Quantummechanica II, Opgavenset 3, 27 januari 1999

Neutrinos and neutrino oscillations

Introduction

On the most fundamental level physicists have found that matter consist of various species elementary particles. Because of quantum mechanics the usual, classical distinction between particles (say, the electron) and fields (say, the electromagnetic field) ceases to exist, all particles and fields are described by the same mathematical entities called quantum-fields who's quanta can be interpreted as 'particles'. It is in that framework for example that the particles associated with the electromagnetic field, the socalled 'photons' do arise. In spite of the above we still do distinguish the particles associated to the four fundamental forces we know about, i.e. the photon, the weak bosons ($W^{\pm}$ and Z^0 particle) and the gluons mediating the strong interactions and finally the graviton. These *force carrying particles* have integral spin and are bosons, the photon and graviton are believed to have strictly zero mass, which explains their long rang character. At the other hand there are the *constituent particles* which we divide up into *quarks* (which do interact strongly) and *leptons* (not strongly interacting) like the electron (e) and its relatives, the muon (μ) and the tau-lepton (τ) and their respective neutrinos ν_e, ν_μ and ν_τ. These all have spin 1/2 and are thus fermions.

Figure 1. *Today's 'periodic system' of the most elementary particles*

Of all these elementary objects the neutrinos are probably the most puzzling as they only interact via the weak and the gravitational interactions; they are electrically neutral and were long time believed to be massless. This causes that most of them fly through the earth without interacting, in fact we are permanently bombarded by neutrinos without noticing anything. To detect them we need gigantically massive detectors; either build up of an enormous number of iron slabs, or tanks filled with millions of liters of water and thousands of light detectors. It is this long-standing belief that neutrinos would be massless that has been thoroughly destroyed last year, as it was discovered that at least the muon and/or tau neutrino species do have mass, through a rather subtle but remarkable effect called *neutrino oscillations*.

a. We have attached a preprint of a Scientific American style article on neutrinos and their oscillations so you can get familiar with some of the facts and ideas behind this spectacular discovery. Make sure you read some of it as to appreciate the calculations we are about to do.

The basics of this oscillation effect is in fact not all that hard to understand, and furnish a beautiful illustration of the things that can happen in quantum mechanics and only involve some calculations for a simple two level quantum mechanical system. This will be the problem we want to consider next.

Calculation of the oscillation probability

We know that neutrinos are produced by weak interactions, as for example the prototype weak decay $n \to p + e^- + \bar{\nu}_e$ (i.e. *neutron* $\longrightarrow$ *proton* + *electron*+*anti*−*electron*−*neutrino*) or in fundamental terms the elementary decay process $W^+ \to e^+ + \nu_e$ etc. Now the interesting thing is that the neutrino states (denoted as $|\nu_\ell>$, $\ell = e, \mu, \tau$) that are produced this way do not coincide with the mass eigenstates (i.e. the states of well defined mass describing the free neutrinos we observe) these we denote by $|\nu_i>$, $i = 1, 2, 3$. However the two sets of states are related by a unitary basis transformation U, so that each $|\nu_\ell>$ can be written as a linear combination of the $|\nu_i>$'s:

$$|\nu_\ell >= U_{\ell i}|\nu_i > \tag{1}$$

In the present context we are mainly interested in the $\mu - \tau$ states, so we may write:

$$\begin{pmatrix} |\nu_\mu > \\ |\nu_\tau > \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1 > \\ |\nu_2 > \end{pmatrix} \tag{2}$$

We furthermore know that the mass eigenstates will have the usual quantummechanical time evolution with energies E_i.

b. Assuming that at $t = 0$ we produce a muon through a weak decay satisfying eqn(2), what does the muon state look like at a later time t?

c. Using the above, calculate the transition probability $P(\nu_\mu \to \nu_\tau; t)$.

In all cases of interest $|p| \gg m_i$, so that we may write $E_i \simeq |p| + \frac{m_i^2}{2|p|}$, with $|p| \equiv |E_\nu|$ being essentially the neutrino energy. Furthermore the time t in the equation for the transition probability can in this case be replaced by the distance L travelled $t = L/c$ (but we have set $c = 1$).

d. Make use of the above relations to derive the expression below, for the probability for a muon-neutrino to have turned into a tau-neutrino after travelling a distance L due to the oscillations.

$$P(\nu_\mu \to \nu_\tau; t) = \sin^2 2\theta \sin^2 \left[\frac{(m_1^2 - m_2^2)L}{4E_\nu} \right] \tag{3}$$

Figure 2.*Results of the Super-Kamiokande Experiment as they were for the first time presented by the team on the* **XVIII International Conference on Neutrino Astrophysics** *in Japan*

The results of the Super Kamiokande Experiment

e. It is gratifying to see that in the seminal paper in the *Physical Review Letters* (Phys.Rev.Lett. 81 (1998) 1562-1567) in which the experimental observation of neutrino oscillations is presented by the socalled Super-Kamiokande Collaboration, basically only a single equation concerning the physics is used. Show that their 'equation (1)' is obtaines exactly from the result you have just derived by substituting the correct units and corresponding numbers.

f. Check now what has been measured and what the meaning of the plots at the end of the paper in PRL is. How is the L dependence realized in the experiment? What is exactly the conclusion about the masses of the three neutrino types? What is the meaning of Figure 2?

g. It is mentioned that the massiveness of neutrinos may have important cosmological implications. State briefly what is meant by these implications.

Attachments:

- Little Articles from NRC Handelsblad (June 5, 98)
- Popular article of Professor John Learned on neutrinos and neutrino oscillations (http://www.phys.hawaii.edu:80/~ jgl/nuosc_story.html)
- The publication of the Kamiokande group in the Physical Review Letters (PRL 81 (1998) 1562-1567)