MP362: Mathematical Methods II

Problem Sheet 2

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(1) Find a particular solution of the following inhomogeneous equations

$$
\begin{array}{lll}
(i) & (1-x^2)y'' - 2xy' = 3x^2 + 5x - 1 & \text{(Legendre)} \\
(ii) & xy'' + (1-x)y' = 2x^2 - 4x & \text{(Laguerre)} \\
(iii) & y'' - 2xy' + 3y = (x-2)^2 & \text{(Hermite).}
\end{array}
$$

(2) Solve the one-dimensional wave equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{a^2}\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

on the interval $0 \le x \le L$ with initial profile

$$u(x,0) = \begin{cases} \frac{2d}{L}x, & 0 \le x \le \frac{L}{2} \\ 2d\left(1 - \frac{x}{L}\right), & \frac{L}{2} \le x \le L, \end{cases}$$

where d is a constant, and $\frac{\partial u(x,t)}{\partial t}\Big|_{t=0} = 0$.

(3) Solve the one dimensional heat equation with the boundary conditions

$$u(0,t) = u(L,t) = 0, \qquad u(x,0) = T\sin^2\left(\frac{\pi x}{L}\right), \qquad (T \text{ constant}).$$

Note: for positive integers m and n

$$\int_0^\pi \sin(mx)\cos(nx)dx = \frac{\pi}{2}\delta_{nm}, \qquad \int_0^\pi \cos(mx)\sin(nx)dx = \begin{cases} 0 & \text{if m+n is even} \\ \frac{2n}{n^2-m^2} & \text{if m+n is odd.} \end{cases}$$

(4) Solve the three dimensional Laplace equation in the box

$$0 < x < a, \quad 0 < y < b, \quad 0 < z < c$$

with the boundary conditions

$$u(0,y,z) = u(a,y,z) = u(x,0,z) = u(x,b,z) = u(x,y,0) = 0$$

$$u(x,y,c) = V\sin(\pi x/a)\sin(\pi y/b).$$

where V is a constant.