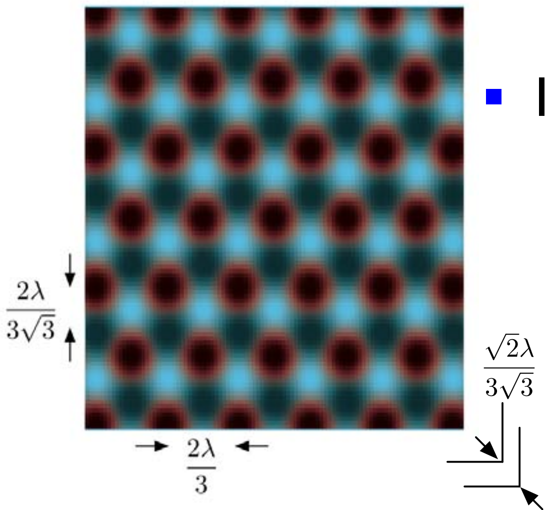
Results: Design and verification on 3x3 lattice

- Noise resilience as measured by rms magnetization in ground manifold
  - as function of the detuning
  - give worst case scenario for logical bit flip errors / phase flip errors
  - protected region near 2_g

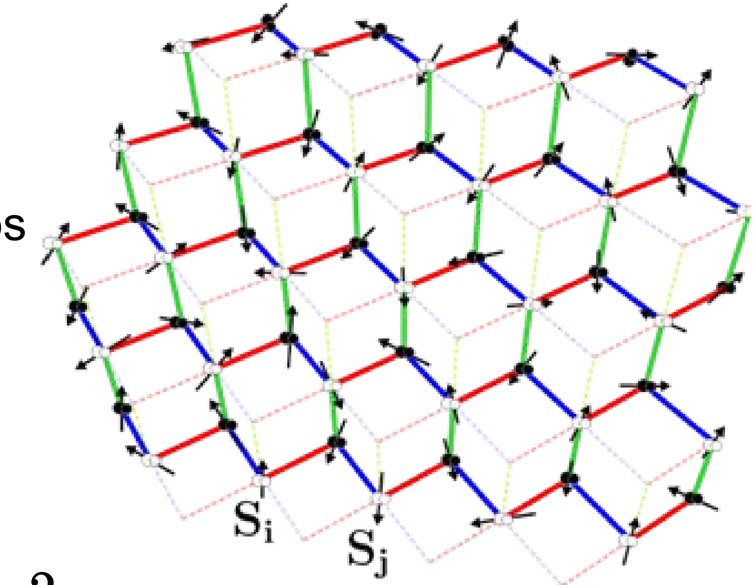


- Verification by absorption spectroscopy
  - Field polarization out of plane
  - Probe gap at J/2
  - Field polarization in plane
  - Gap disappears and excitations are spin-waves S^x

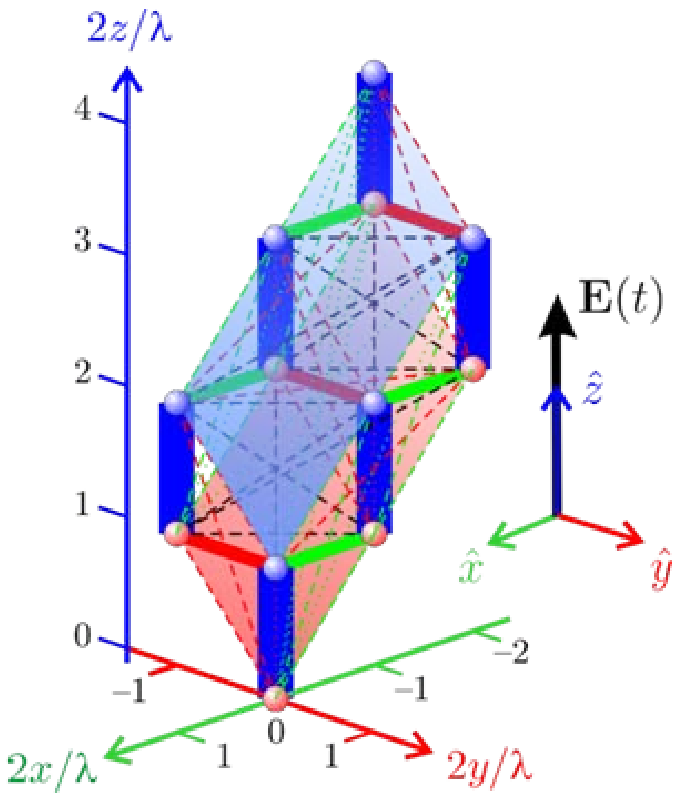




- Implementation in *Q*bert lattice*:
  - Two staggered triangular lattices
  - Nearest neighbors give honeycombs
  - their edges form orthogonal triads



- Realization with 3 fields: (several possible choices) shown when all 3 being z polarized, resp. near 0_g, 1_g, 2_g



Spin pattern

- $\sigma^z\sigma^z$
- $\sigma^x\sigma^x$
- $\sigma^y\sigma^y$
- Other

$|J_\perp| = 0.4|J_z|$

Residual long range coupling strengths $|J_{lr}|$

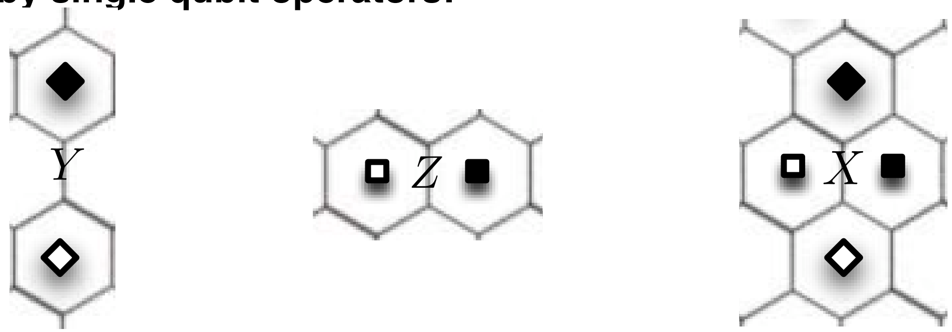
- – – – $< 10^{-2}\ |J_z|$
- .... $< 10^{-3}\ |J_z|$

Operator fidelity (on a 4 spin configuration)

$$\sup\left[||H_{\text{spin}} - H^{(\text{II})}_{\text{spin}}|\psi\rangle||_2;\ \langle\psi|\psi\rangle = 1\right] = 10^{-4}\ |J_z|$$

Observing anyonic statistics

- **Excitations created by spin flips (along a z-link)**
  - **Effective interaction** $H_{\text{eff}} = -J_{\text{eff}} \sum_{\diamond} Y_{\text{left}} Z_{\text{up}} Y_{\text{right}} Z_{\text{down}}$
  - **Anyons created by single qubit operators:**



  - Fusion rules (as obtained from the action of the Pauli operators):

$$\square \times \square = 1 \qquad \diamond \times \diamond = 1 \qquad \square\diamond \times \square\diamond = 1$$
$$\square \times \diamond = \square\diamond \qquad \square \times \square\diamond = \diamond \qquad \diamond \times \square\diamond = \square$$

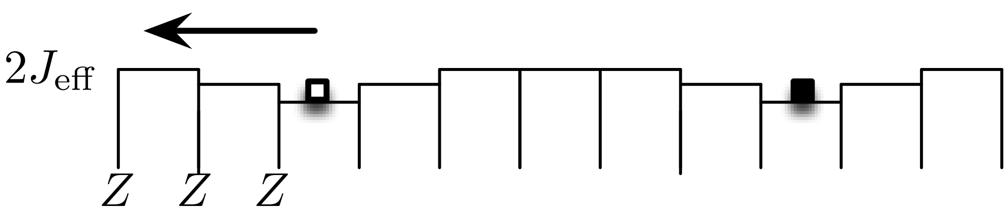
  - Relative statistics under braiding:

Particles	Statistical phase
$\square \ \square$	0
$\diamond \ \diamond$	0
$\square \ \diamond$	π
$\square\diamond \ \square\diamond$	0

Braiding

- **Prepare two quasiparticle pairs**
  $$|\Psi(1)\rangle = S_A^Y S_B^Z |\lambda_g\rangle$$
- **Beam splitter at I**
- **Adiabatically drag □ left**

$$H'(t) = H + \sum_{e \in Path} \delta J_e(t)(\sigma_1^z \sigma_2^z)_e + \kappa(t) Z_e(t)$$

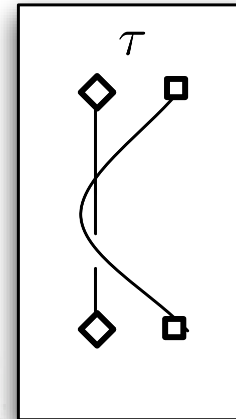


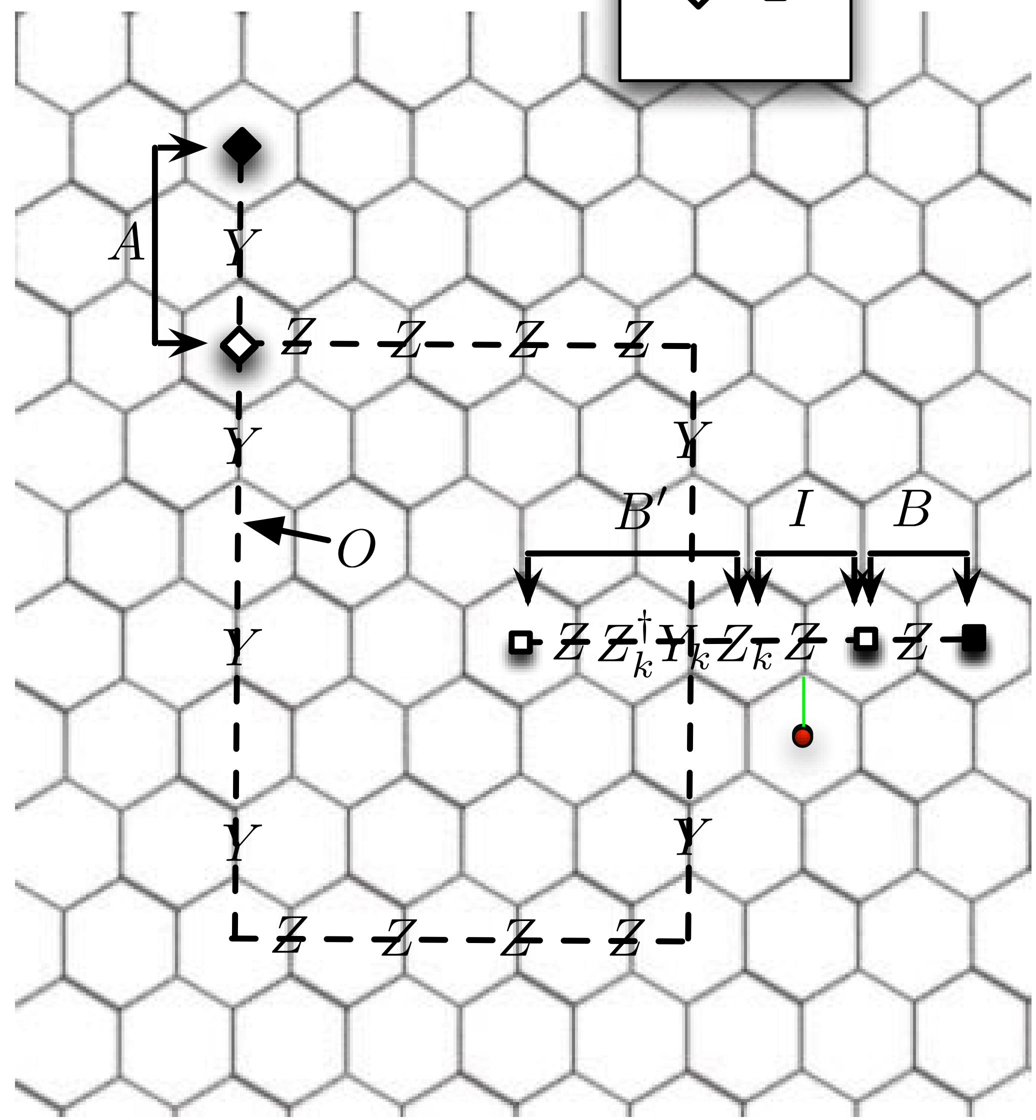
- **Adiabatically drag ◇ CCW around □**
- **Adiabatically drag □ right**
- **Inverse Beam splitter at I**
- **Measure location of □**

$$\langle S_I^Z \rangle = \sin(\beta + \pi)$$

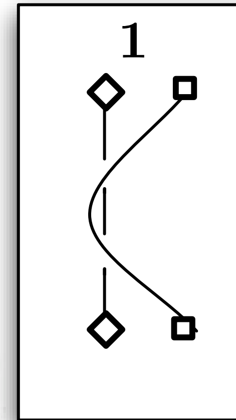
Dynamical+Berry phases

Statistical phase



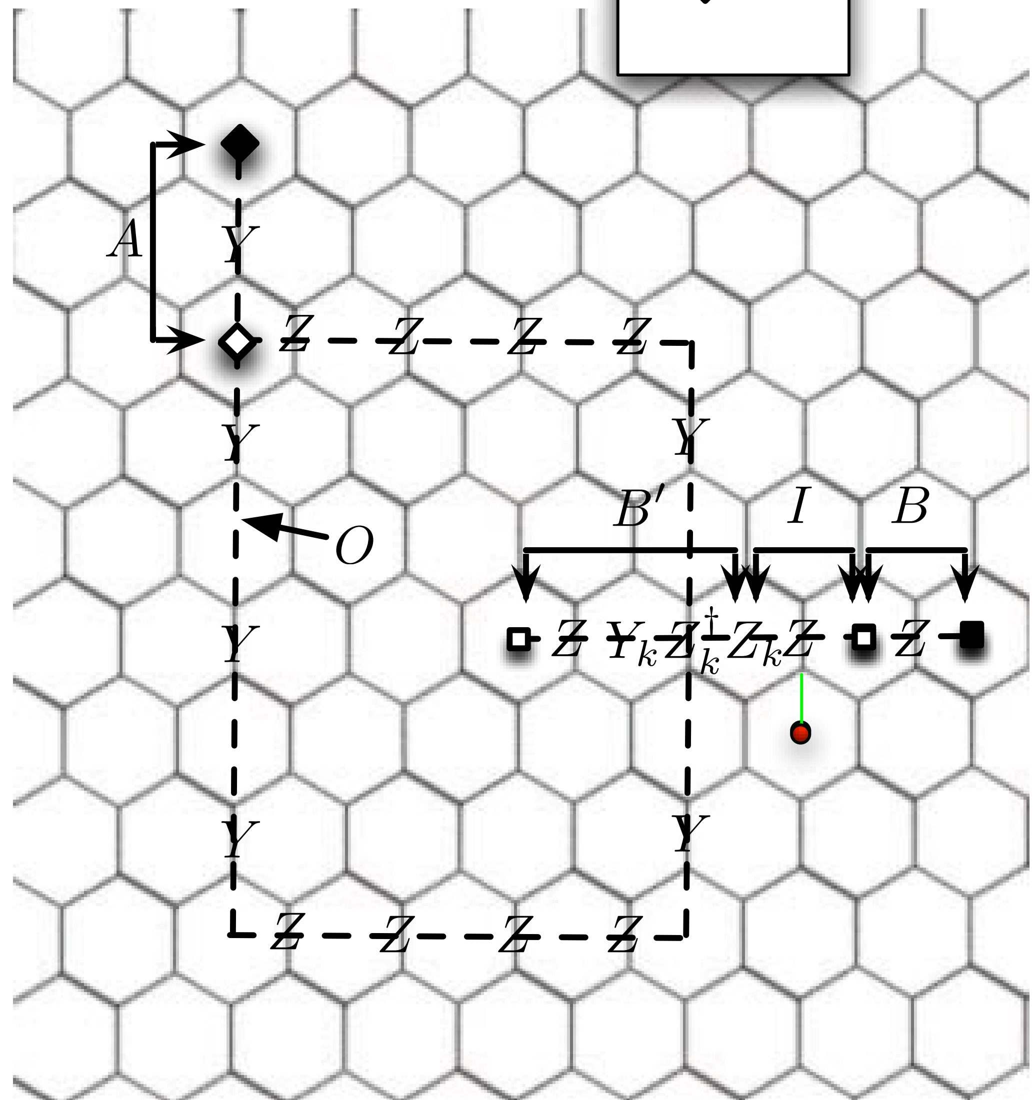


For trivial braid use same steps but in different order



- Adiabatically drag ◇ CCW around □
- Adiabatically drag □ left
- Adiabatically drag □ right
- Measure location of □

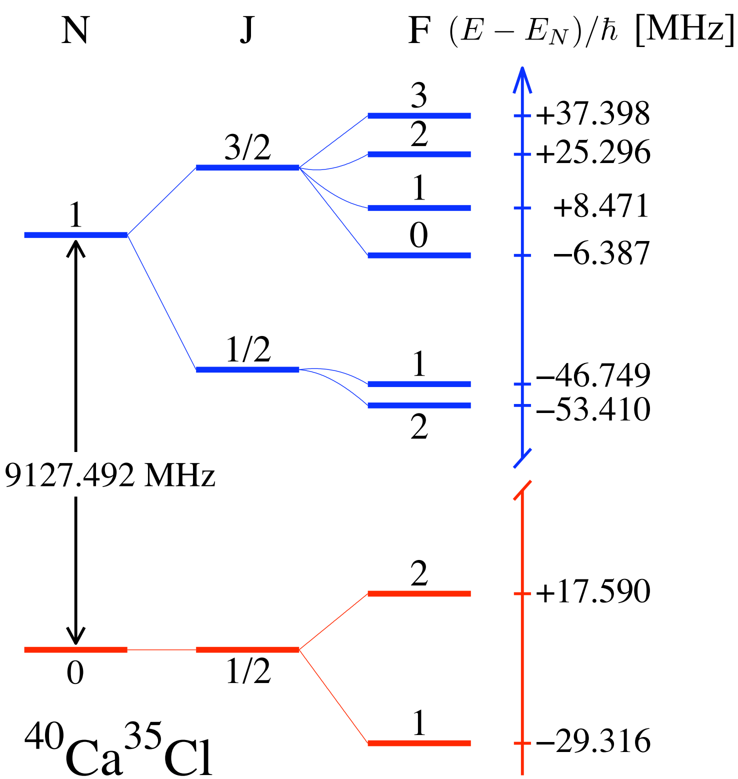
$$\langle S_I^Z \rangle = \sin(\beta)$$



S.S. Bullock, GKB quant-ph/0609070

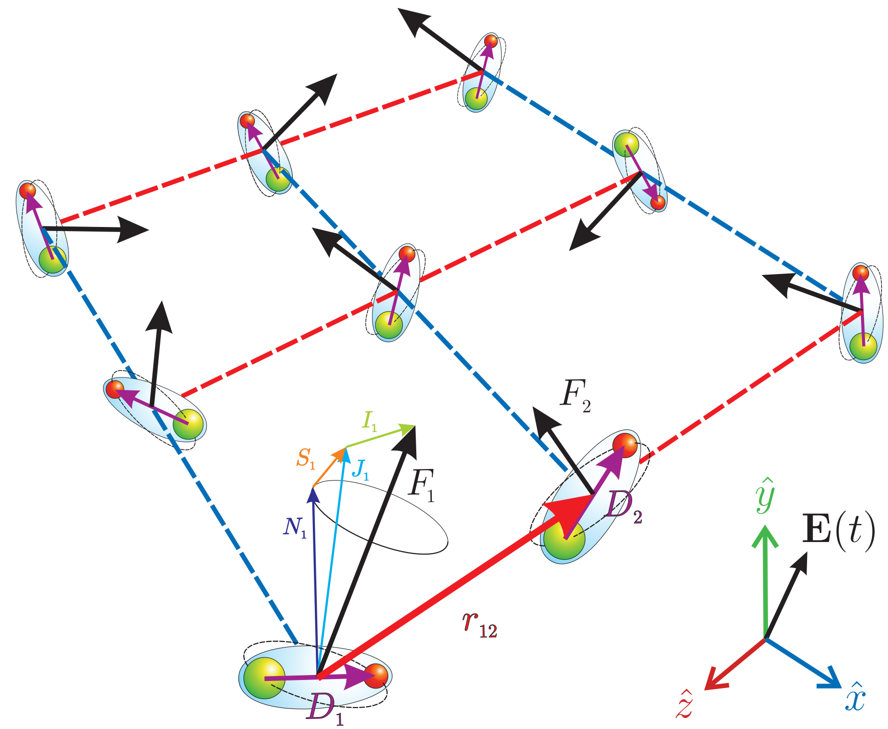
see also
J. Pachos, quant-ph/0511273;
C. Zhang, V.W. Scarola, S. Tewari, and
S. Das Sarma, quant-ph/0609101

Integer spin lattice models



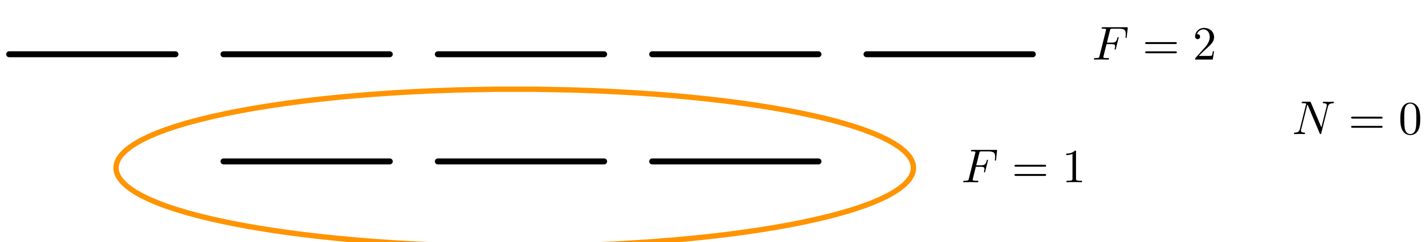
$^{40}Ca^{35}Cl$

$I = 3/2$



$$H_{\rm m} = B\mathbf{N}^2 + \gamma \mathbf{N}\cdot\mathbf{S} + b\mathbf{I}\cdot\mathbf{S} + cI^z S^z + eQq\frac{3I^{z2} - I(I+1)}{4I(2I-1)}$$

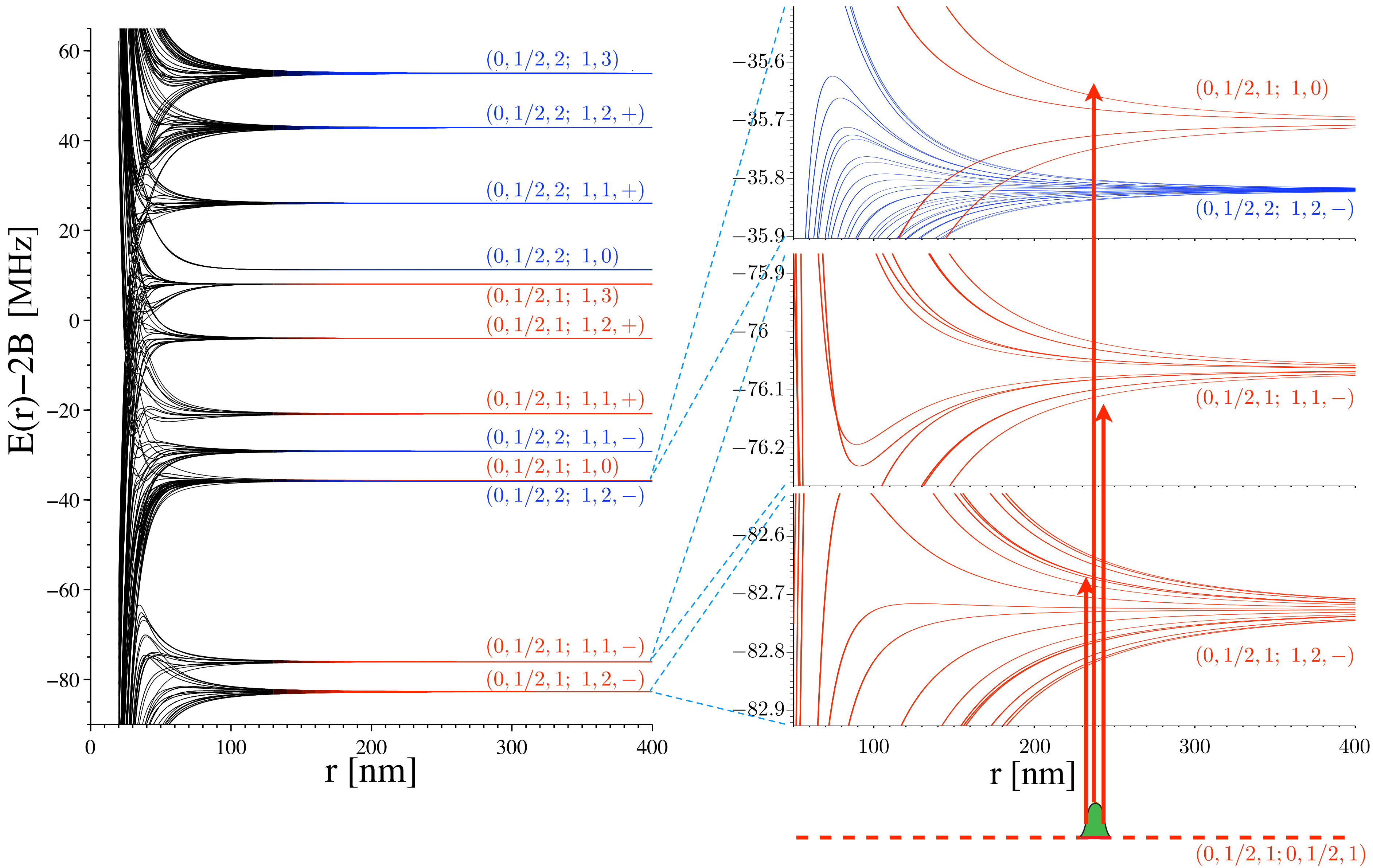
Fermi contact **Dipolar** **Electric Quadrupole**



Encode here

GKB, A. Micheli, and P. Zoller, quant-ph/0612180

Dipole-dipole with hyperfine

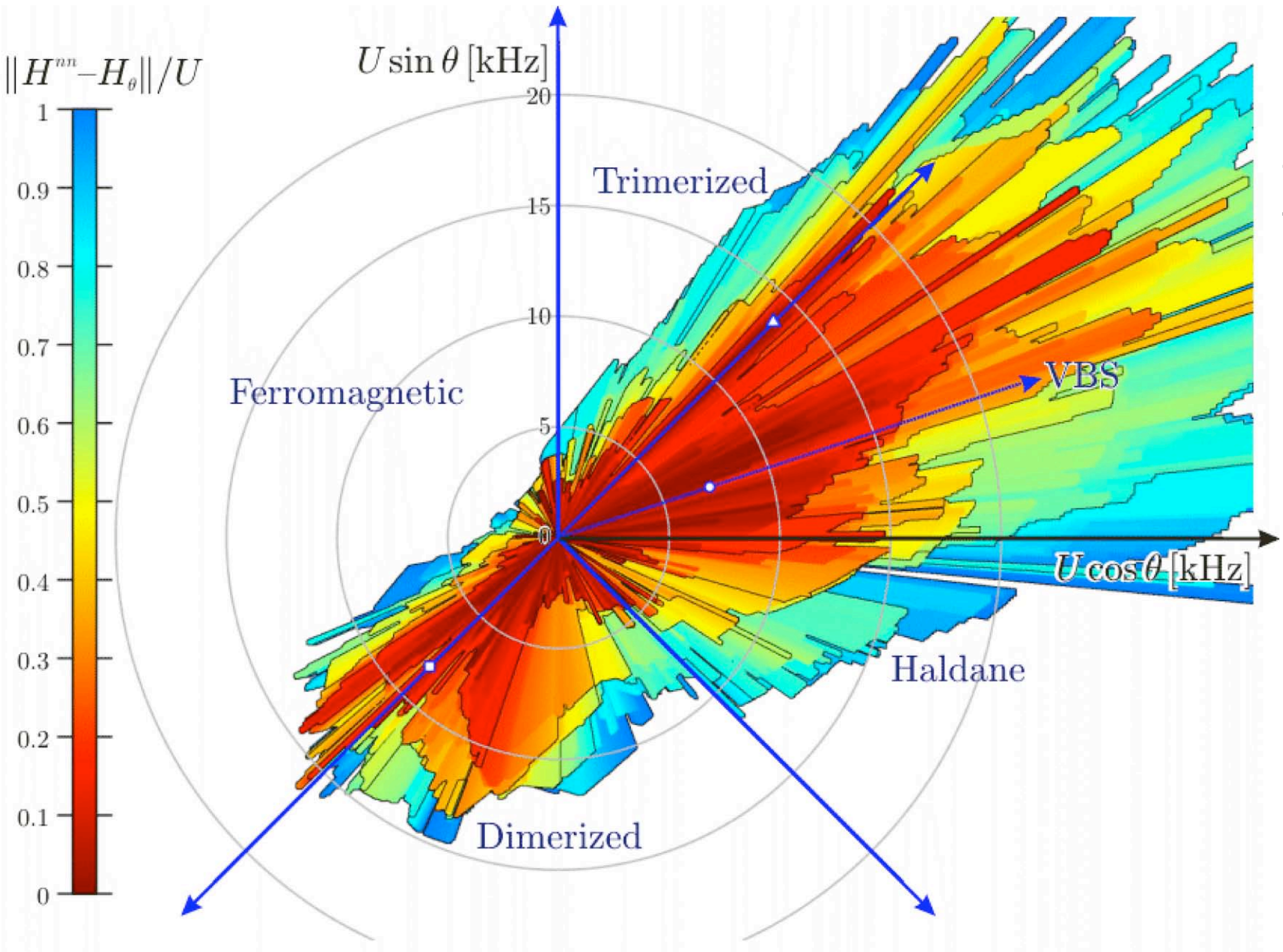


Asymptotic couplings solvable

Can't build generic two body Hamiltonians but can build a large class

Ex: 1D Generalized Haldane Model

$$H_\theta = U\sum_j(\cos\theta\vec{S}_j\cdot\vec{S}_{j+1} + \sin\theta(\vec{S}_j\cdot\vec{S}_{j+1})^2)$$



Numerical optimization over 4 fields

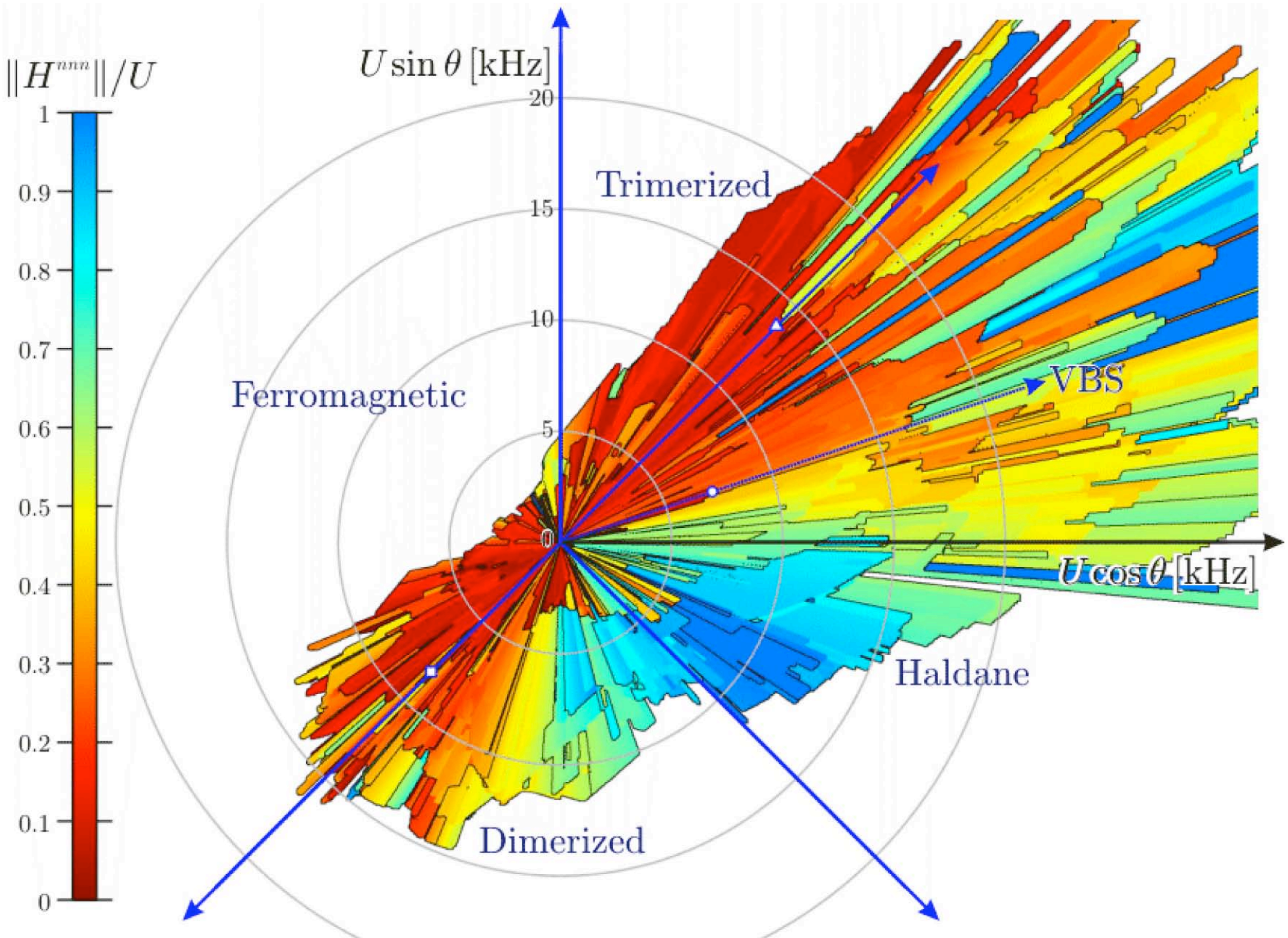
Realizations $\triangle, \square, \bigcirc$ have error < 0.05

Lattice spacing:

$\Delta z = 200\text{nm}$

Residual next-nearest neighbor interaction

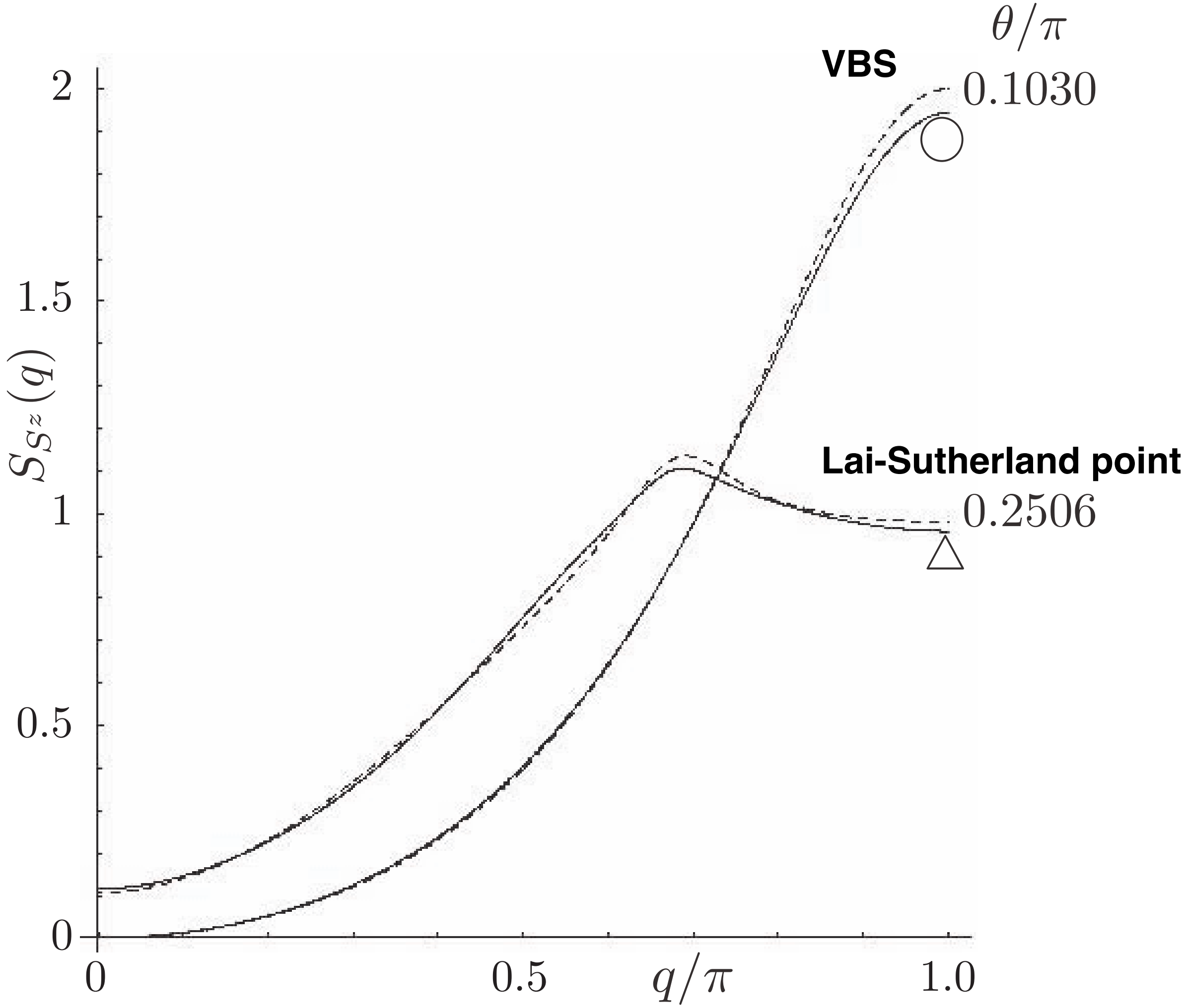
$$H_\theta = U \sum_j (\cos\theta \vec{S}_j \cdot \vec{S}_{j+1} + \sin\theta (\vec{S}_j \cdot \vec{S}_{j+1})^2)$$

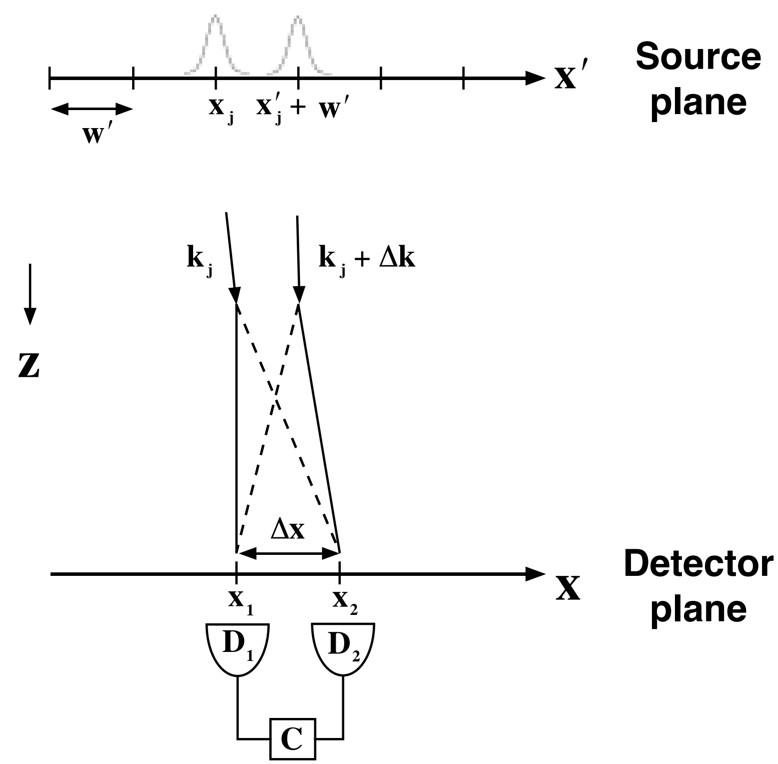


Realizations $\triangle, \square, \bigcirc$ have error < 0.05

Verification

- **Spin structure factor** $S_O(q) = \frac{1}{N}\sum_{j,j'=1}^{N} e^{iq(j-j')}\langle O_j O_{j'}^\dagger \rangle$
  - **obtainable from time of flight measurements (coincidence measurements)**



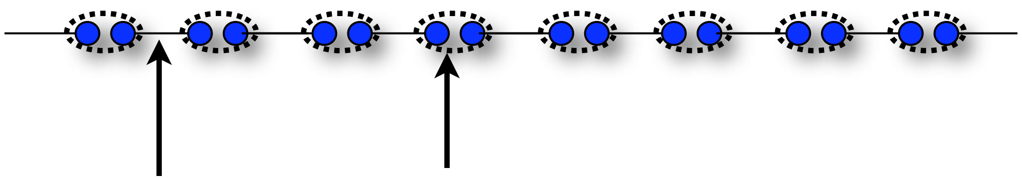


Teleportation with VBS*

- **Valence bond state is ground state of**

$$H = U\Big(\sum_j \vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3}(\vec{S}_j \cdot \vec{S}_{j+1})^2 + \frac{2}{3}\mathbf{1}_9\Big) = U\sum_j P^{S_{\rm tot}=2}(j,j+1)$$

  – **represent state as subspace of chain of virtual spin-1/2 particles**



  – **teleportation from one end to the other by *single particle measurements only* in basis** $\{|0\rangle, (|-1\rangle \pm |1\rangle)/\sqrt{2}\}$

- **Also serves as verification of ground state**

***I. Affleck, T. Kennedy, E.H. Lieb, H. Tasaki, CMP 115, 477 (1988)**

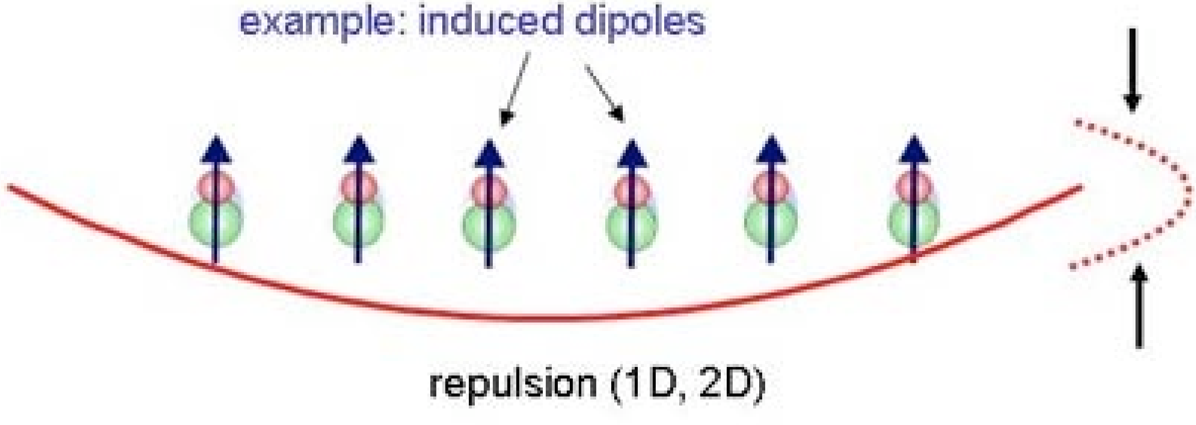
F. Verstraete, M.A. Martin-Delgado, J.I. Cirac, PRL 92, 087201 (2004).

Summary & Outlook

- We can design a large class spin-spin interactions with polar molecules
  - Tunable range and anisotropy
  - Large coherence to decoherence ratio   Q~800-10000 for reasonable trapping parameters
- Examples of Lattice Spin Model with TO
  - The Kitaev Model
    - Gapped system with abelian excitations
    - Strategy for measuring quasiparticle statistics
- Higher spin models
  - Isotropic model: rich phase diagram, quantum communication channel
- Coupling strength, hence gap for topological protection, limited by lattice spacing. But optical lattices limited to lattice spacings at optical transition wavelengths. Strategies for improvement
- Many useful TO models built up from three-body projectors (e.g. that enforce fusion rules). In principle electromagnetic interactions can be used to build these directly

Stronger correlations: self-assembled crystals

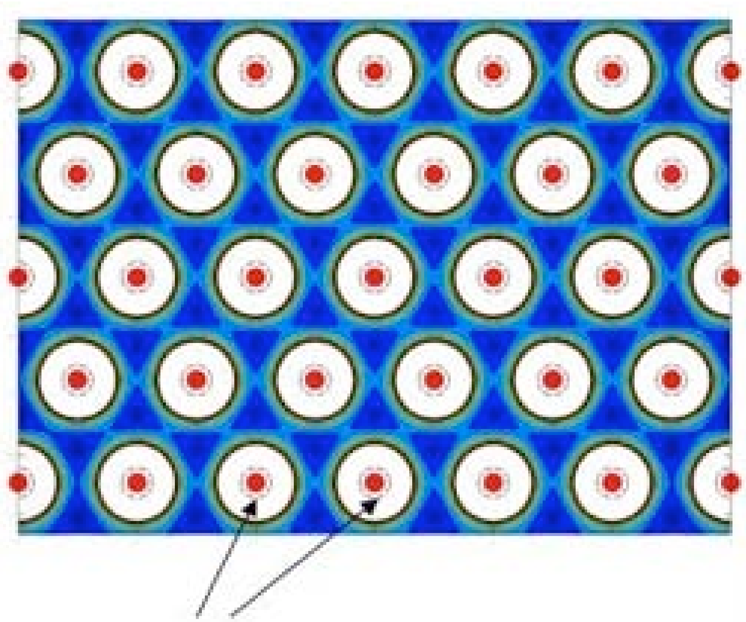
- **Engineer repulsive interactions***



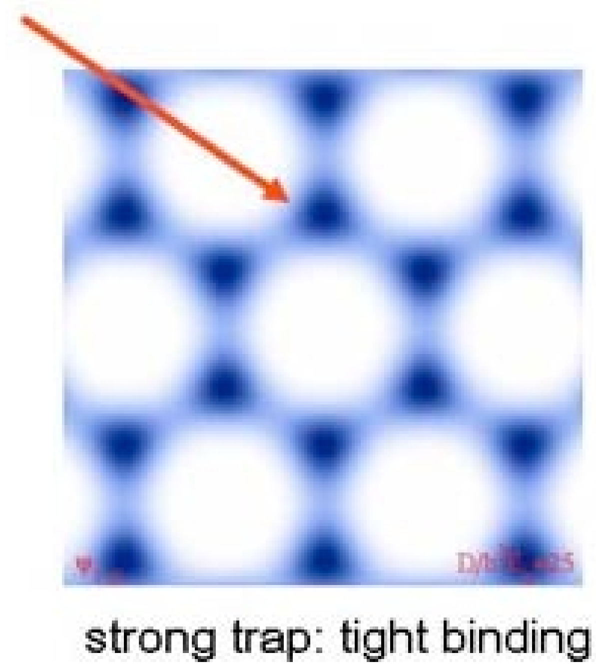
$$\hbar\omega_z \gg E_{\text{int}} \sim \frac{d^2}{a^3}$$

$$a \sim 100\text{nm}$$

other particles see honeycomb lattice

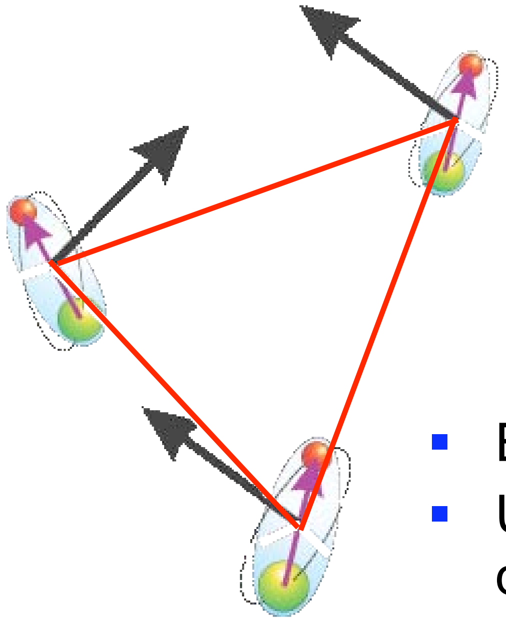


dipoles generate 2D triangular lattice



H.P. Büchler, et al., *cond-mat/0607294*

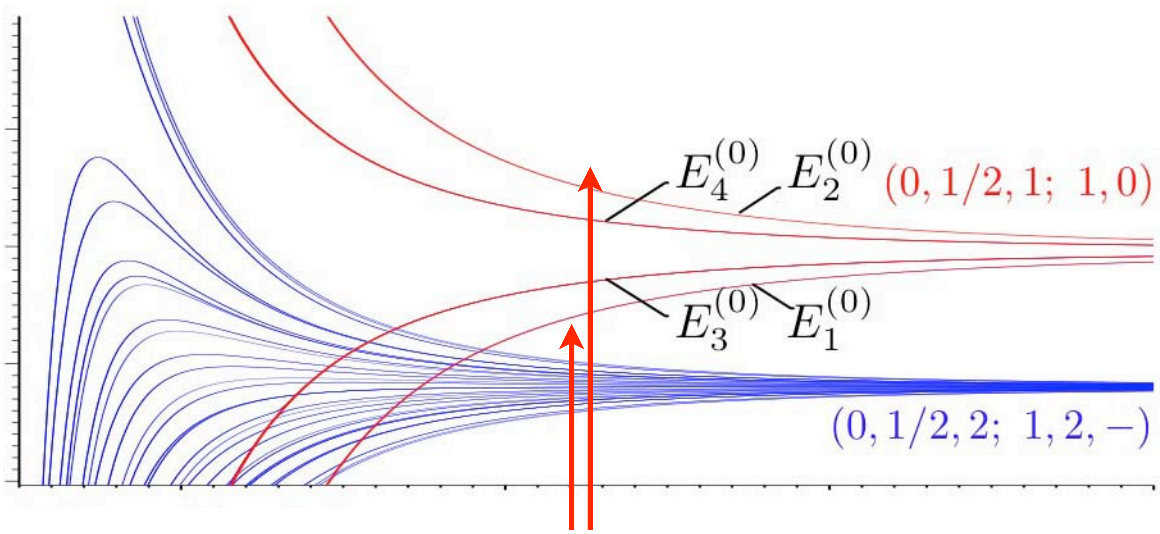
Three-body interactions



$$H_{\rm dd} = f(\mathbf{r}_1,\mathbf{r}_2)S_1^+S_2^- + f(\mathbf{r}_1,\mathbf{r}_3)S_1^+S_3^- + f(\mathbf{r}_2,\mathbf{r}_3)S_2^+S_3^- + h.c.$$

$$f(\mathbf{r}_i,\mathbf{r}_j) = \frac{\mathbf{d}_i\cdot\mathbf{d}_j - 3\mathbf{d}_i\cdot\hat{\mathbf{r}}_{i,j}\mathbf{d}_j\cdot\hat{\mathbf{r}}_{i,j}}{r_{i,j}^3}$$

- Eigenstates labeled by total number of excited rotational quanta
- Under low saturation couple to states with one shared rotational quanta. These states carry 3-body entanglement (e.g. W states)
- Typically, the non scalar (i.e. spin dependent) piece of the interaction is dominately pairwise but may be engineered to be dominately three body



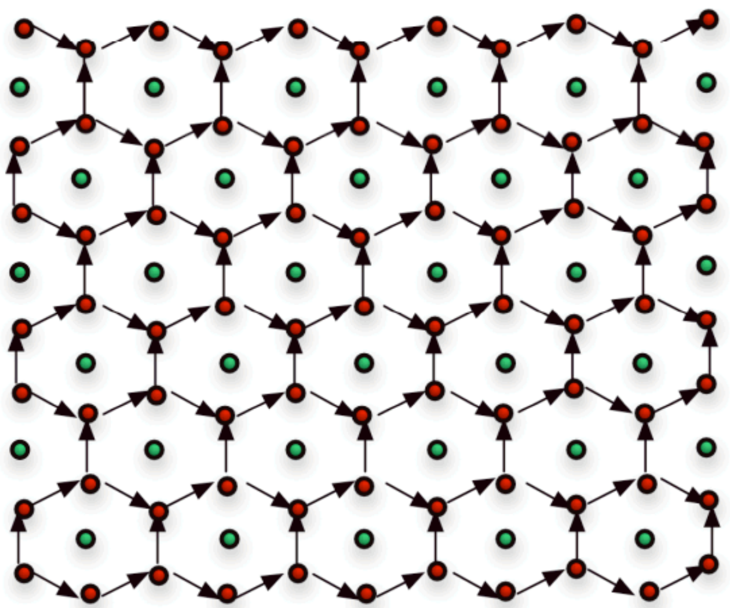
Spin-1 coupled to F=0 excited states

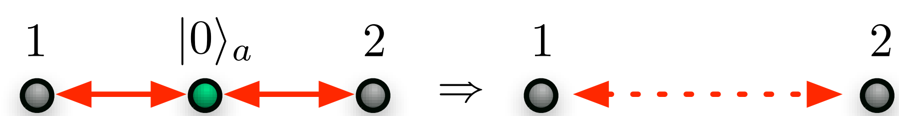
$$H_{\rm eff} = g(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)|000\rangle\langle 000| + 2-\text{body}$$

Open questions

- Most spin-lattice models with TO have many body interactions (k-local for k>3). Can we build effective Hamiltonians using mediator particles?

System spins represented by edges. Edges oriented to account for neighboring interactions. Vertex(face) ancilla can mediate vertex(face) operators





$$H_{a,1} = JA_1 \otimes X_a$$
$$H_{a,2} = JB_2 \otimes X_a$$
$$H_a = -E_a|0\rangle\langle 0|$$

$$H_{\text{eff}} = -\frac{J^2}{E_a} A_1 \otimes B_2$$
$$E_a \gg |J|$$

Qudit ancilla

$$H_a = -E_a |0\rangle_a \langle 0|$$
$$V_a = J_v \sum_{r=1}^{k} (Z_{e_r}^{o_r} \otimes |r-1\rangle \langle r| + h.c.)$$
$$H_{v\text{eff}} = U \prod_{e=[*,v]} Z_e \prod_{e=[v,*]} Z_e^{-1} + O(\epsilon)$$
$$U = (-1)^k E_a (J_v/E_a)^k \qquad ||\epsilon|| \ll 1$$

- Can we find efficient construction of observables for TO in spin lattices?
    - Ground state degeneracy
    - Topological entanglement
    - Mutual statistics for non-abelian anyons
    - ...