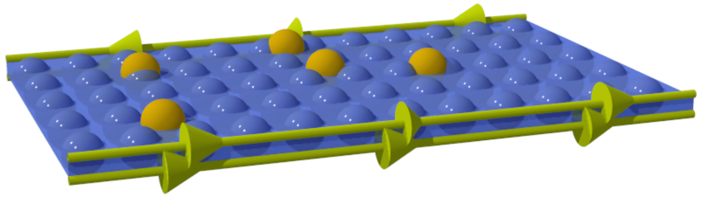
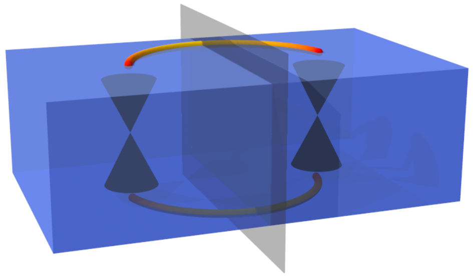


Frustration meets topology:

from C>1 fractional Chern insulators to tilted Weyl semimetals



Emil J. Bergholtz

Freie Universität Berlin

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

HELMHOLTZ ASSOCIATION
Virtual Institute: New States of Matter and their Excitations

Nano Scale

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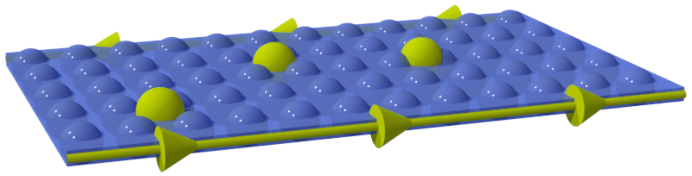
Mathematical Physics Seminar
Maynooth University, Ireland
February 2016

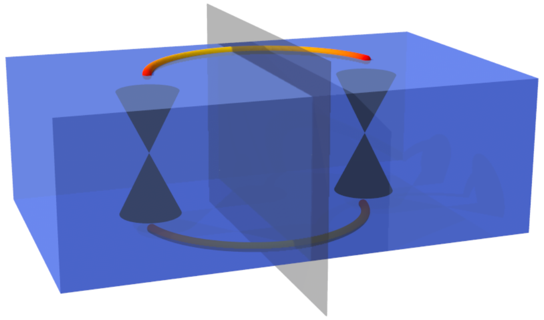
Today, I will

- Briefly introduce two frontiers of condensed matter physics

1) *Fractional Chern insulators*

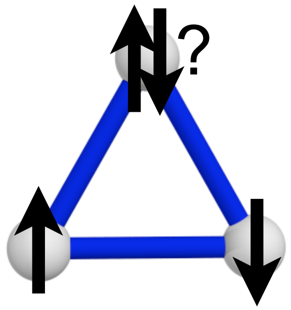


2) *Weyl semimetals*

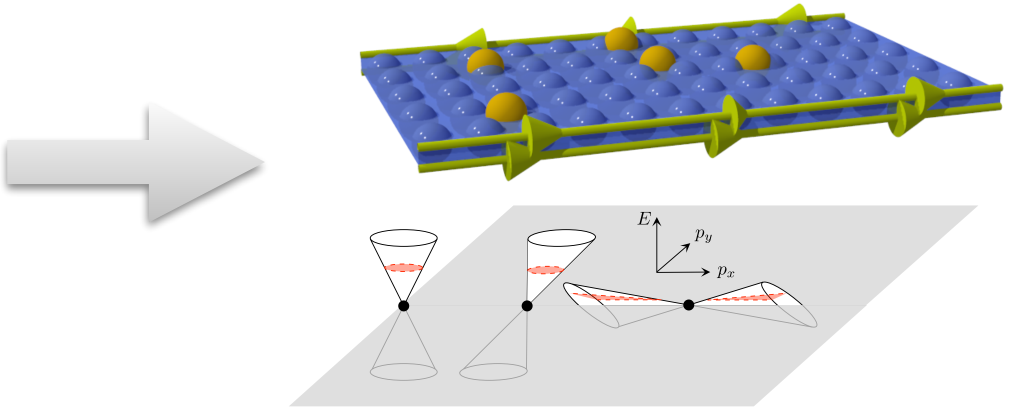


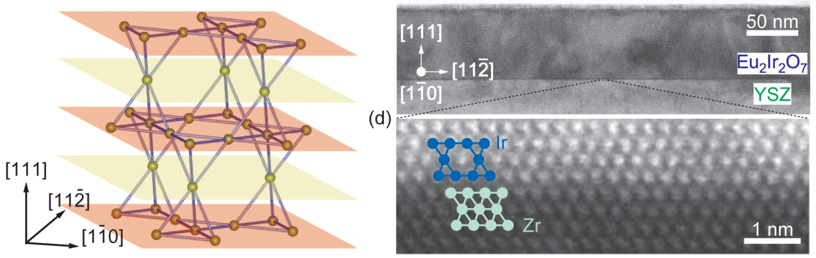
- Report on related progress on both topics

Key ingredient: ***Geometrical frustration*** + interactions and spin-orbit coupling



New phenomena





… and intriguing first experiments (by others)

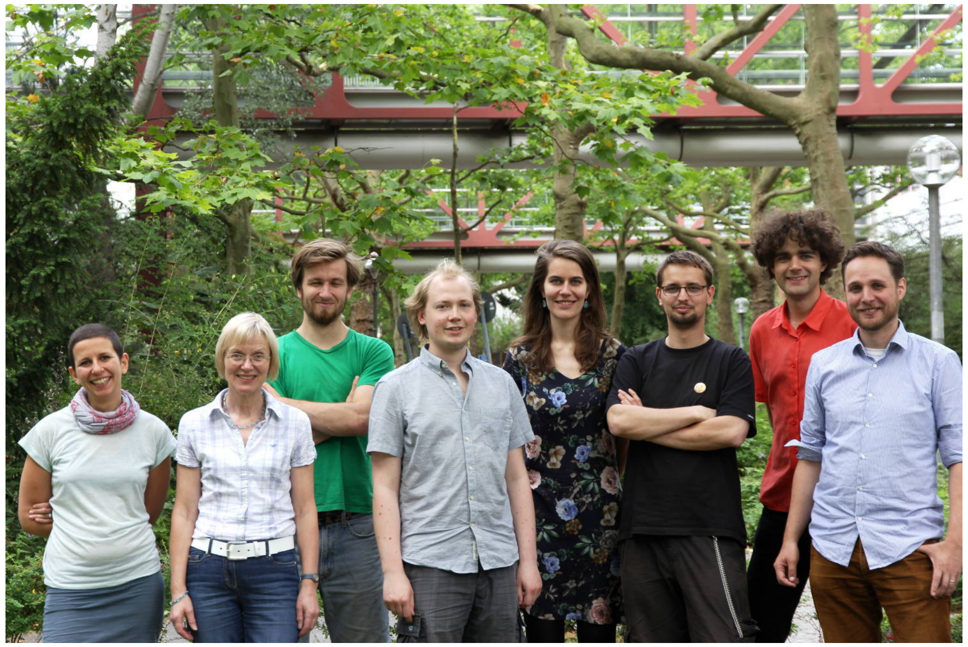
First: My collaborators on these topics

- In Berlin

Jörg Behrmann
Piet Brouwer
Jens Eisert
Irina Gancheva
Flore Kunst
Kevin Madsen
Gregor Pohl
Björn Sbierski
Maximilian Trescher

- External

Jan Budich, Innsbruck
Eliot Kapit, Oxford/New York
Dmitry Kovrizhin, Cambridge
Zhao Liu, Princeton -> Berlin
Andreas Läuchli, Innsbruck
Roderich Moessner, Dresden
Masaaki Nakamura, Tokyo
Masafumi Udagawa, Tokyo



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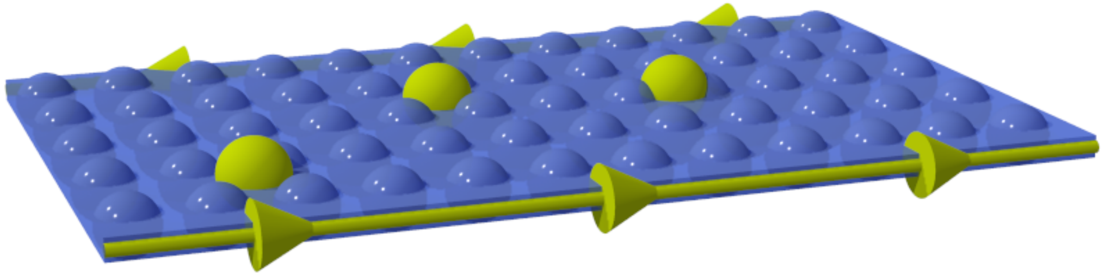


Fractional Chern insulators

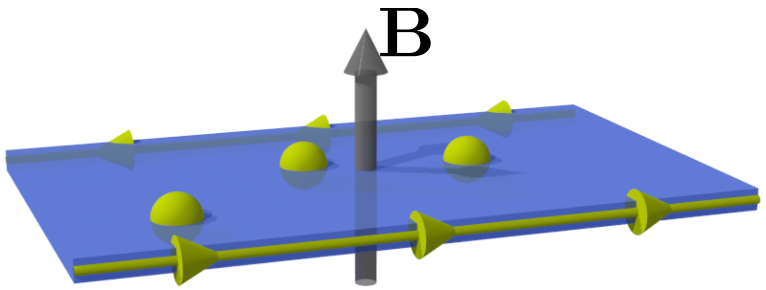
Reviews:

E. J. Bergholtz & Z. Liu
Topological Flat Band Models and Fractional Chern Insulators
Int. J. Mod. Phys. B 27, 1330017 (2013) [arXiv:1308.0343]

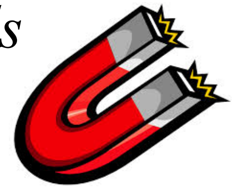
S. A. Parameswaran, R. Roy & S. L. Sondhi
Fractional Quantum Hall Physics in Topological Flat Bands
C. R. Physique 14, 816 (2013) [arXiv:1302.6606]

Fractional Chern insulators — motivation

- Fractional quantum Hall states in a strong magnetic field are truly amazing!
  - Quantized conductance & chiral edge states
  - Abelian and non-Abelian anyon excitations with fractional charge and statistics



Very strong magnetic fields



$|\mathbf{B}| \sim 10 - 30$ Tesla

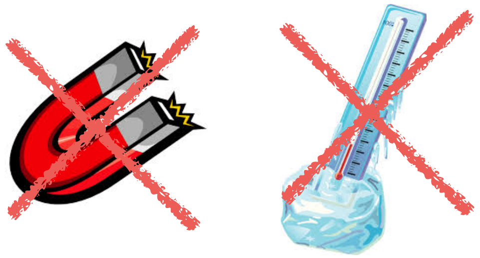
Extremely low temperatures

$T \lesssim 1$ Kelvin

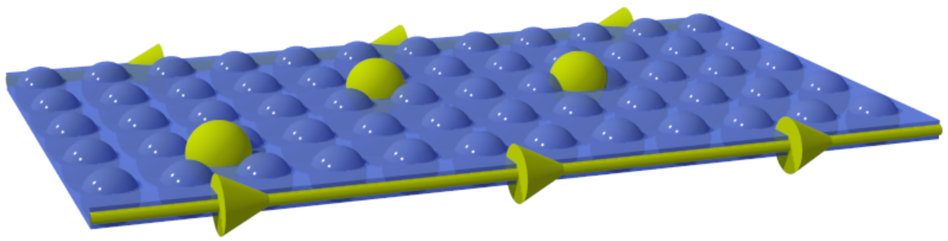
$$\Delta E \sim e^2/\ell_B \propto \sqrt{B}$$

- But no "topological quantum computer" in service, no Nobel prize for non-Abelian anyons,…

- Lattice scale realizations?

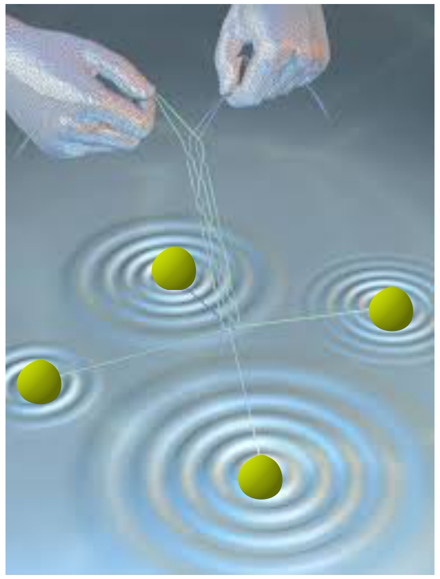


Fractional Chern insulators!?



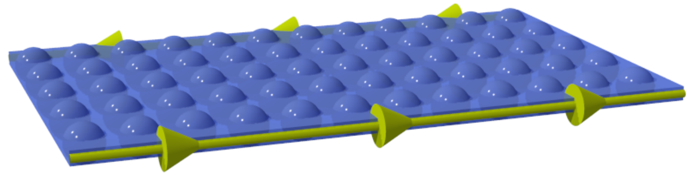
Robust experiments?

Topological quantum computation?

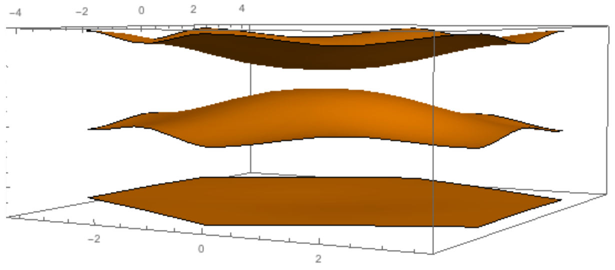


Fractional Chern insulators

- Integer Chern insulators recently realized!
  - Magnetic topological insulator slabs (2013), cold atoms (2014),…



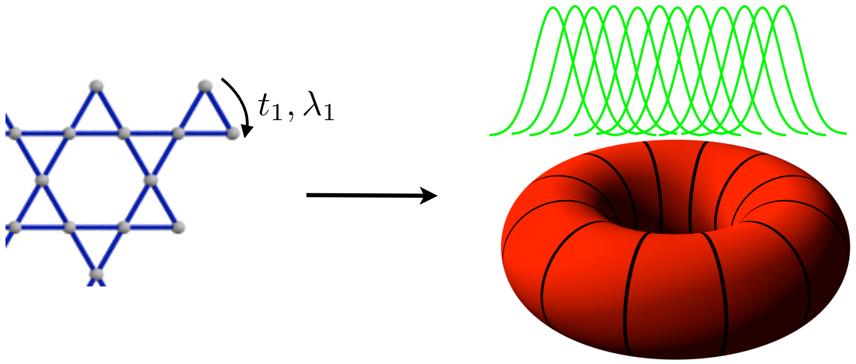
- How about strongly interacting versions?
  - Flat bands with Chern number C=1 similar to Landau levels quite easy to find



Theory: *FQH/FCI states survive can despite strong lattice effects*

- Interesting differences compared to the continuum

- But all known FCIs in C=1 bands are adiabatically connected to corresponding FQH states!



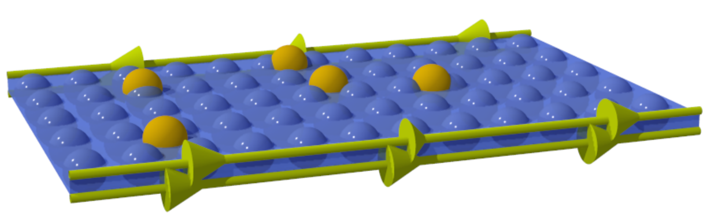
Z. Liu and E.J. Bergholtz,
Phys. Rev. B 87, 035306 (2013)

Questions:

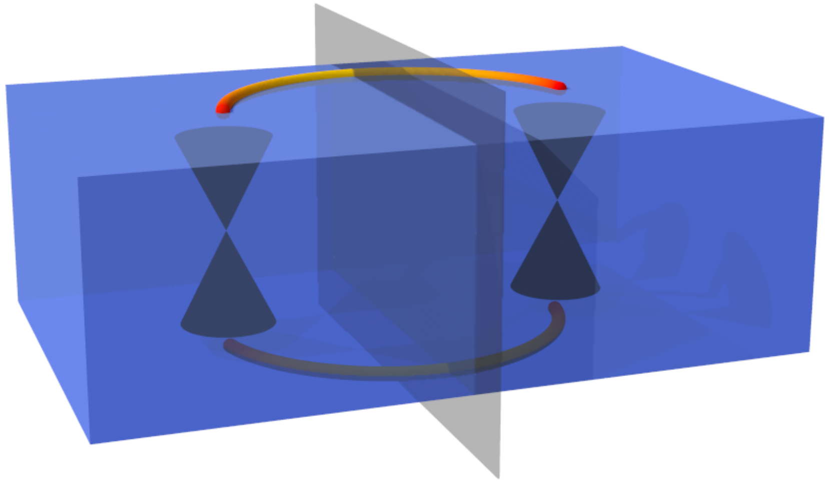
1) Where are FCIs likely to form?

2) Are there topologically ordered states qualitatively different from the FQH states?

- How about flat C>1 bands?



?



Weyl semimetals

Reviews:

P. Hosur and X.-L. Qi,
Recent developments in transport phenomena in Weyl semimetals,
arXiv:1309:4464

A.M. Turner and A. Vishwanath,
Beyond Band Insulators: Topology of Semi-metals and Interacting Phases,
arXiv:1301.0330

Weyl semimetal basics

- Topological gapless phase in three dimensions

  - half a gapless Dirac low-energy theory, linear crossing of two *non*-degenerate bands

$$H_{\mathrm{Weyl}} = \sum_{i,j} v_{ij} k_i \sigma_j \qquad (= \mathbf{d}(\mathbf{k}) \cdot \sigma)$$

  - identical to the surface theory of a 4D QH state

- Broken symmetry

  - time-reversal *and* inversion symmetry would imply degenerate bands

- Robust nodal points

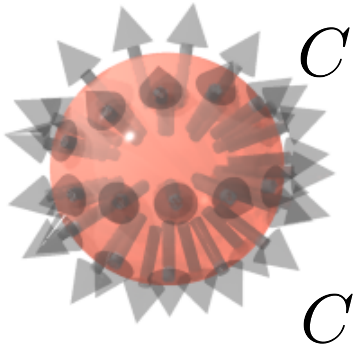
  - there is no 4th Pauli matrix
  - striking difference to 2d!

$$E = \pm \sqrt{\sum_{m,n,l} v_{ml} v_{nl} k_n k_m}$$

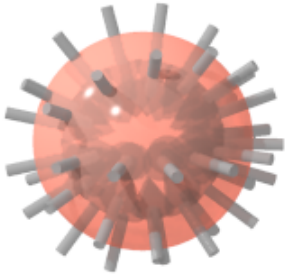
- Topological stability of a Weyl node

  - protected by a Chern number

$$C = \frac{1}{4\pi} \int dk_x \int dk_y \; \hat{\mathbf{d}} \cdot \left(\frac{\partial \hat{\mathbf{d}}}{\partial k_x} \times \frac{\partial \hat{\mathbf{d}}}{\partial k_y} \right)$$



$C = 1$

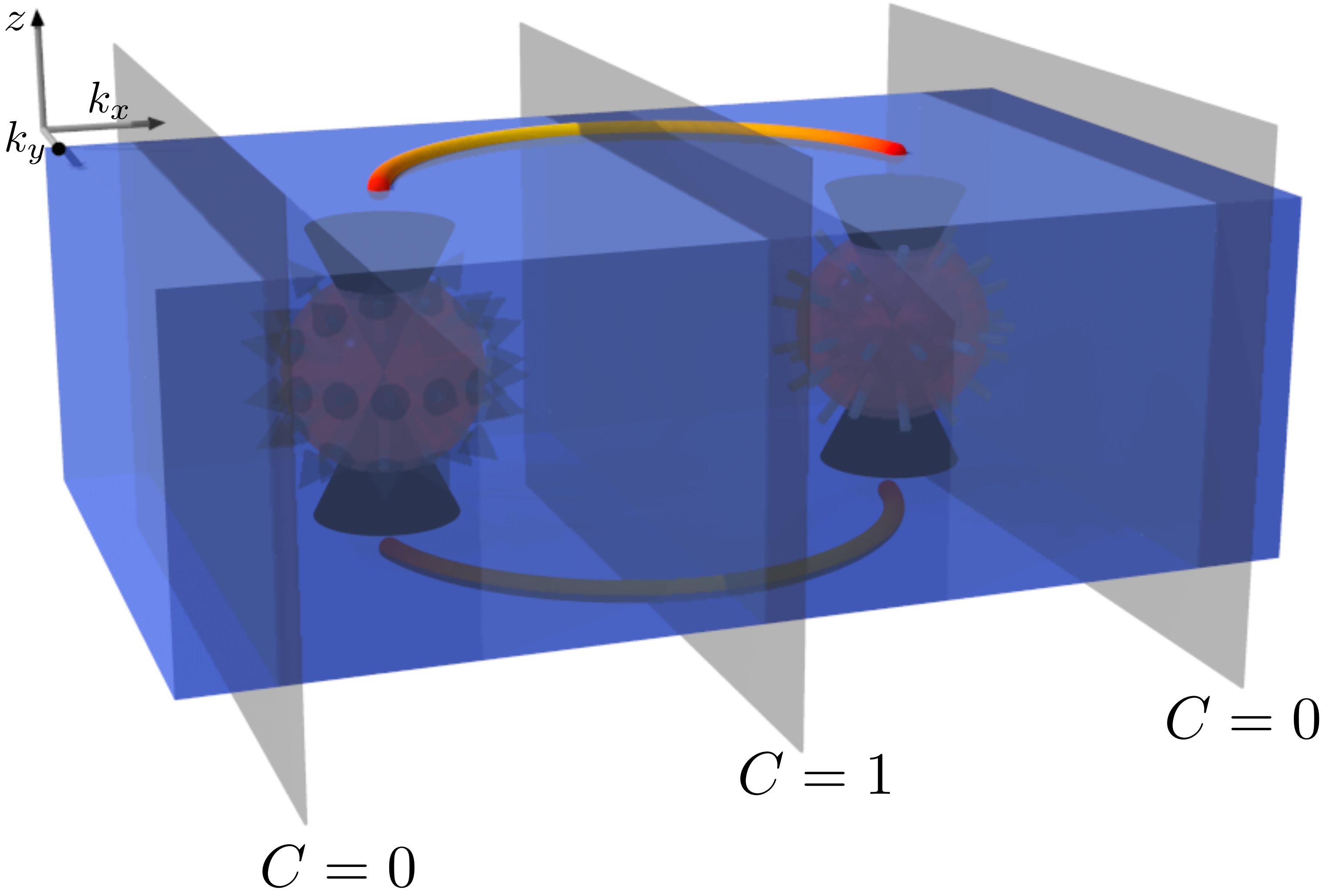


$C = -1$

$$C = \mathrm{sign}(\det(v_{ij})) = \pm 1$$

Global topology & Fermi arcs

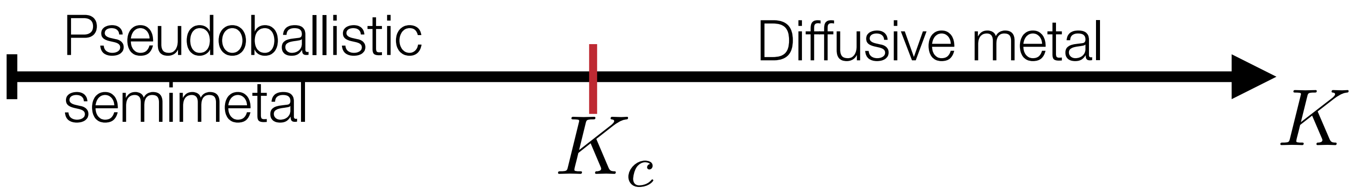
X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B 83, 205101 (2011)



- Zero total Chern flux in any periodic band structure
  - even number of nodes, equal number of each chirality
- The topology is manifested through exotic surface states, “Fermi arcs”
  - remnants of the Chern insulator edge states

Weyl semimetals: recent activity

- Theory first
  - early work by Volovik and others decades ago — much increased interest since ~2011
  - many intriguing transport phenomena predicted, including novel disorder induced phase transitions, …



Pseudoballistic semimetal

Diffusive metal

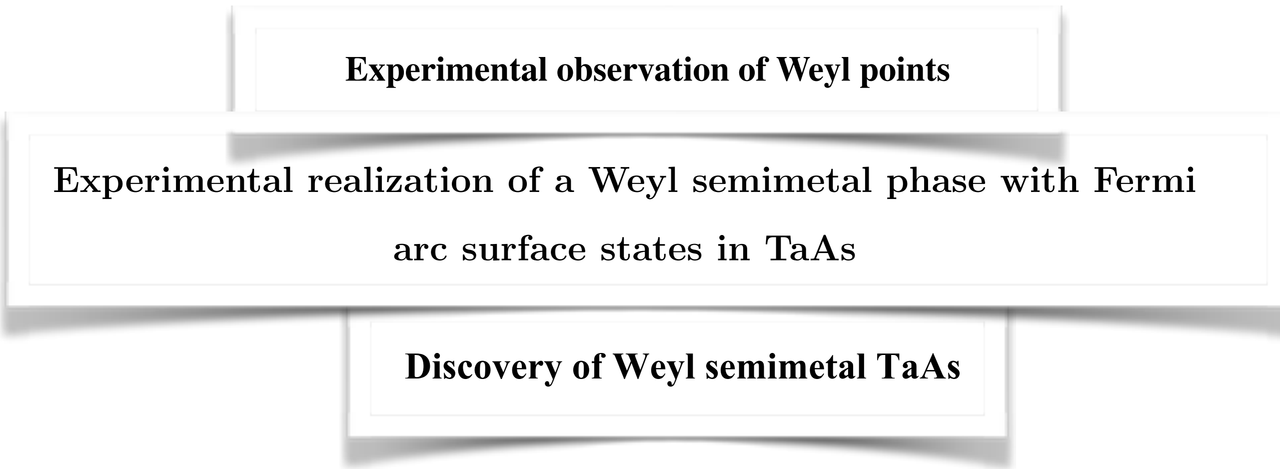
K_c

K

B. Sbierski, G. Pohl, E. J. Bergholtz, and P. W. Brouwer
Phys. Rev. Lett. 113, 026602 (2014)
… and many others

- Now with an avalanche of experiments!
  - First observations reported in 2015

Lu et. al. arXiv:1502.03438 (photonic crystals @ MIT)
Xu et. al. arXiv:1502.03807 (TaAs @ Princeton)
Lv et. al. arXiv:1502.04684 (TaAs @ Beijing)



Experimental observation of Weyl points

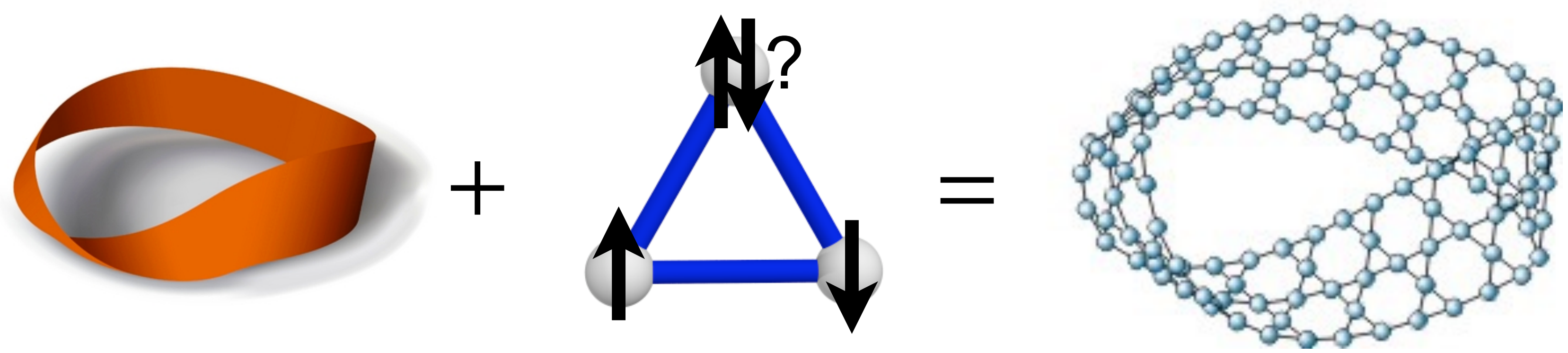
Experimental realization of a Weyl semimetal phase with Fermi arc surface states in TaAs

Discovery of Weyl semimetal TaAs

Questions: *1) How about interaction effects?*

2) Is the correspondence between bulk and surface one-to-one?

3) Breaking of Lorentz invariance?



Topology meets frustration

References:

M. Trescher and E.J. Bergholtz,
Flat bands with higher Chern number in pyrochlore slabs
Phys. Rev. B 86, 241111(R) (2012)

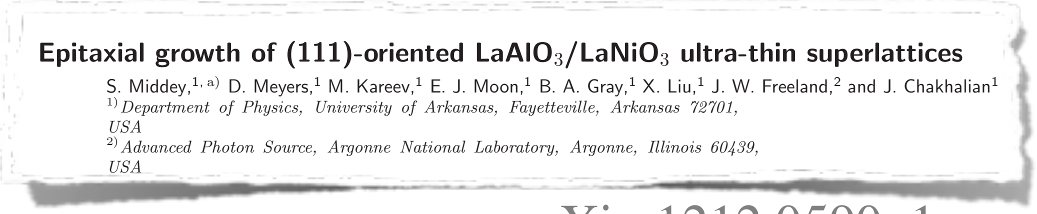
Z. Liu, E.J. Bergholtz, H. Fan, and A. M. Läuchli,
Fractional Chern Insulators in Topological Flat bands with Higher Chern Number
Phys. Rev. Lett. 109, 186805 (2012)

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa,
Topology and Interactions in a Frustrated Slab: Tuning from Weyl Semimetals to $C > 1$ *Fractional Chern Insulators*
Phys. Rev. Lett. 114, 016806 (2015)

Materials motivation

- Perovskite materials, ABO_3, routinely grown in sandwich structures in the [100] direction

  - Instead (111) slabs would be good for topological physics (relatively flat C=1 bands).



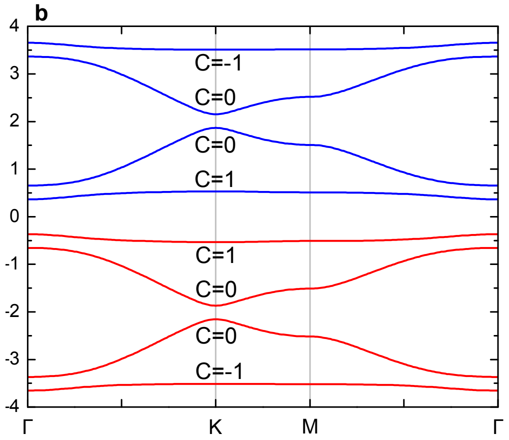
Epitaxial growth of (111)-oriented $LaAlO_3/LaNiO_3$ ultra-thin superlattices

S. Middey,[1, a)] D. Meyers,[1] M. Kareev,[1] E. J. Moon,[1] B. A. Gray,[1] X. Liu,[1] J. W. Freeland,[2] and J. Chakhalian[1]

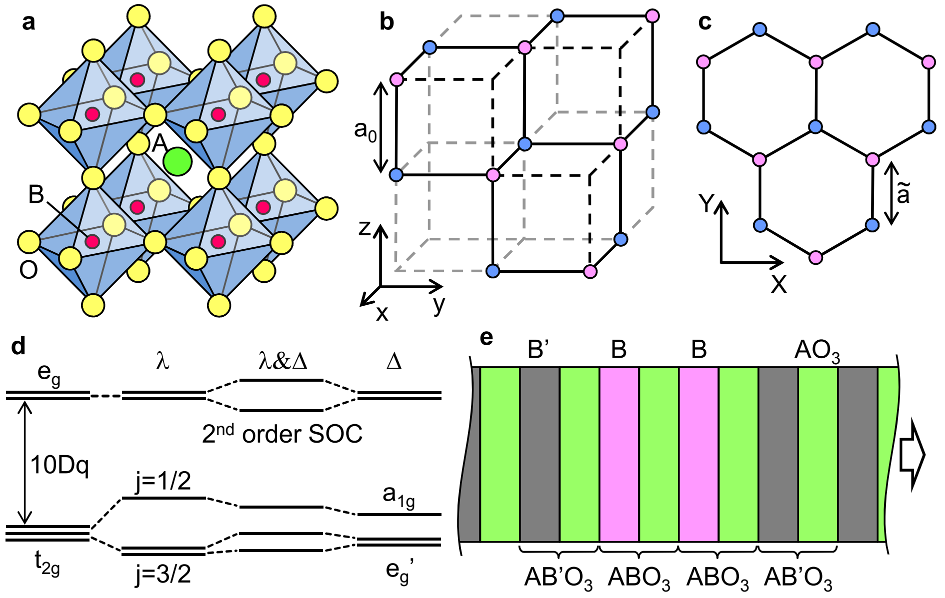
[1)] *Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA*

[2)] *Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA*

arXiv:1212.0590v1



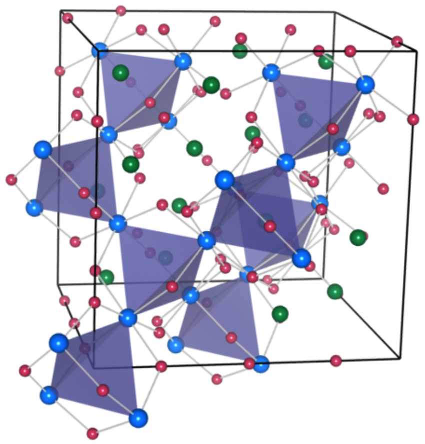
  - Fractional Chern insulators!?
  - But [111] is not a natural cleavage/growth direction...

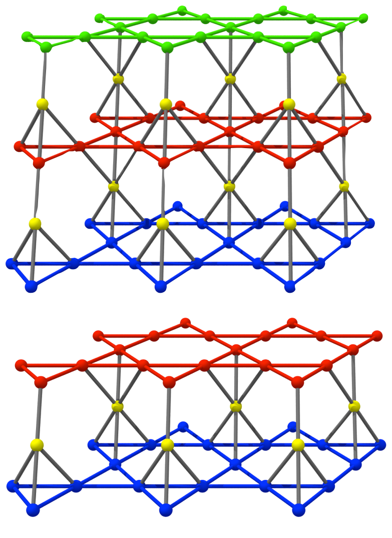


D. Xiao, W. Zhu, Y. Ran, N. Nagaosa, and S. Okamoto, Nature Commun. 2, 596 (2011).

- **Our suggestion:** *Consider (111) slabs of pyrochlore transition metal oxides, in particular* $A_2Ir_2O_7$ *iridate thin films*

  - Natural cleavage/growth direction!
  - Strong spin-orbit coupling
  - Even richer physics…?





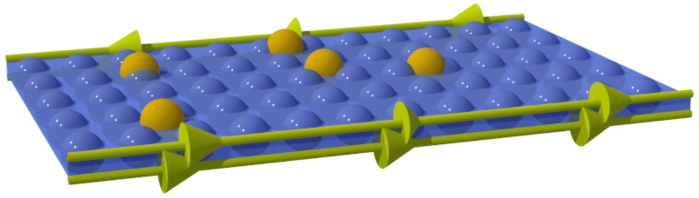
M. Trescher and E.J. Bergholtz, Phys. Rev. B 86, 241111(R) (2012)

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, Phys. Rev. Lett. 114, 016806 (2015)

Conceptual motivation

- Why did nobody report on fractional Chern insulators in C>1 bands?

  - It is the obvious thing to look for as they would be unique to the lattice setting: Landau levels always have C=1!

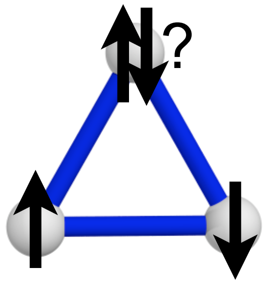


?

- Frustrated lattices are especially promising

  - Natural platform for flat bands

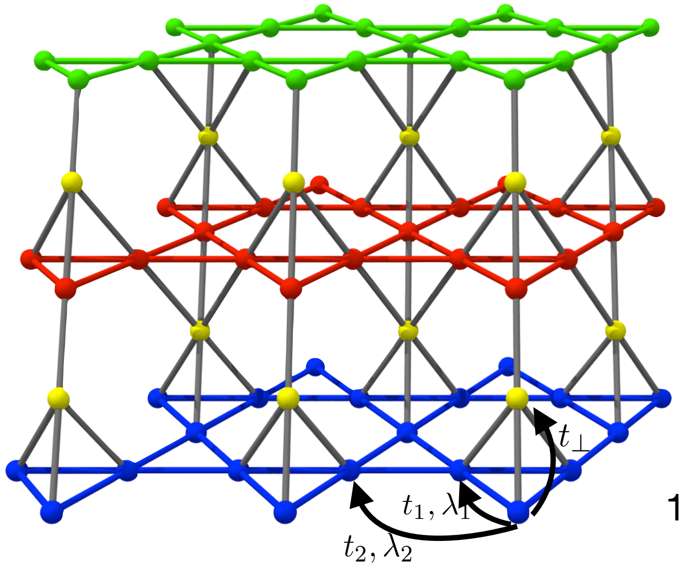
  - Frustrates the main FCI competitors such as CDWs



- Is it possible to make N C=1 bands hybridize so that one band absorbs all the topology (C=N) while the others become trivial (C=0)?

$$1+1\to 2+0?$$

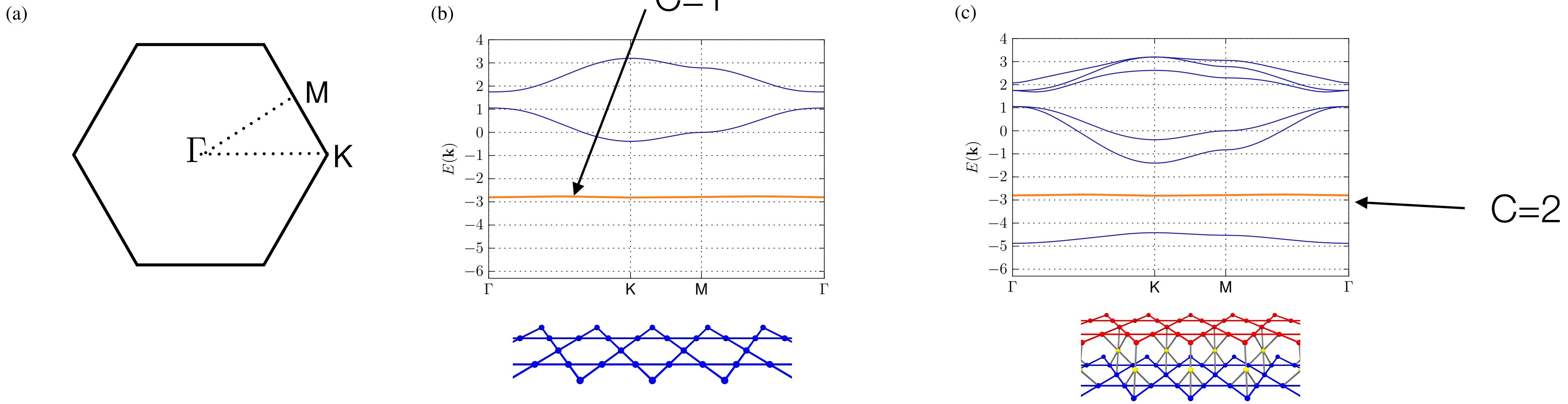
Consider *frustrated systems with a layered structure*!



Tight binding results: bulk dispersion and Chern numbers

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

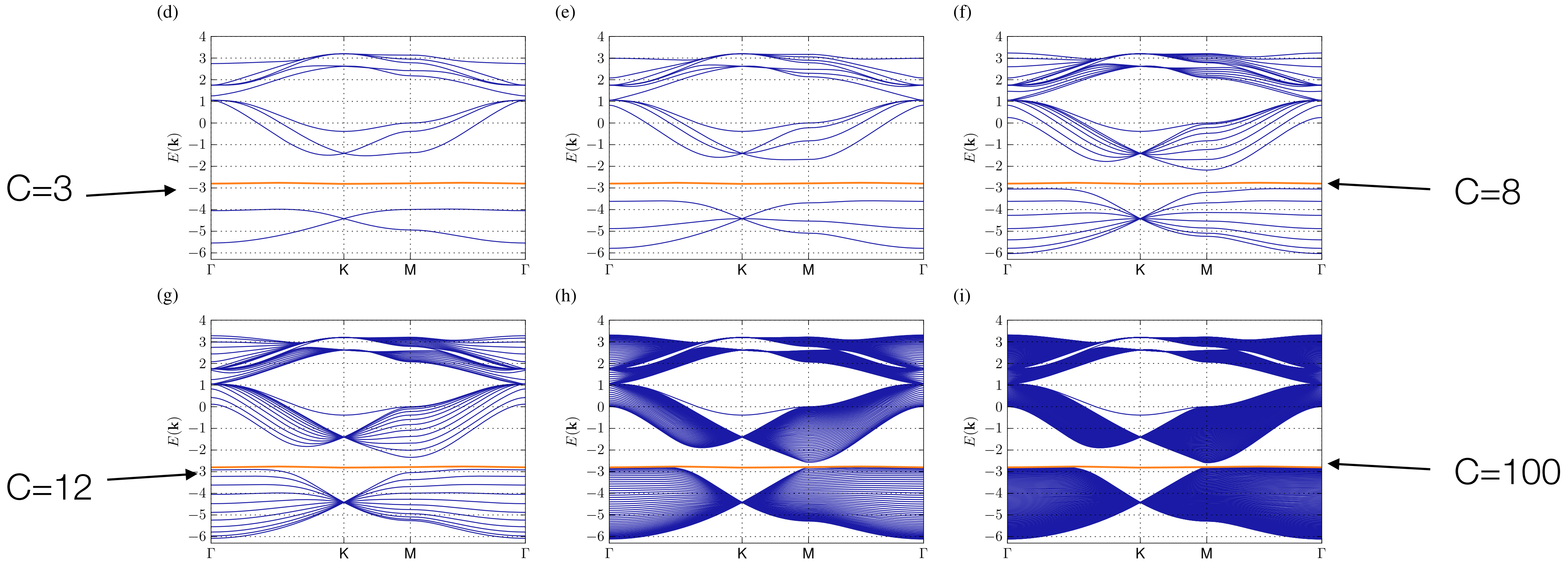
- Dispersion for one layer
- Dispersion with two layers



C=1

C=2

- For N kagome layers we find an almost flat band with C=N!



C=3

C=8

C=12

C=100

Does local interactions give new FCI phases within the C>1 bands?

Z. Liu, E.J. Bergholtz, H. Fan, A. M. Läuchli
Phys. Rev. Lett. 109, 186805 (2012)

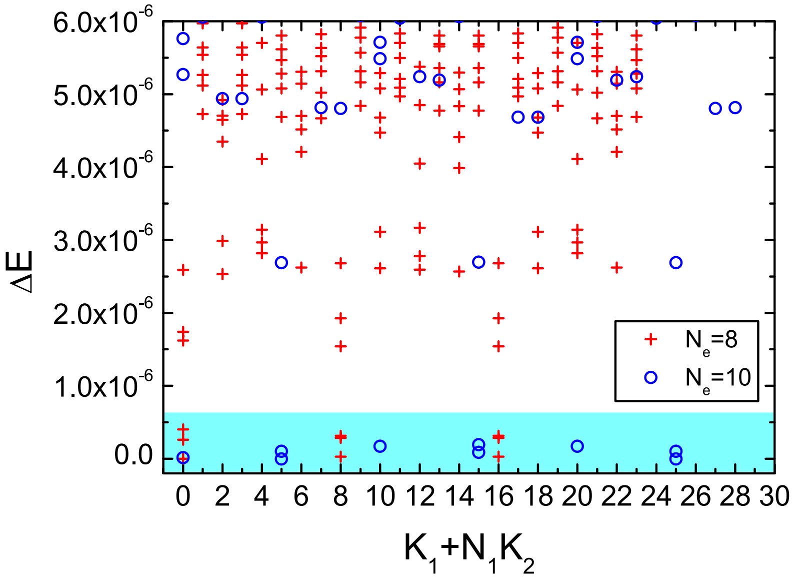
- Fermionic FCIs at $\nu_f = 1/(2C+1)$

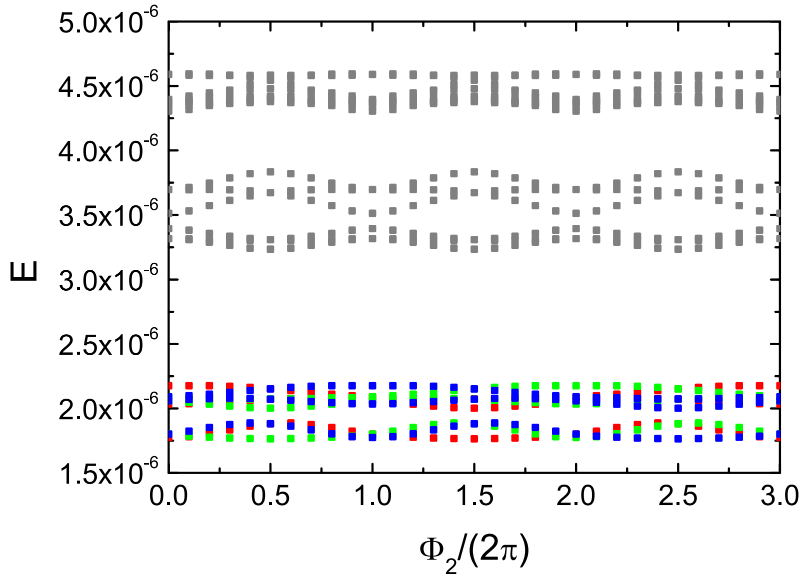
but absent at higher filling fractions!

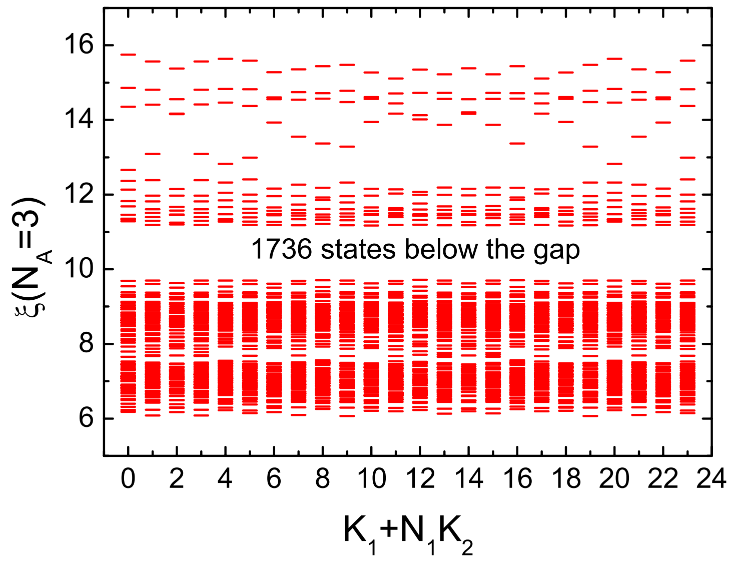
- Bosonic FCIs at $\nu_b = 1/(C+1)$
- Strong evidence also for C>1 generalizations of non-Abelian FQH states found in this model!

E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa, Phys. Rev. Lett. 114, 016806 (2015)

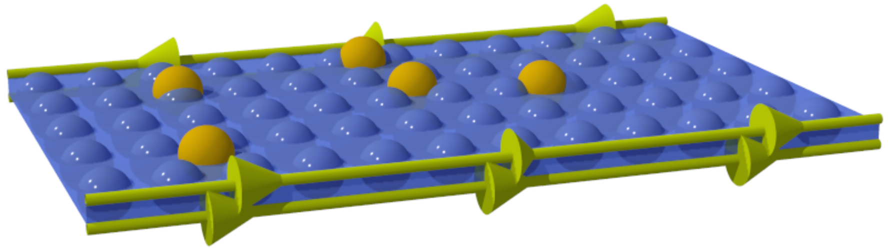
A. Sterdyniak, C. Repellin, B.A. Bernevig, and N. Regnault, Phys. Rev. B 87, 205137 (2013)







- Different also from conventional multi-layer FQH systems

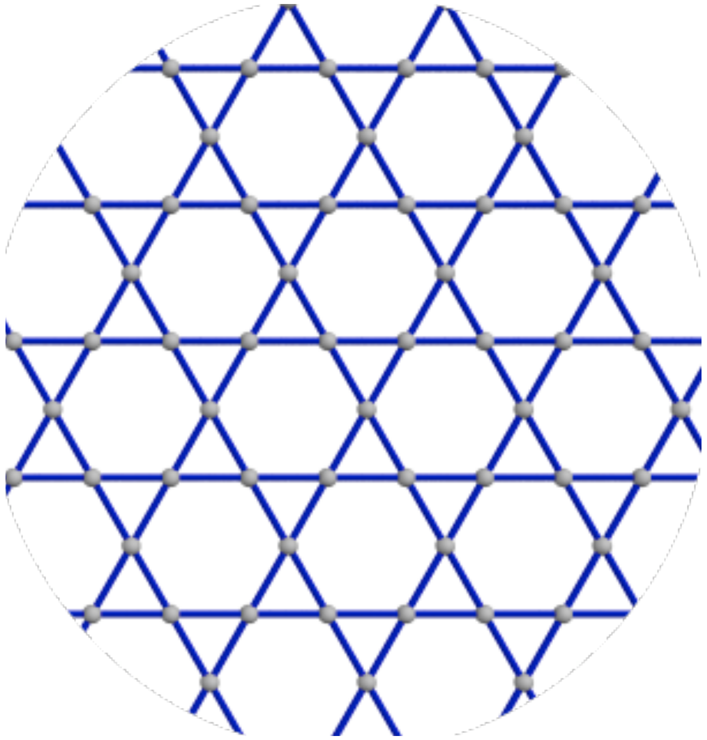


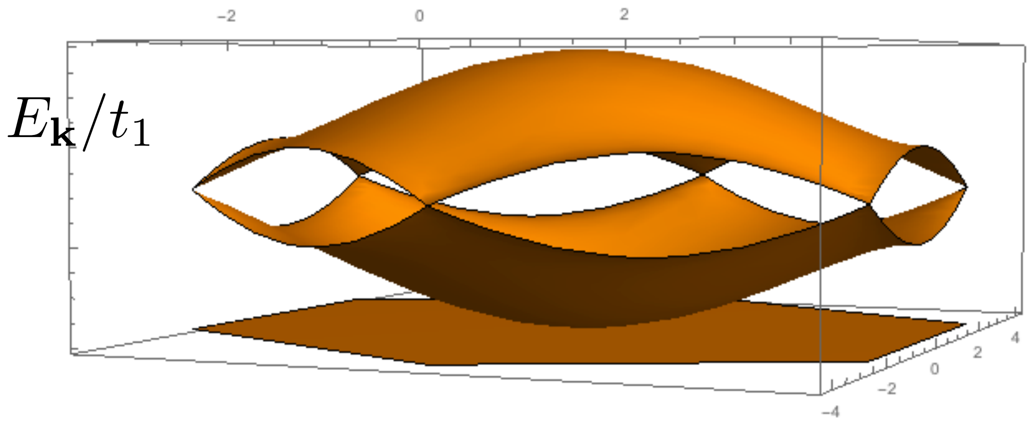
Yes!

Can we understand the microscopic structure of the C=N states?

- A brief interlude: Flat bands and localized modes on frustrated lattices
- Example: nearest neighbor hopping on a kagome lattice

$$H = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j$$

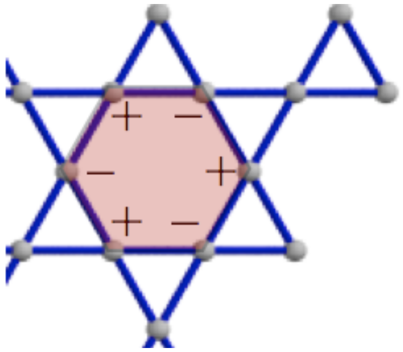




"Graphene + a flat band"

Localized modes explain the flat band

$$|\psi\rangle = \frac{1}{\sqrt{6}} \sum_{n \in \hexagon} (-1)^n |n\rangle$$



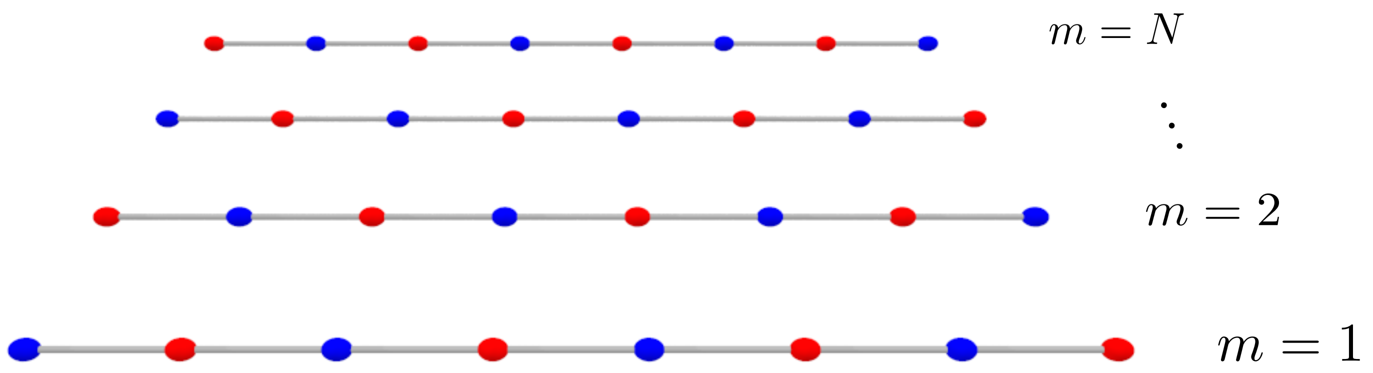
- But these states are neither topological nor Wannier functions!
  - Quadratic touching point
  - We need a refined concept that accommodates spin-orbit coupling…

Frustrated lattices with spin-orbit coupling

- Start by considering a single chain

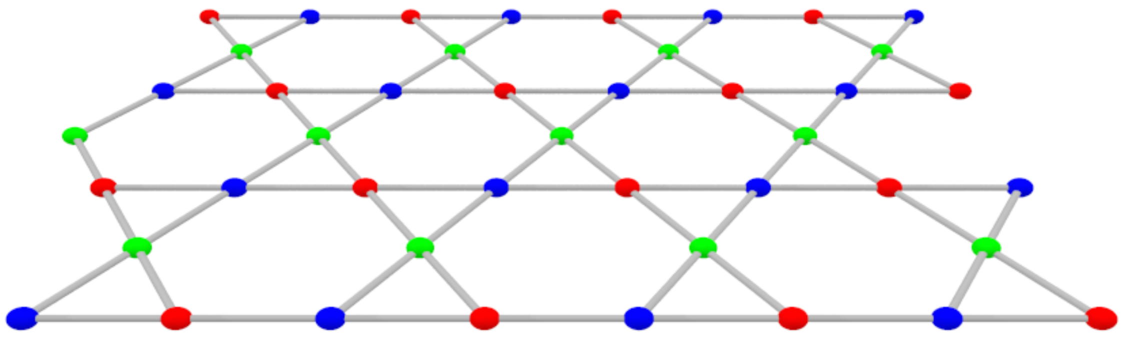
$$H(k_x) = \mathbf{d}(k_x) \cdot \sigma \qquad E_\pm(k_x) = \pm|\mathbf{d}(k_x)|$$

- Stack *N* identical chains



  - Look for eigenstates of the form

$$|\psi_\pm(k_x)\rangle = \sum_m \left(r_\pm(k_x)\right)^m |\phi_\pm(k_x)\rangle_m$$



  - **Local constraint**, zero total hopping amplitude to the green sites

$$\Rightarrow \quad r_\pm(k_x) = -\frac{\phi^1_\pm(k_x) + \phi^2_\pm(k_x)}{\phi^1_\pm(k_x) + e^{ik_x}\phi^2_\pm(k_x)}$$

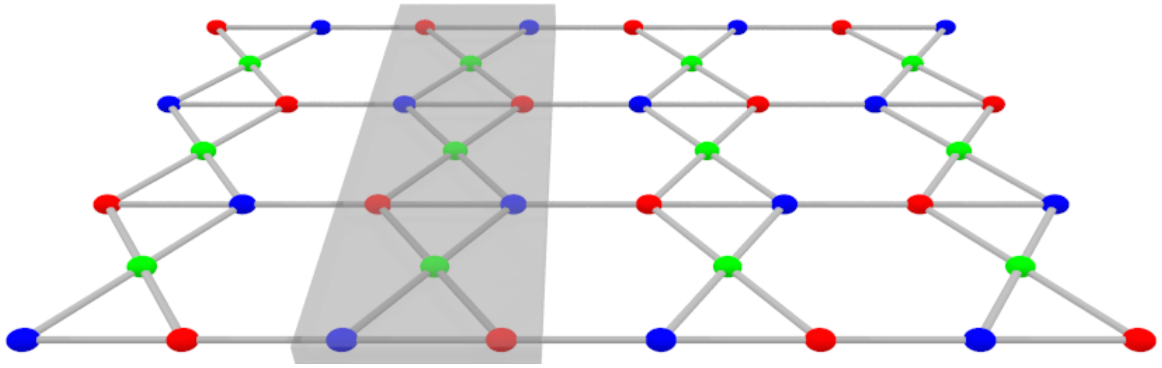
- A suitable gauge choice making the hopping to the intermediate (green sites) real always exists.

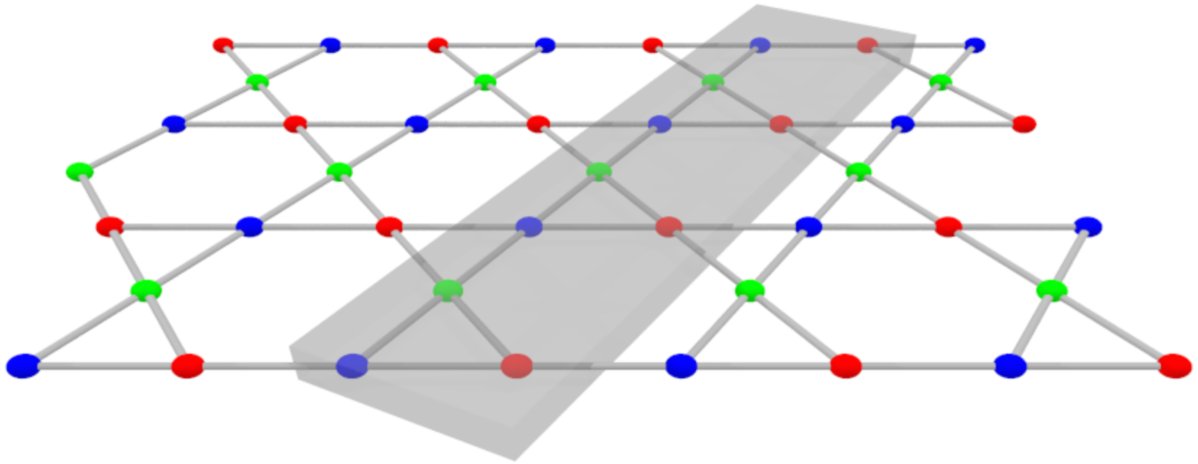
  - Completely generic, works for any single-chain Hamiltonian with spin-orbit coupling and in presence of magnetic fields

F. Kunst, M. Trescher and E.J. Bergholtz, in preparation

Exact expression for the topological edge states

- No spin-orbit coupling or magnetic fields $|r(k_x)| = 1$ (no edge state!)
- With spin-orbit coupling there are two cases:





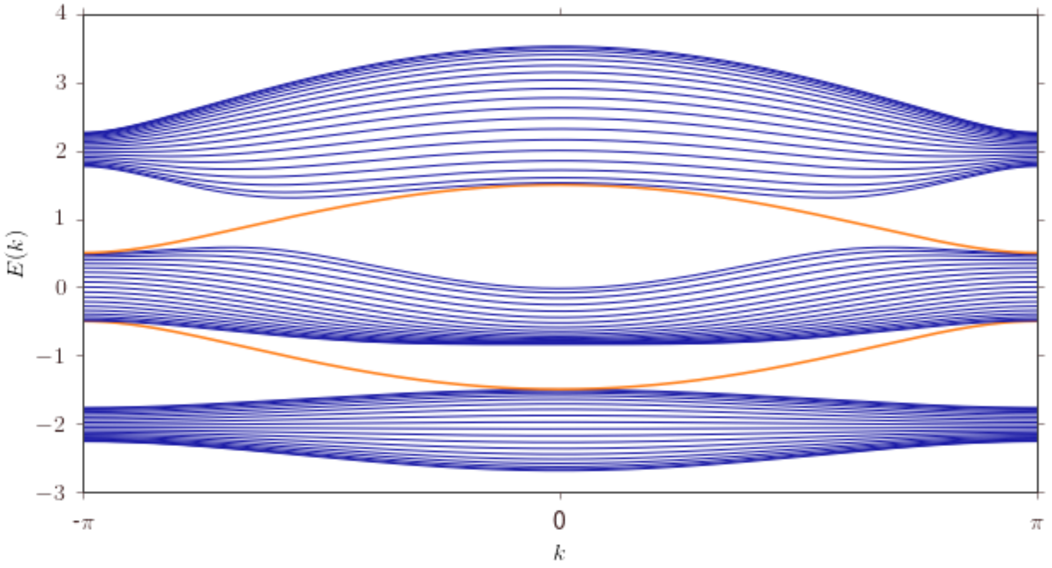
- Constraint within the unit cell

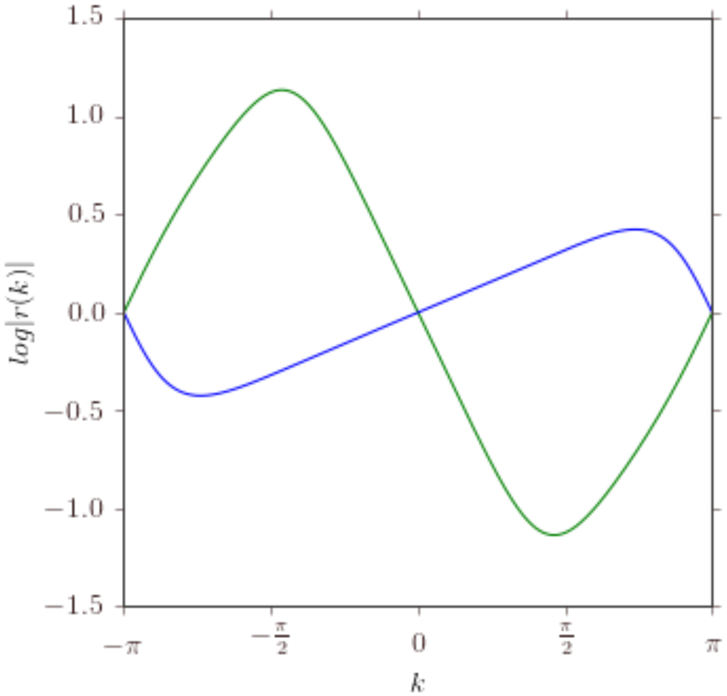
$|r(k_x)| = 1$ (no edge state!)

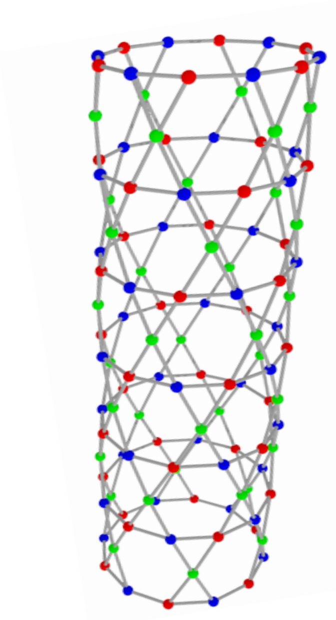
- The local constraint necessarily involves multiple unit cells

$|r(k_x)| \neq 1$!

- Cylinder spectra and edge localization



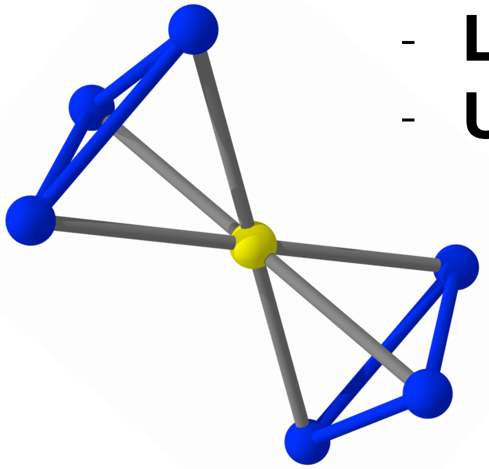




Back to Pyrochlore: localize in the third dimension

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

- Surface bands localized to the kagome layers iff the total hopping amplitude to the intermediate triangular layer vanish.

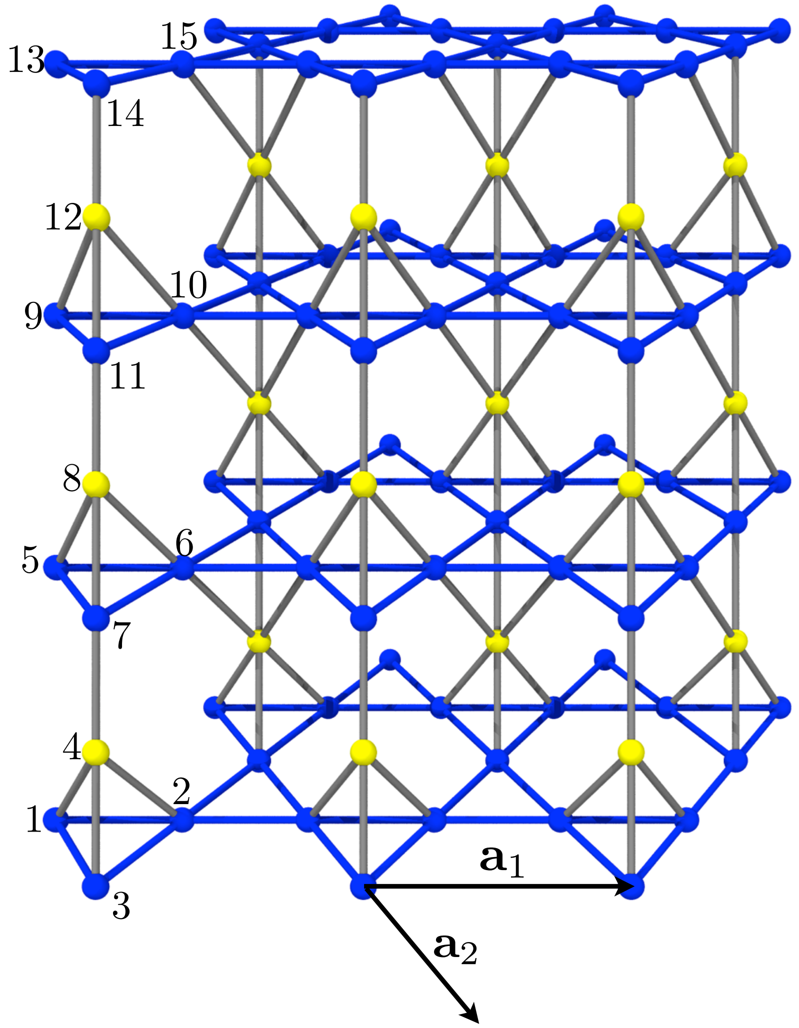


  - **Local constraint**, destructive interference
  - **Unique solution**, independent of details!

$$|\psi^i(\mathbf{k})\rangle = \mathcal{N}(\mathbf{k}) \sum_{m=1}^{N} \Big(r(\mathbf{k})\Big)^m |\phi^i(\mathbf{k})\rangle_m$$

$$r(\mathbf{k}) = -\frac{\phi_1^i(\mathbf{k})+\phi_2^i(\mathbf{k})+\phi_3^i(\mathbf{k})}{e^{-ik_2}\phi_1^i(\mathbf{k})+e^{i(k_1-k_2)}\phi_2^i(\mathbf{k})+\phi_3^i(\mathbf{k})}$$

components of the single-layer Bloch spinor



- Inherits the dispersion of the single layer model — *precisely what we need*!

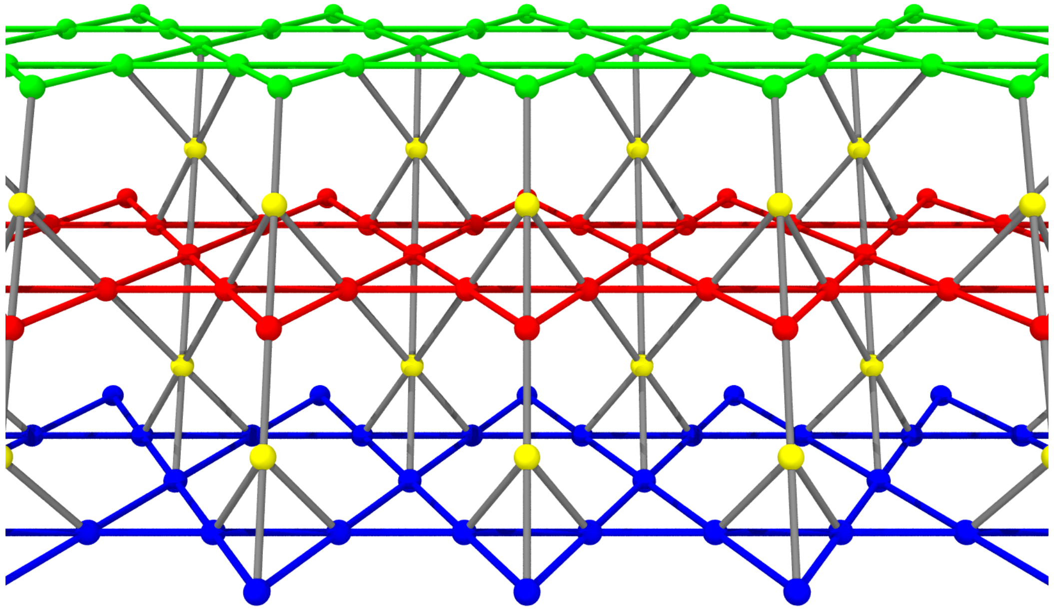
- Localized to top or bottom layer, depending on $|r(\mathbf{k})|$

- Reminiscent of Fermi arcs…..

Illuminating, in color…

M. Trescher and E.J. Bergholtz,
Phys. Rev. B 86, 241111(R) (2012)

$|r(\mathbf{k})| > 1$ state localized to the top



$|r(\mathbf{k})| < 1$ state localized to the bottom

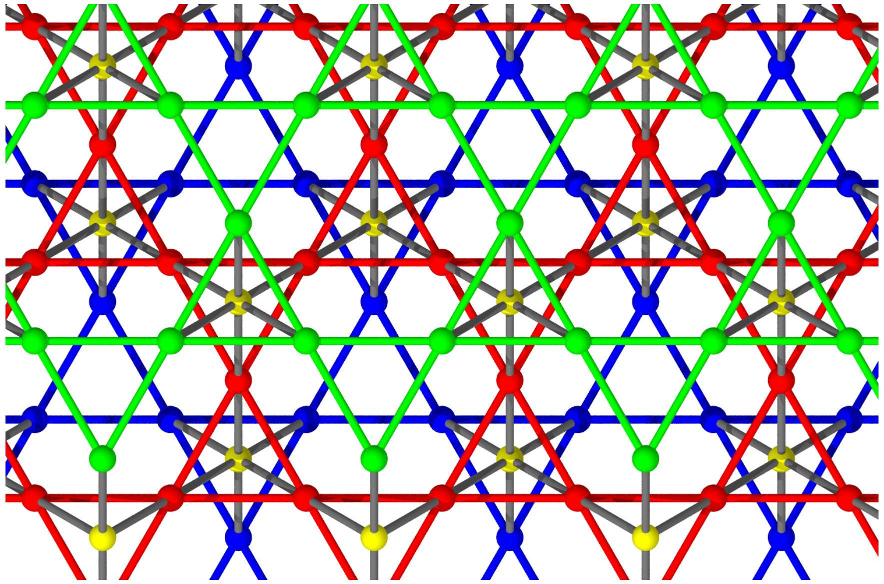
$|r(\mathbf{k})| = 1$ state delocalized!

$$\Psi(\mathbf{k}) = \mathcal{N}(\mathbf{k}) \begin{pmatrix} r^2(\mathbf{k})\phi_1(\mathbf{k}) \\ r^2(\mathbf{k})\phi_2(\mathbf{k}) \\ r^2(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ r(\mathbf{k})\phi_1(\mathbf{k}) \\ r(\mathbf{k})\phi_2(\mathbf{k}) \\ r(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ \phi_1(\mathbf{k}) \\ \phi_2(\mathbf{k}) \\ \phi_3(\mathbf{k}) \end{pmatrix}$$

- Non-trivial $r(\mathbf{k})$ due to the **twisted layer structure**

$$r(\mathbf{k}) = -\frac{\phi_1^i(\mathbf{k}) + \phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}{e^{-ik_2}\phi_1^i(\mathbf{k}) + e^{i(k_1-k_2)}\phi_2^i(\mathbf{k}) + \phi_3^i(\mathbf{k})}$$

- generalizes to many other *frustrated* lattices!



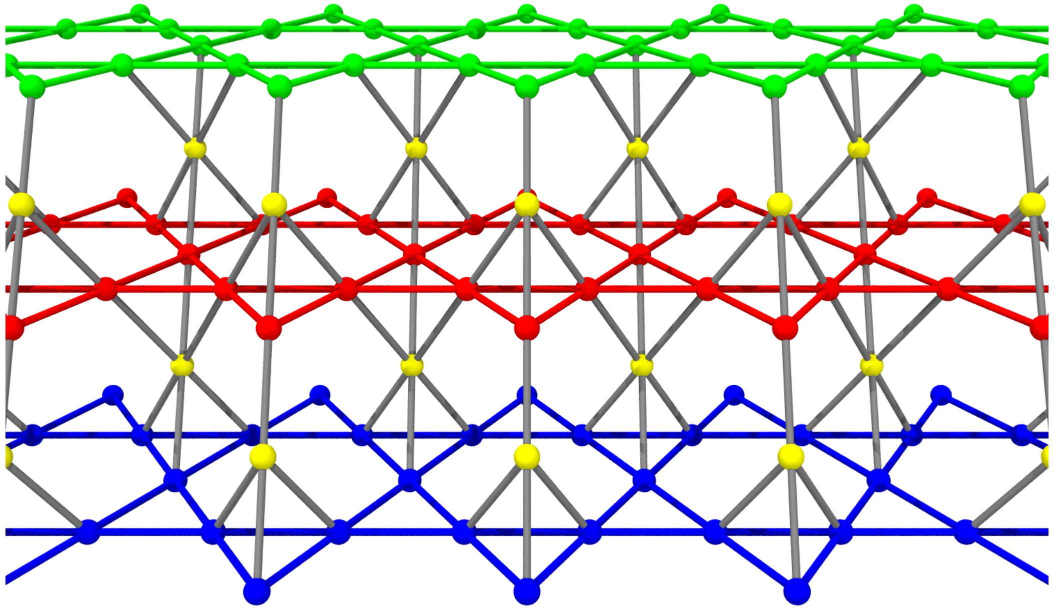
top view

Topological selection rule

- N bands, each with C=1, hybridize so that the surface band absorbs all the topology (C=N) while the others become trivial (C=0)

  - Simple way of generating (flat) bands with any Chern number

$$|\psi^i(\mathbf{k})\rangle = \mathcal{N}(\mathbf{k}) \sum_{m=1}^{N} \Big(r(\mathbf{k})\Big)^m |\phi^i(\mathbf{k})\rangle_m$$



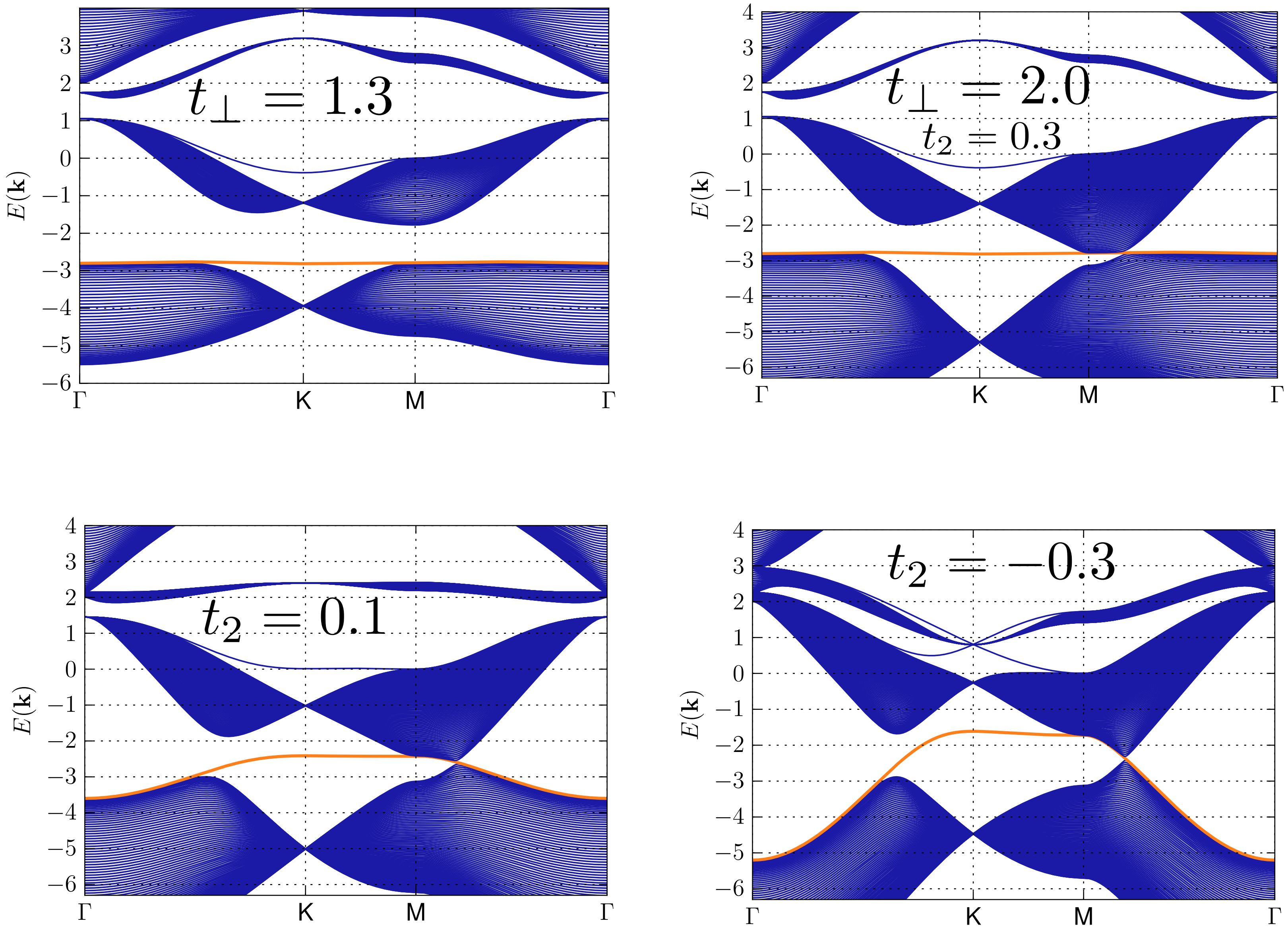
$$\Psi(\mathbf{k}) = \mathcal{N}(\mathbf{k}) \begin{pmatrix} r^2(\mathbf{k})\phi_1(\mathbf{k}) \\ r^2(\mathbf{k})\phi_2(\mathbf{k}) \\ r^2(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ r(\mathbf{k})\phi_1(\mathbf{k}) \\ r(\mathbf{k})\phi_2(\mathbf{k}) \\ r(\mathbf{k})\phi_3(\mathbf{k}) \\ 0 \\ \phi_1(\mathbf{k}) \\ \phi_2(\mathbf{k}) \\ \phi_3(\mathbf{k}) \end{pmatrix}$$

$$1 + 1 + 1 \rightarrow 3 + 0 + 0$$ etc.

What’s the connection to Weyl semimetals?

E.J. Bergholtz, Z. Liu, M. Trescher,
R. Moessner, and M. Udagawa,
Phys. Rev. Lett. 114, 016806 (2015)

- Another look at the bulk spectrum…
- Increase the interlayer tunnelling —> bulk phase transition with surface band unchanged!
- Change the nearest neighbor hopping (no change in topology)



- Band touching described by a *tilted* Weyl Hamiltonian

$$H_{\mathrm{Weyl}} = \sum_{i,j} v_{ij} k_i \sigma_j + E_0(\mathbf{k})\mathbb{1}$$

- Nb. this holds in each case, also when the touching cone is nearly flat, or even “over-tilted”

Fermi arcs in the pyrochlore slab

E.J. Bergholtz, Z. Liu, M. Trescher,
R. Moessner, and M. Udagawa,
Phys. Rev. Lett. 114, 016806 (2015)

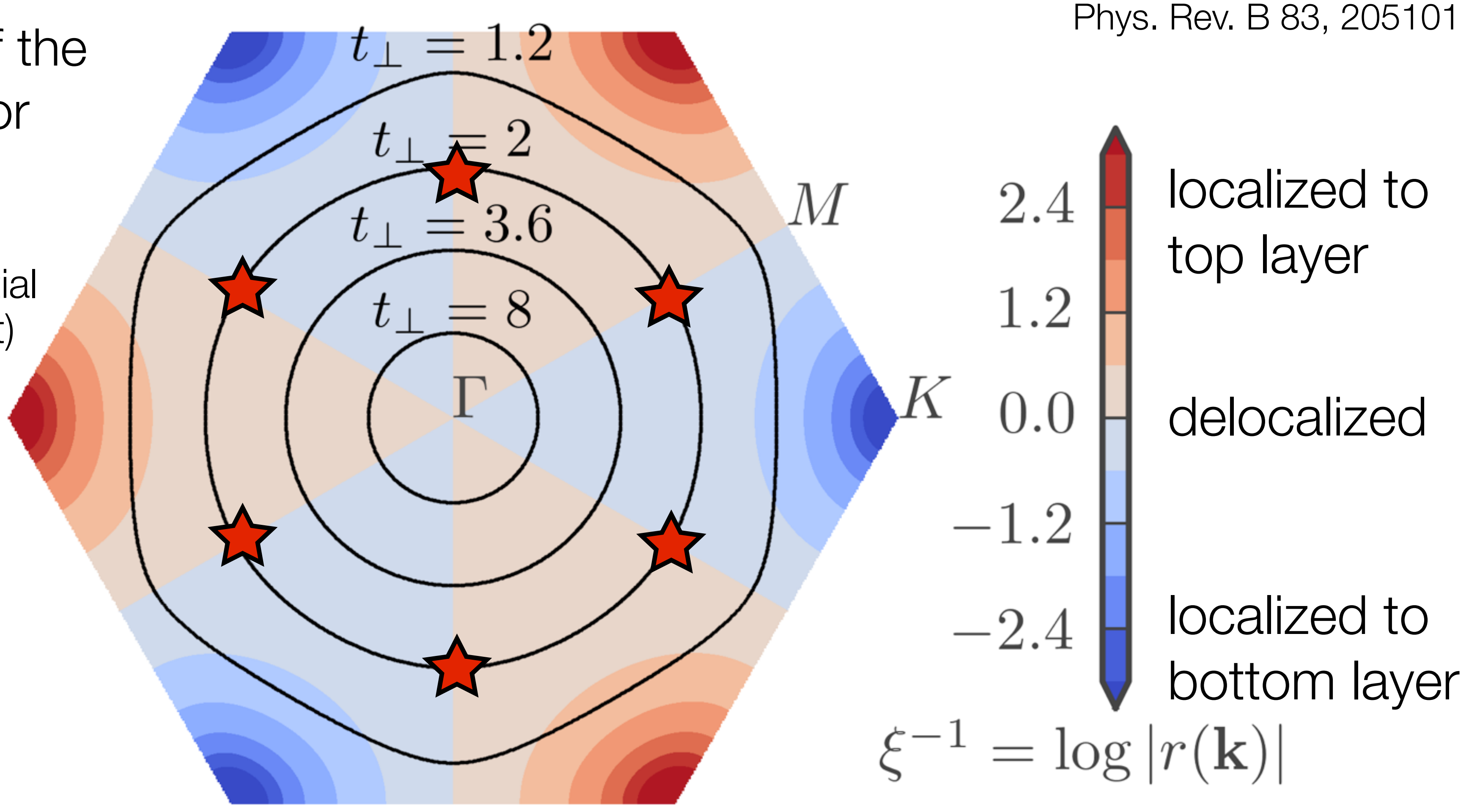
- Constant energy lines, “Fermi circles”, are split into Fermi arcs

X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B 83, 205101 (2011).

★ Projections of the Weyl points for

$$t_\perp = 2$$

(chemical potential at the Weyl point)

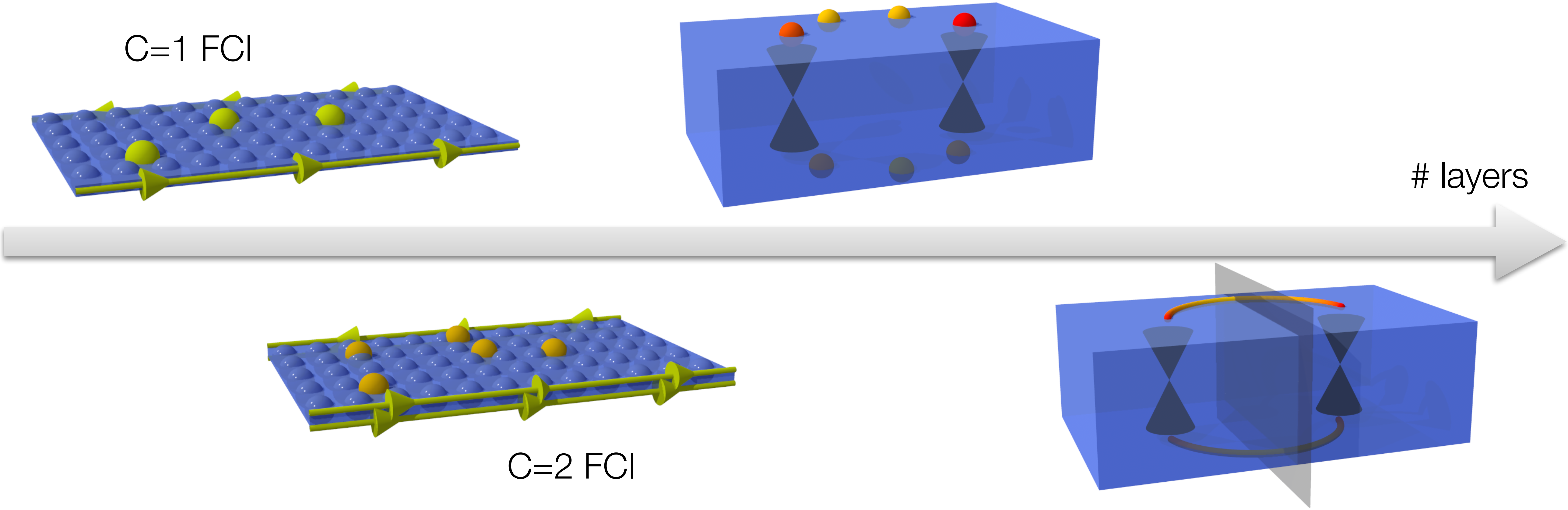


- Here we have an exact solutions for the Fermi arcs, and seen as a family, they carry a huge Chern number.
- The Fermi arcs also exist in absence of Weyl nodes in the bulk!

2D -> 3D with strong interactions

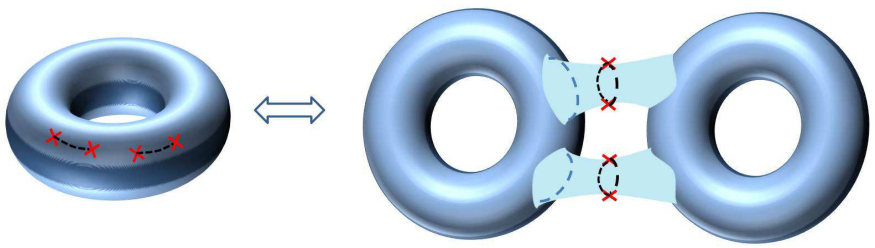
E.J. Bergholtz, Z. Liu, M. Trescher, R. Moessner, and M. Udagawa,
Phys. Rev. Lett. 114, 016806 (2015)

- Intriguing dimensional crossover



(Tilted) Weyl semimetal or layered Chern insulator in the large C limit

- Generic absence of FCIs in the 3D limit
- New type of fractionalization in the C>1 FCIs?



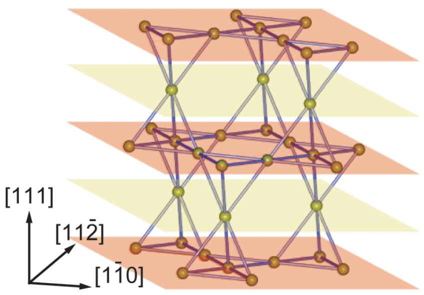
+ ??

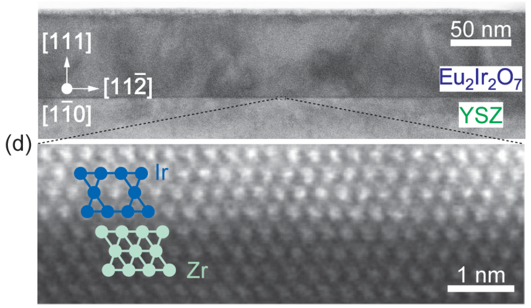
M. Barkeshli and X.-L. Qi,
Phys. Rev. X 2, 031013 (2012)

First experiments

- Very clean (111) slabs of $Eu_2Ir_2O_7$ recently grown!

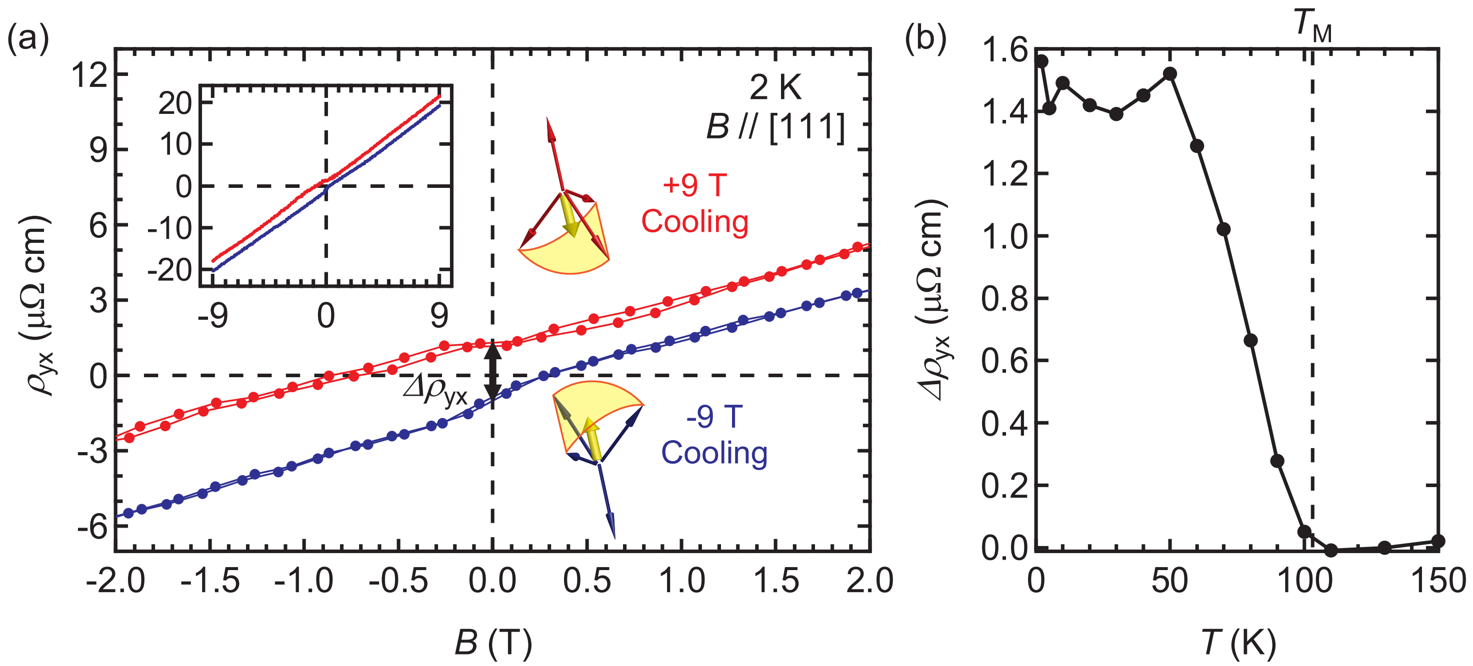
Fujita et. al., arXiv:1508.01318





- Spontaneously time-reversal and shows a sizeable Hall effect at zero B-field!

- Effect survives to high temperatures



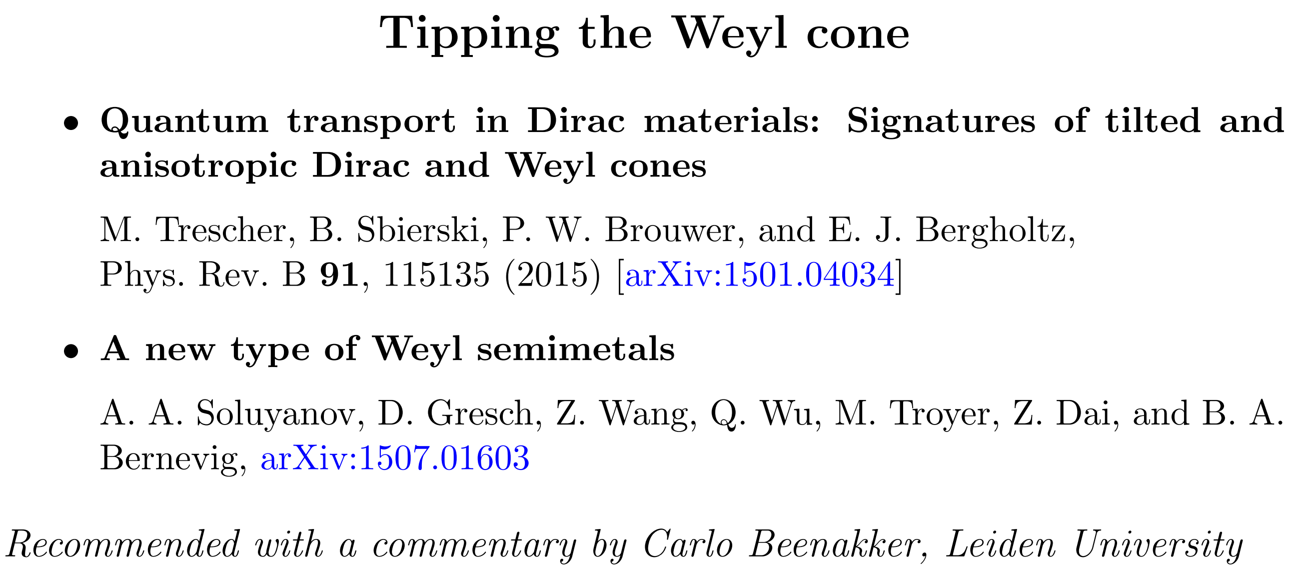
- Many open questions….

Transport and tilted Weyl cones

- *Tilts* of the Weyl cones are forbidden by Lorentz invariance

  - but tilt is generic in Weyl semimetals

  - and has striking consequences in transport!



Tipping the Weyl cone

- **Quantum transport in Dirac materials: Signatures of tilted and anisotropic Dirac and Weyl cones**

  M. Trescher, B. Sbierski, P. W. Brouwer, and E. J. Bergholtz,
  Phys. Rev. B **91**, 115135 (2015) [arXiv:1501.04034]

- **A new type of Weyl semimetals**

  A. A. Soluyanov, D. Gresch, Z. Wang, Q. Wu, M. Troyer, Z. Dai, and B. A. Bernevig, arXiv:1507.01603

Recommended with a commentary by Carlo Beenakker, Leiden University

- *Suggestion:* probe the controversial disorder induced phase transition by tilting the Weyl cones

  - this could be done by applying strain or mechanical pressure!

Pseudoballistic semimetal

Diffusive metal

K_c

K (disorder strength)

tilt

See also
B. Sbierski, E.J Bergholtz and P.W. Brouwer,
Phys. Rev. B 92, 115145 (2015)

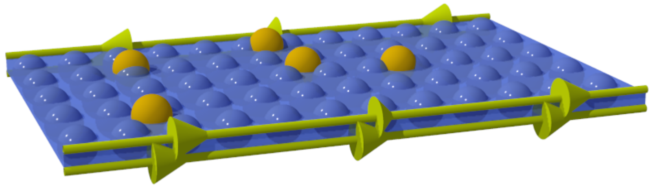
Conclusions

- Frustration & topology combine well

  - Microscopic insight
  - **Exact solutions for topological surface bands**
  - Fermi arcs “for free”
  - **Topological selection rule**
  - Fermi arcs also in absence of Weyl nodes
  - **New topologically ordered states in C>1 bands**
  - Interaction induced topological order in the Fermi arc surface bands of thin Weyl semimetal slabs

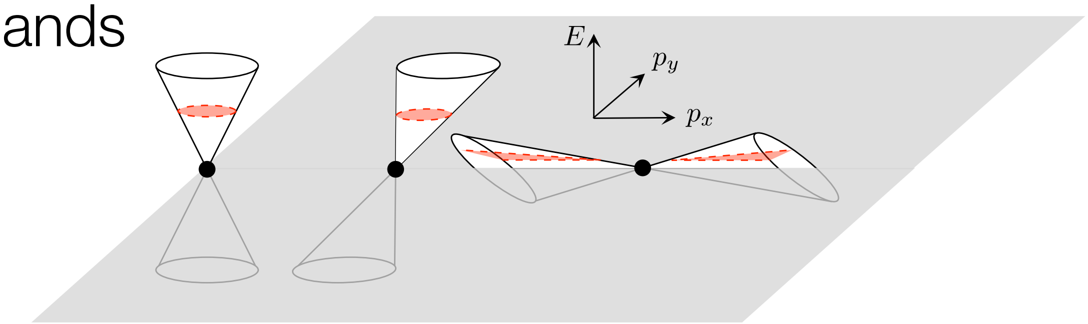
$$|\psi^i(\mathbf{k})\rangle = \mathcal{N}(\mathbf{k}) \sum_{m=1}^{N} \Big(r(\mathbf{k})\Big)^m |\phi^i(\mathbf{k})\rangle_m$$

$$1 + 1 + 1 \rightarrow 3 + 0 + 0 \quad \text{etc.}$$



- Less symmetry gives richer physics!

  - Interaction induced gapless states in flat Chern bands
  - C>1 phenomena
  - **Tilted Weyl cones**
  - Novel disorder induced criticality



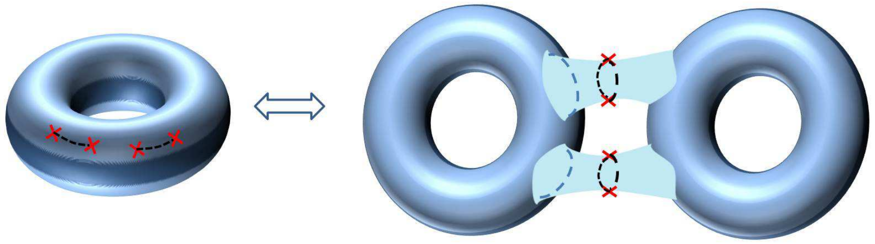
Outlook

- Experiments!

    - Several groups are presently studying thin [111] slabs of pyrochlore iridates

- “Second generation” of fractionalization in C>1 FCIs — phenomenology essentially unexplored — how about proximity effects?

- Dislocations as non-Abelian wormholes? Microscopic picture?



M. Barkeshli and X.-L. Qi,
Phys. Rev. X 2, 031013 (2012)

- Tilted Weyl cones:

    - Possible relevance for “titanic magnetoresistance” in WTe2
    - Gravitational analogues, Hawking radiation?

- Higher Chern number generalizations of Weyl cones: transport, defects, …

- Frustrated layer construction in other dimensions and symmetry classes