Opgaven Quantumfysica II, Week 4, 26 januari 2000

Inleveren voor maandag 31 januari 2000, 12.00 uur, bij de portier van de Diamantslijperij. Met vragen over stof en opgaven kun je terecht bij de werkcollegedocenten:

- *Alejandro Arrizabalaga (ITF kamer 333, tel. 5255006, arrizaba@wins.uva.nl)*
- *Joost Slingerland (ITF kamer 356, tel. 5255920, slinger@wins.uva.nl)*
- *Martijn Wubs (WZI kamer B045, tel 5255646, wubs@wins.uva.nl).*

Opmerking:
Deze week worden Joost en Alejandro vervangen door

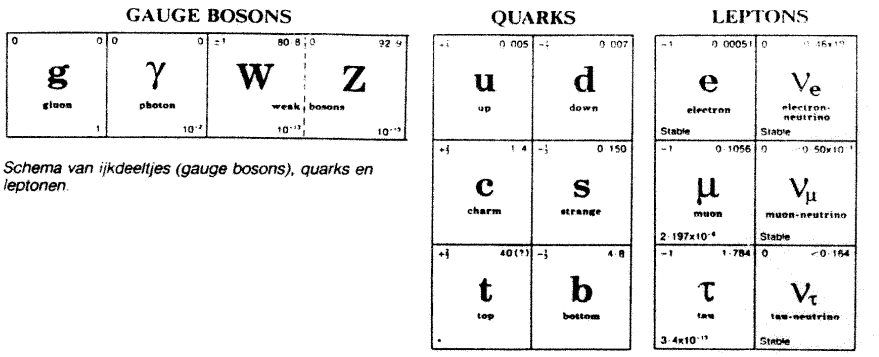
- *Eddy Ardonne (ITF kamer 358, tel. 5255922, ardonne@wins.uva.nl)*
- *Ronald van Elburg (ITF kamer 386, tel. 5255975, elburg@wins.uva.nl)*

Om chaos te voorkomen zouden we het op prijs stellen als je de naam van je werkcollegedocent (niet die van de vervanger !) op je blaadje vermeldt. Succes met de opgaven !

Exercise 4.1: Neutrinos and neutrino oscillations

Introduction

On the most fundamental level physicists have found that matter consists of various species of elementary particles. Because in quantum mechanics the usual, classical distinction between particles (say, the electron) and fields (say, the electromagnetic field) ceases to exist, all particles and fields are described by the same mathematical entities called quantum-fields whose quanta can be interpreted as 'particles'. It is in that framework for example that the particles associated with the electromagnetic field, the so called 'photons' do arise. In spite of the above we still do distinguish the particles associated to the four fundamental forces we know about, i.e. the photon, the weak bosons ($W^{\pm}$ and Z^0 particle), the gluons mediating the strong interactions and finally the graviton. These *force carrying particles* have integral spin and are bosons, the photon and graviton are believed to have strictly zero mass, which explains their long range character. Apart from these, there are the *constituent particles* which we divide up into *quarks* (which do interact strongly) and *leptons* (not strongly interacting) like the electron (e) and its relatives, the muon (μ) and the tau-lepton (τ) and their respective neutrinos ν_e, ν_μ and ν_τ. These all have spin 1/2 and are thus fermions.



Schema van ijkdeeltjes (gauge bosons), quarks en leptonen.

Figure 1: Today's 'periodic system' of the most elementary particles

Of all these elementary objects the neutrinos are probably the most puzzling as they only interact via the weak and the gravitational interactions; they are electrically neutral and were for a long time believed to be massless. Because neutrinos feel only gravity and the weak force, most of them fly through the earth without interacting significantly. In fact we are permanently bombarded by neutrinos without noticing anything. To detect them we need gigantically massive detectors; either detectors built up of an enormous number of iron slabs, or tanks filled with millions of liters of water and thousands of light detectors. It is the long-standing belief that neutrinos would be massless that has been thoroughly destroyed in 1998, as it was discovered that at least the muon and/or tau neutrino species do have mass, through a rather subtle but remarkable effect called *neutrino oscillations*.

a. We have attached a preprint of a Scientific American style article on neutrinos and their oscillations so you can get familiar with some of the facts and ideas behind this spectacular discovery. Make sure you read some of it as to appreciate the calculations we are about to do.

The basics of the oscillation effect are in fact not all that hard to understand, and furnish a beautiful illustration of the things that can happen in quantum mechanics and only involve some calculations for a simple two level quantum mechanical system. This will be the problem we want to consider next.

Calculation of the oscillation probability

We know that neutrinos are produced by weak interactions, as for example the prototype weak decay $n \to p + e^- + \bar{\nu}_e$ (i.e. *neutron* $\longrightarrow$ *proton+electron+anti−electron−neutrino*) or in fundamental terms the elementary decay process $W^+ \to e^+ + \nu_e$ etc. Now the interesting thing is that the neutrino states (denoted as $|\nu_\ell>$, $\ell = e, \mu, \tau$) that are produced this way do not coincide with the mass eigenstates (i.e. the states of well defined mass describing the free neutrinos we observe) these we denote by $|\nu_i>$, $i = 1, 2, 3$. However the two sets of states are related by a unitary basis transformation U, so that each $|\nu_\ell>$ can be written as a linear combination of the $|\nu_i>$'s:

$$|\nu_\ell> = U_{\ell i}|\nu_i> \tag{1}$$

In the present context we are mainly interested in the $\nu_\mu - \nu_\tau$ states, so we may write:

$$\begin{pmatrix} |\nu_\mu> \\ |\nu_\tau> \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1> \\ |\nu_2> \end{pmatrix} \tag{2}$$

We furthermore know that the mass eigenstates will have the usual quantum mechanical time evolution with energies E_i.

b. Assuming that at $t = 0$ we produce a muon neutrino through a weak decay satisfying eqn(2), what does the muon neutrino state look like at a later time t?

c. Using the above, calculate the transition probability $P(\nu_\mu \to \nu_\tau; t)$.

In all cases of interest $|p| \gg m_i$, so that we may write $E_i \simeq |p|c + \frac{m_i^2 c^3}{2|p|}$, with $|p| \equiv |E_\nu/c|$ being essentially the neutrino energy. Furthermore the time t in the equation for the transition probability can in this case be replaced by the distance L travelled $t = L/c$.

d. Make use of the above relations to derive the expression below, for the probability for a muon-neutrino to have turned into a tau-neutrino after travelling a distance L due to the oscillations.

$$P(\nu_\mu \to \nu_\tau; t) = \sin^2 2\theta \sin^2 \left[\frac{c^3(m_1^2 - m_2^2)L}{4\hbar E_\nu} \right] \tag{3}$$

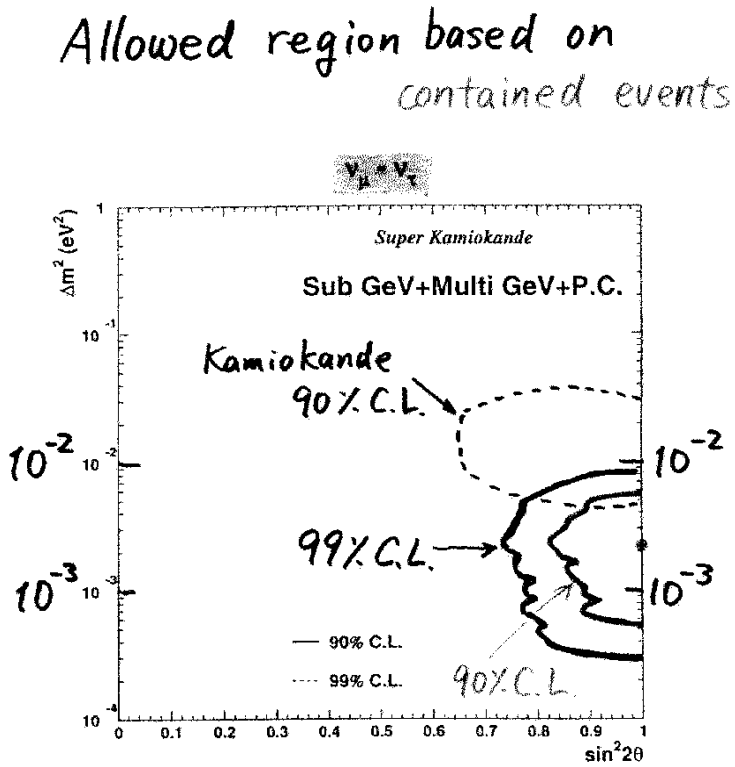


Figure 2: Results of the Super-Kamiokande Experiment as they were for the first time presented by the team on the **XVIII International Conference on Neutrino Astrophysics** in Japan

The results of the Super Kamiokande Experiment

e. It is gratifying to see that in the seminal paper in the *Physical Review Letters* (Phys.Rev.Lett. 81 (1998) 1562-1567) in which the experimental observation of neutrino oscillations is presented by the socalled Super-Kamiokande Collaboration, basically only a single equation concerning the physics is used. Show that their 'equation (1)' is obtained exactly from the result you have just derived by substituting the correct units and corresponding numbers. Note that the unit of mass used in the article should really be eV/c^2 rather than eV.

f. Check now what has been measured and what the meaning of the plots at the end of the paper in PRL is. How is the L dependence realized in the experiment? What is exactly the conclusion about the masses of the three neutrino types? What is the meaning of Figure 2?

g. It is mentioned that the massiveness of neutrinos may have important cosmological implications. State briefly what is meant by these implications.

Attachments:

- Little Articles from NRC Handelsblad (June 5, 98)
- Popular article of Professor John Learned on neutrinos and neutrino oscillations (http://www.phys.hawaii.edu:80/$\sim$ jgl/nuosc_story.html)
- The publication of the Kamiokande group in the Physical Review Letters (PRL 81 (1998) 1562-1567)