Anyonic/FQH-Interferometry
the current status

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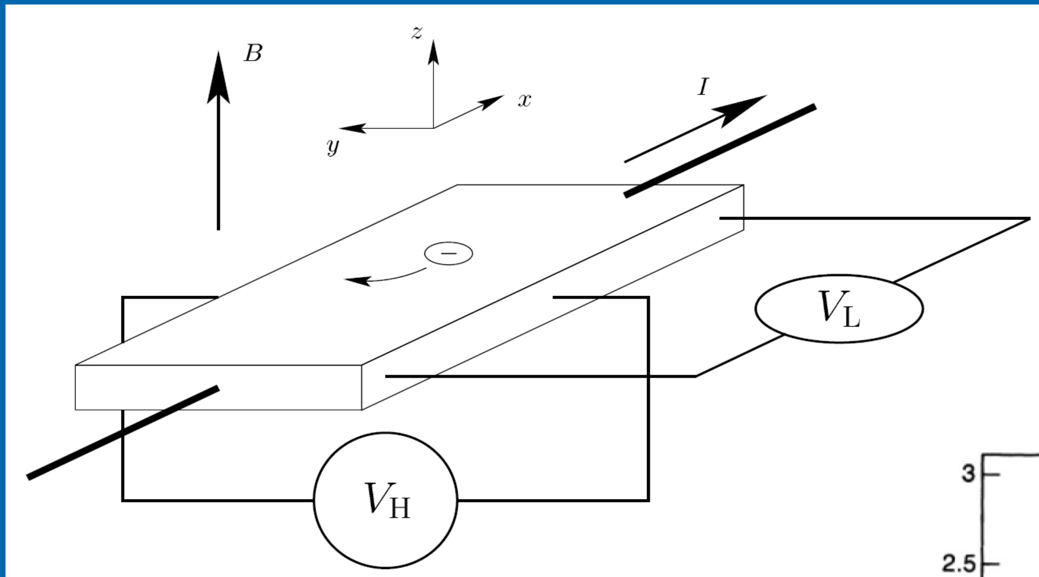
Maynooth, September 2009

Mostly not my work, but:
partly based on joint work with
Parsa Bonderson, Kirill Shtengel, Waheb Bishara, Chetan Nayak

- Bishara, Bonderson, Nayak, Shtengel, JKS, arXiv:0903.3108 (PRB)
- Bonderson, Shtengel, JKS. Ann. Phys. 323:2709-2755 (2008)
- Bonderson, Shtengel, JKS. PRL 98, 070401 (2007)
- Bonderson, Shtengel, JKS. PRL 97, 016401 (2006)
- Bonderson, Kitaev, Shtengel. PRL 96, 016803 (2006)

The Quantum Hall Effect



B ~ 10 Tesla
T ~ 10 mK

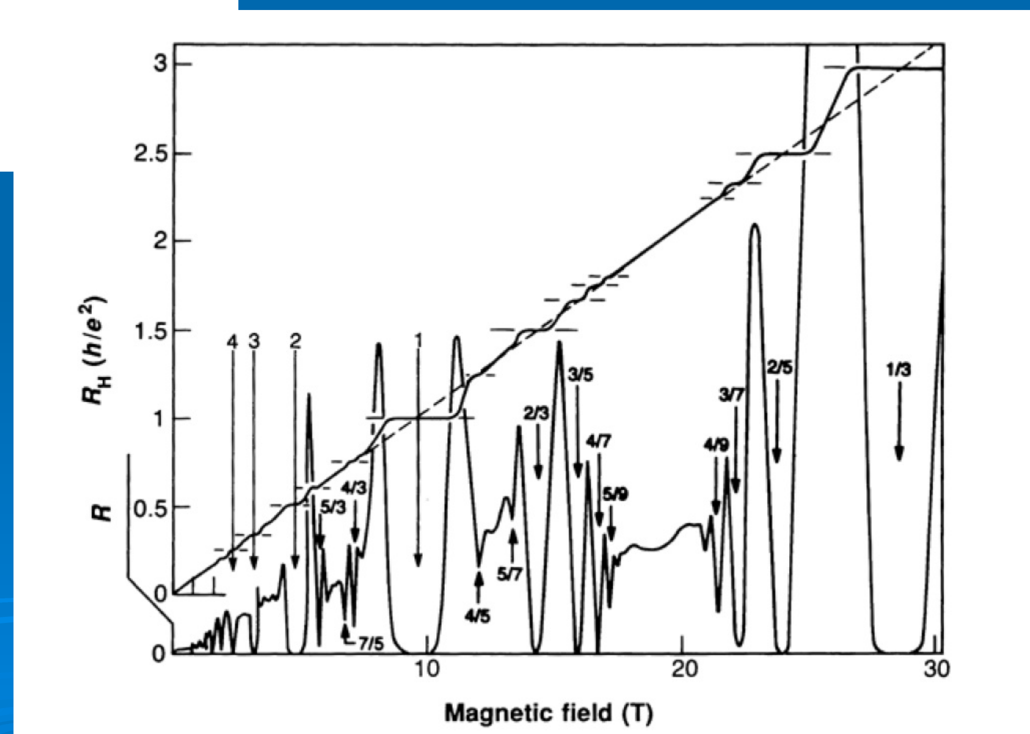
On the Plateaus:

- Incompressible electron liquids
- Off-diagonal conductance:

values $\nu \frac{e^2}{h}$ Filling fraction $\nu = \frac{p}{q}$

- Vortices with fractional charge
- +AB-effect: fractional statistics

(Abelian) **ANYONS!**



Eisenstein, Stormer, Science 248, 1990

Some more quantum Hall background follows... (4/5 slides)

The one particle problem (notation and some scales)

$$H = \frac{1}{2m}\left((p_x - \frac{e}{c}A_x)^2 + (p_y - \frac{e}{c}A_y)^2\right)$$

$$\mathbf{B} = (0, 0, B), \qquad \mathbf{A} = (-\frac{B}{2}y, \frac{B}{2}x, 0).$$

Note:
we are ignoring
- Disorder (plateaus...)
- Interactions (fractions...)
- Spin (assume polarized...)
- Finite size (for now)

Introduce dimensionless complex coordinates (units of magnetic length)

$$z = (x + iy)/\ell$$ with $$\ell = \sqrt{\frac{\hbar c}{eB}} \approx 10nm$$

Then Hamiltonian, angular momentum become

$$\begin{aligned} H &= \frac{1}{2}\hbar\omega_c(-4\partial_z\partial_{\bar{z}} - z\partial_z + \bar{z}\partial_{\bar{z}} + \frac{1}{4}z\bar{z}) \\ L &= \hbar(z\partial_z - \bar{z}\partial_{\bar{z}}) \end{aligned}$$

$$\omega_c = eB/mc, \ \hbar\omega_c \approx 100K$$

- H is 'similar' to a harmonic oscillator
- L counts powers
- cyclotron frequency comes out naturally

note: fractional plateaus appear at T of order 1K

Landau levels

$$a = -\partial_{\bar{z}} - \frac{z}{4} \quad a^\dagger = \partial_z - \frac{\bar{z}}{4}$$

$$b = -\partial_z - \frac{\bar{z}}{4} \quad b^\dagger = \partial_{\bar{z}} - \frac{z}{4},$$

Solve the 1-particle problem algebraically...

$$[H, a^\dagger] = \hbar\omega_c a^\dagger \qquad [H, b^\dagger] = 0$$

This gives

$$\psi_{m,n}(z) = (\partial_{\bar{z}} - \frac{z}{4})^m (\partial_z - \frac{\bar{z}}{4})^n e^{-z\bar{z}/4} = e^{z\bar{z}/4} \partial_{\bar{z}}^m \partial_z^n e^{-z\bar{z}/2}$$

$$E_n = \hbar\omega_c (n + \frac{1}{2})$$

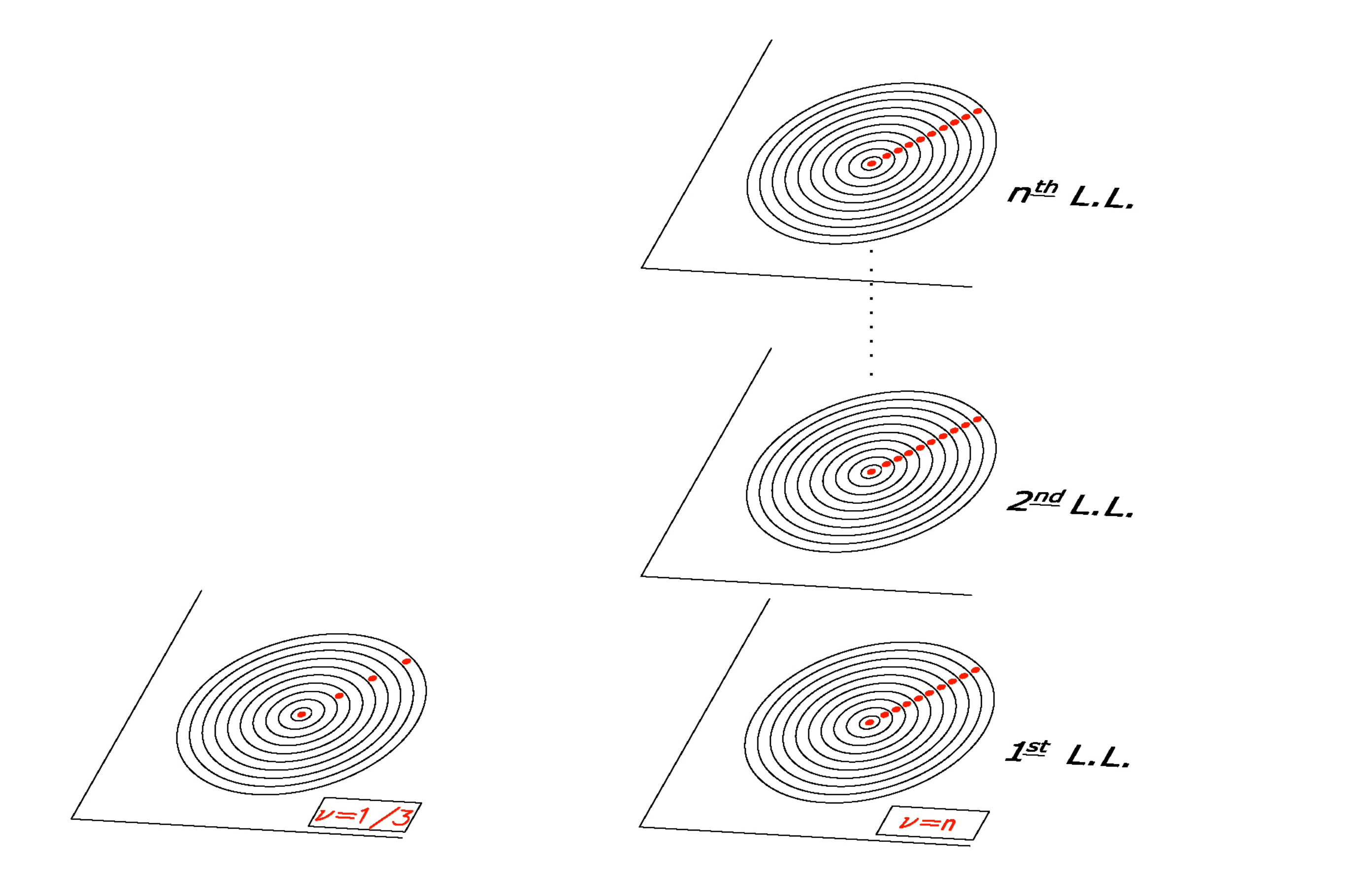
Independent of m, so infinitely degenerate.

With finite surface area A have Landau level degeneracy $\frac{eBA}{hc} = N_\Phi$

Now can define the filling fraction $\nu = \frac{N_e}{N_\Phi}$

Note: lowest LL wave functions are holomorphic (polynomial) times gaussian

Landau levels and filling fractions
(stolen from Ivan Rodriguez)



Laughlin's 'variational' wave function

Want variational ansatz for ground state wave functions on the plateaus

Reasonable/Necessary requirements:

- Lowest LL approximation, i.e. holomorphic function times exponential
- Antisymmetry (electrons are fermions)
- Polynomial part is homogeneous (eigenstate of total angular momentum)

Need to put in interaction (repulsion), try Jastrow form:

$$\Psi(z_1, \ldots, z_N) = \prod_{i<j} f(z_i - z_j)$$

This eliminates all continuous parameters!

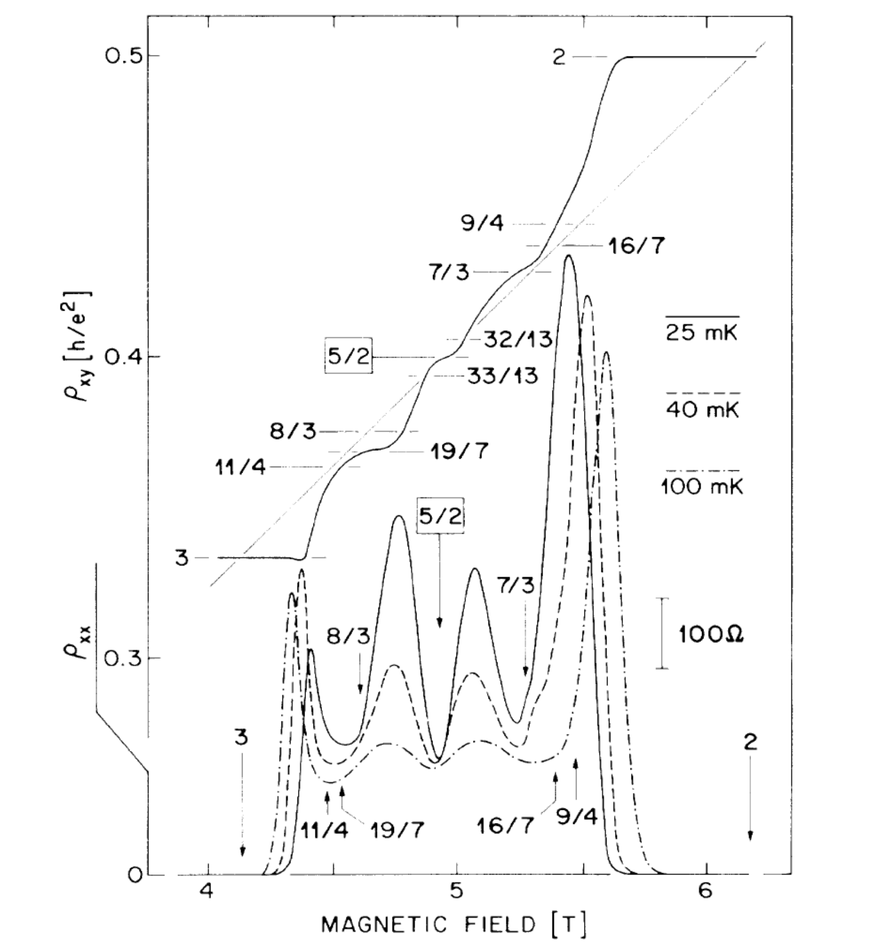
Result 'predicts' filling fractions 1, 1/3, 1/5, 1/7, ... (power counting)

$$\Psi_N^m(z_1, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^{2m+1} \, e^{-\left(\frac{1}{4}\sum_i z_i \bar{z}_i\right)}$$

Can insert fractionally charged quasiholes by piercing the sample with extra flux quanta.

$$\Psi_{\nu=\frac{1}{3}} = \prod_{i<j}^{N_e} (z_i - z_j)^3 \prod_{k,l}^{N_e, N_h} (z_k - w_l) \prod_{m<n}^{N_h} (w_m - w_n)^{1/3} e^{-\sum_p \frac{|z_p|^2}{4}}$$

An Unusual Hall Effect



Willett et al. PRL 59, 1776, 1987

Filling fraction 5/2: even denominator!

Now believed to have

- electrons paired in ground state (exotic p-wave 'superconductor')
- halved flux quantum
- charge e/4 quasiholes (vortices) which are **Non-Abelian Anyons**

(exchanges implement non-commuting unitaries)

Moore, Read, Nucl. Phys. B360, 362, 1991

Can use braiding interaction for Topological Quantum Computation

(not universal for 5/2 state, but see later)

Some interesting papers

(a small and unfair selection, papers up to some time in 2006)

- **Proposals for Hall States with non-Abelian anyons**

  Moore, Read, Nucl. Phys. B360, 1990 (trial wave functions from CFT, filling 5/2, not universal)
  Read, Rezayi, PRB 59, 1999, cond-mat/9809384 (filling 12/5, universal for QC, clustered)
  Ardonne, Schoutens, PRL 82, 1999, cond-mat/9811352 (filling 4/7, universal, paired)
  Others: Wen, Ludwig, van Lankvelt,…

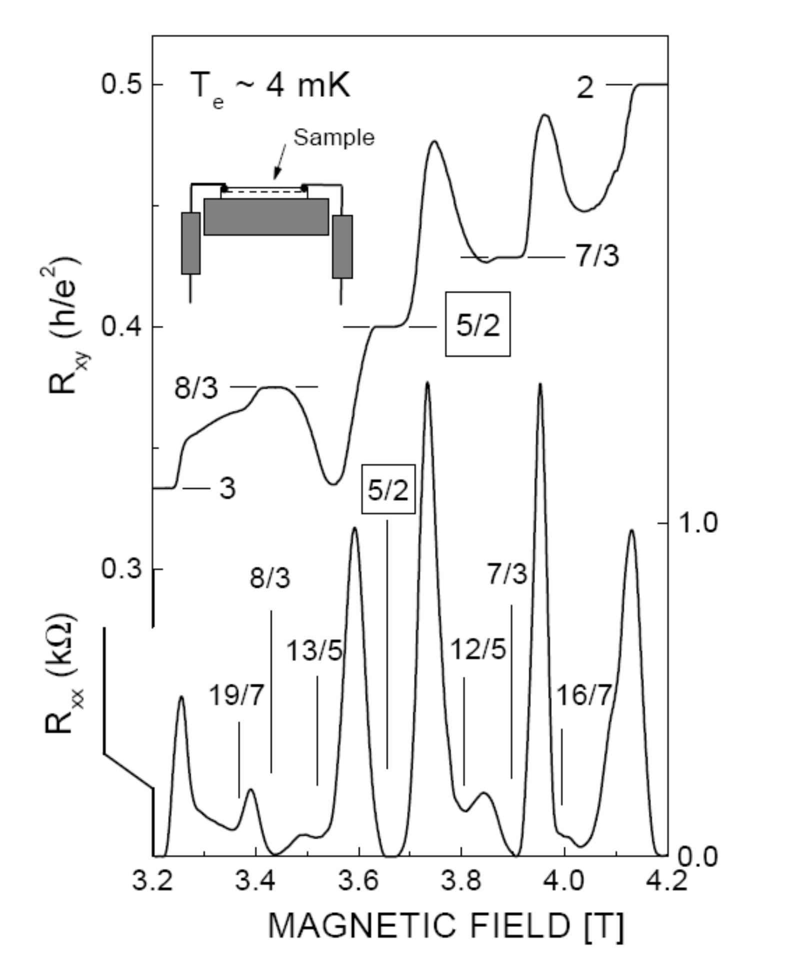
- **Work on Braiding interaction in these states**

  Nayak, Wilczek, Nucl. Phys. B479, 529, 1996 (filling 5/2, n-quasihole braiding, from CFT)
  JKS, Bais, Nucl. Phys. B612, 2001, cond-mat/0104035 (filling 12/5, algebraic framework/Qgroups)
  Ardonne, Schoutens cond-mat/0606217 (filling 4/7),
  Freedman, Larsen, Wang, Commun. Math. Phys., 227+228, 2002 (universality)

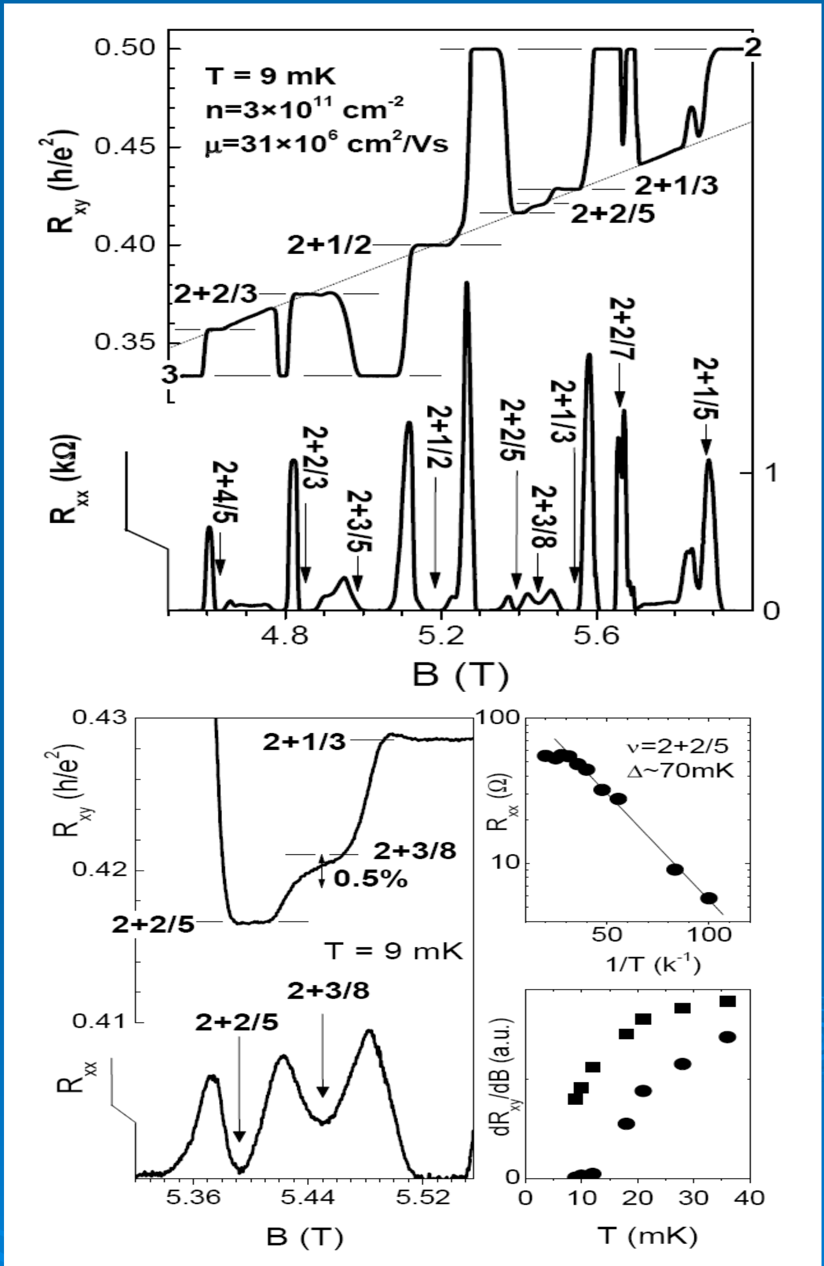
- **Non-Abelian Interferometry papers**

  Fradkin, Nayak, Tsvelik, Wilczek, Nucl. Phys. B516, 1998, cond-mat/9711087 (idea, filling 5/2)
  Overbosch, Bais, Phys. Rev. A64, 2001, quant-ph/0105015 (importance of setup, decoherence)
  Das Sarma, Freedman, Nayak, PRL 94, 2005, cond-mat/0412343 (+bit +NOT, filling 5/2)
  Stern, Halperin, PRL 96, 2006, cond-mat/0508447 (filling 5/2)
  Bonderson, Kitaev, Shtengel, PRL 96, 2006, cond-mat/0508616 (filling 5/2)
  Bonderson, Shtengel, JKS, PRL 97, 2006 (all fillings, role of S-matrix)
  Bonderson, Shtengel, JKS, quan-ph/0608119 (decoherence of anyonic charge)
  Also: Hou-Chamon, Chung-Stone, Kitaev-Feldman (2x), all 2006

Experimental Progress

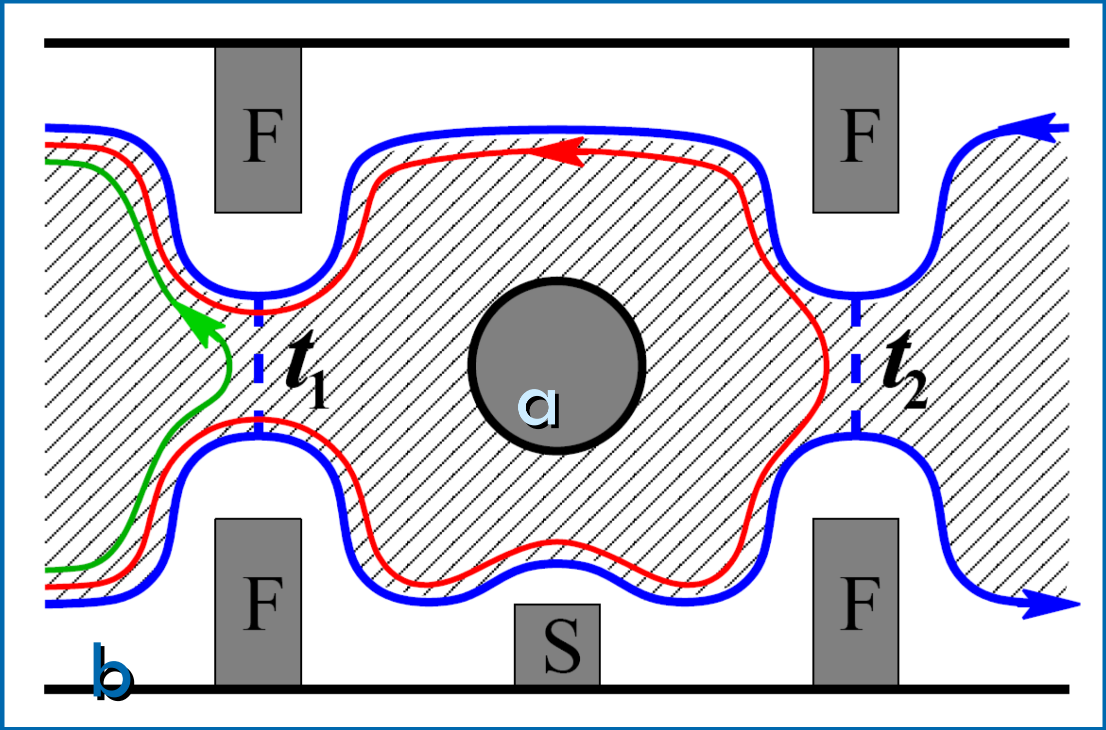


Pan et al. PRL 83, 1999
Gap at 5/2 is 0.11 K



Xia et al. PRL 93, 2004,
Gap at 5/2 is 0.5 K, at 12/5: 0.07 K

Quantum Hall Interferometry



Note: current flows along the edge, except at tunneling contacts. We get

$$\begin{aligned}\sigma_{xx} &\propto |t_1|^2 + |t_2|^2 + 2\mathrm{Re}\left\{t_1^* t_2 \langle \Psi_{ab}| U_1^{-1} U_2 |\Psi_{ab}\rangle\right\} \\ &= |t_1|^2 + |t_2|^2 + 2\,|t_1 t_2|\,|M_{ab}| \cos\left(\beta + \theta_{ab}\right)\ .\end{aligned}$$

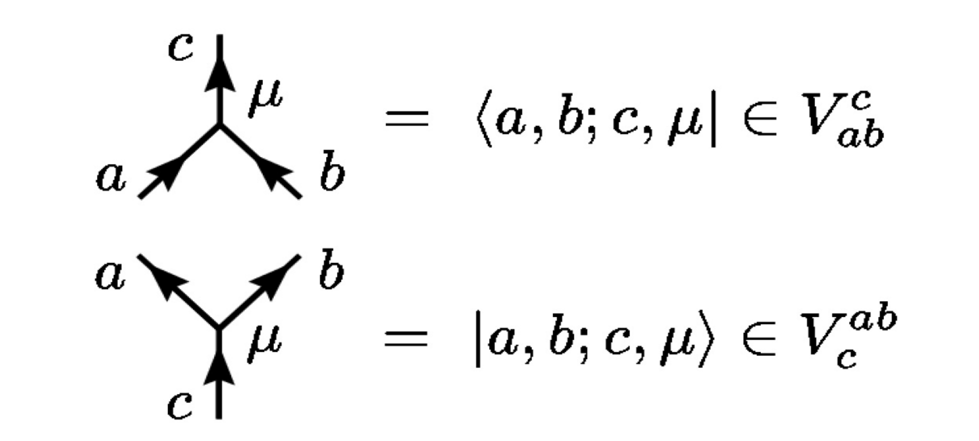
Interference suppressed by |M|: effect from non-Abelian braiding!

(This should actually be easier to observe than the phase shift from Abelian braiding…)

Graphical calculus for Anyonic interferometry

or: where does the M-matrix come from?

Fusion vs. Splitting histories correspond to states, bra vs. ket.
can build up multiparticle states, inner products, operators (“computations”) etc.



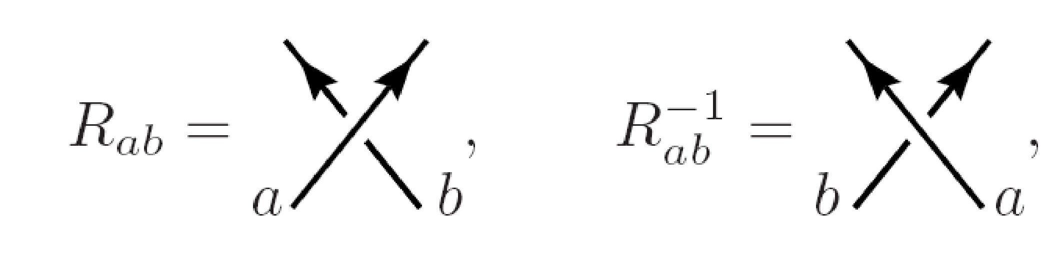
Dimensions of these spaces:

$$N^{ab}_{c}$$

Fusion rules:

$$a \times b = \sum_c N^{ab}_{c}\, c$$

Braiding, R-matrix

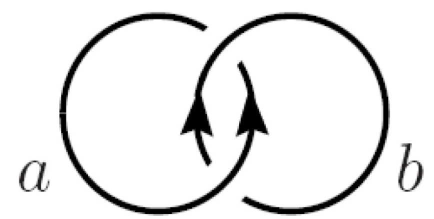


S-matrix and M-matrix

Interferometer superimposes over- and undercrossings.
Topological Interference term proportional to:

Closely related to Verlinde S-matrix:
- Well known for most CFTs/TQFTs (can do all proposed Hall states)
- Determines fusion rules, in fact, almost determines the anyon model completely

$$S_{ab} = \frac{1}{D}$$



Normalized monodromy matrix important for interferometry: $M_{ab} = \frac{S_{ab} S_{11}}{S_{a1} S_{b1}}$

Note $|M_{ab}| \leq 1$ and $M_{ab} = 1$ signals trivial monodromy

$\nu = 5/2$ topological order is believed to be $\mathrm{MR} = \mathrm{U}(1)_4 \times \mathrm{Ising}$

U(1) is an Abelian factor due to electric charge (Aharonov - Bohm)

Ising particle types: I, σ, ψ

Fusion rules: $\sigma \times \sigma = I + \psi, \quad \sigma \times \psi = \sigma, \quad \psi \times \psi = I$

Monodromy: $$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Note especially $\mathrm{M}_{\sigma\sigma} = 0$......interference totally suppressed!

quasiholes carry anyonic charge: $(e/4, \sigma)$

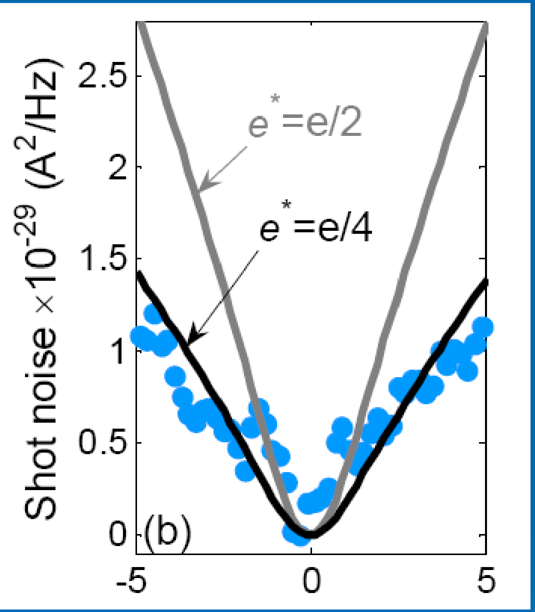
electrons carry anyonic charge: $(-e, \psi)$

n quasiholes carry anyonic charge: $(ne/4, \sigma)$ for n odd

$(ne/4, I \text{ or } \psi)$ for n even

2008: Charge e/4

(2009: charge x/4?)



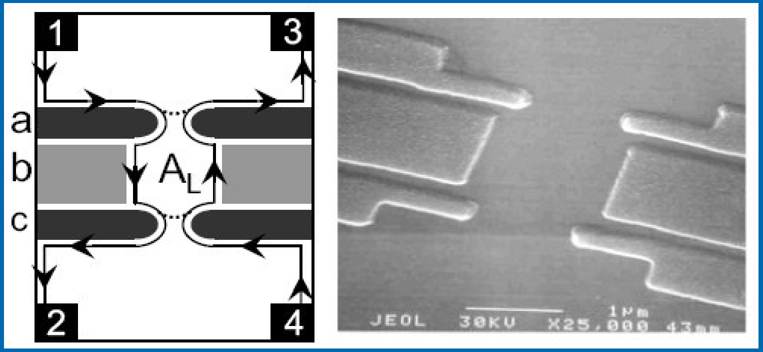
Noise: Dolev et al., Nature 452, 829 (2008) (LEFT)

$$S^{i}(0)_{T=0} = 2e^{*}V\Delta g_{i}\, t_{\nu_i-\nu_{i-1}}(1-t_{\nu_i-\nu_{i-1}})$$

Also, Tunneling: Radu et al., Science 320, 899 (2008)
(was charge x/4 from the start...)

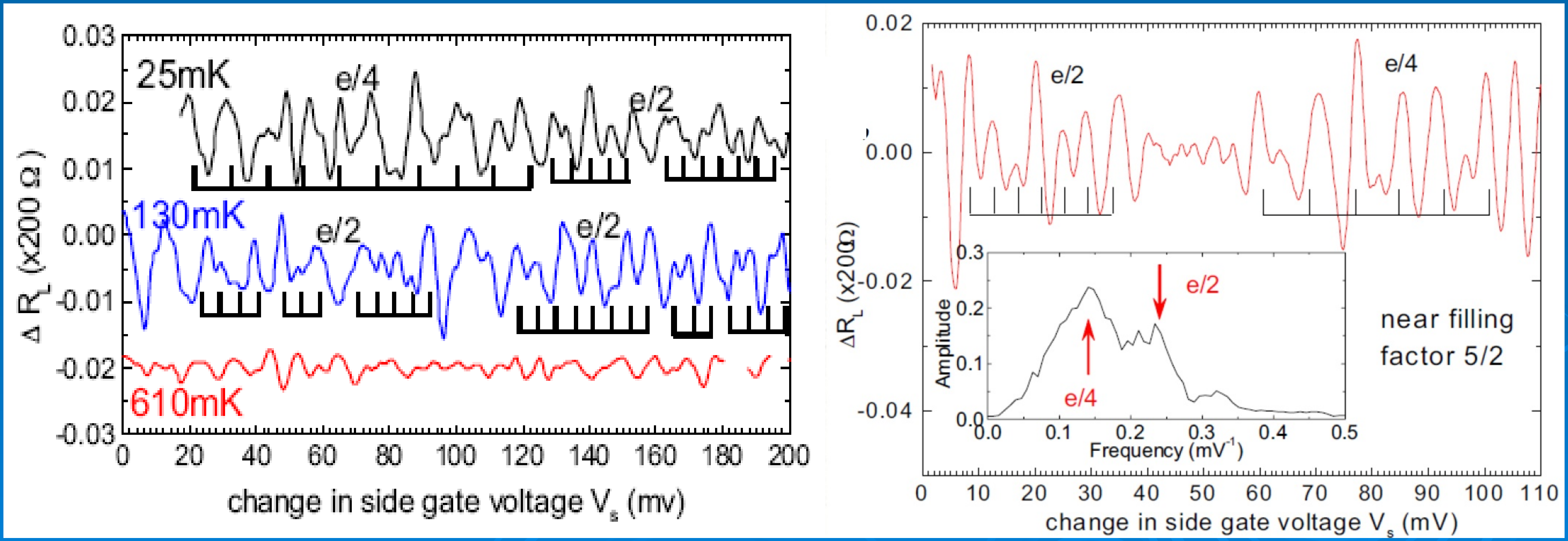
Of course charge e/4 does not prove non-Abelian statistics....

2009: Willett's Wiggles



Willett et al. arXiv:0807.0221, PNAS 2009

With some good will, see
- e/2 and e/4 charges tunneling at low T
- e/2 only at intermediate T
- nothing at "high" T (no good will necessary)



What does it all mean?
(hopefully...)

Remember the interference term: $2\,|t_1 t_2|\,|M_{ab}|\cos\left(\beta + \theta_{ab}\right)$

Naïve idea of the experiment: vary $\beta=(q_t/e)\,(\Phi_{dot}/\Phi_0)$ (in effect, just Φ_{dot})

This should give
- cosine with period ~ e/4 when there is an even number of σ-s in the interferometer
- no interference when there is an odd number of σ-s (since $M_{\sigma\sigma}=0$)

Complication 1 (actually, Feature) :
If we vary β enough, we might shrink/grow the interferometric loop enough to exclude/include An extra (pinned) σ. Then the behavior changes between the alternatives above (on/off interference).

Complication 2:
The e/2 quasiparticle may tunnel in addition to the e/4 quasiparticle
This would explain the half-period oscillations in the “off” regions,
(the e/2 interference does not switch off)

Complication 3:
The contributions of e/2 and e/4 tunneling scale differently with temperature and device size.
This could explain that only e/2 is seen at intermediate T, and also that e/2 is seen at all.
For this must calculate t_1, t_2 - use CFT rather than TQFT.

Single and double point contacts

Single point contact, or non-oscillatory part of current in double point contact

$$I_b^{(qp)} \propto \begin{cases} T^{2g-2}\, V & \text{for small } eV \ll k_B T \\ V^{2g-1} & \text{for small } eV \gg k_B T \end{cases}$$

$$g = g_c + g_n$$

This goes into t_1, t_2

Double point contact: coherence factor for oscillatory term (“thermal smearing”)

$$I_{12}^{(qp)} \propto e^{-T/T^*(L)} = e^{-L/L_\phi(T)}$$

This is not visible in TQFT, so far ignored (CFT gives it)

$$L_\phi(T) = \frac{1}{2\pi T}\left(\frac{g_c}{v_c} + \frac{g_n}{v_n}\right)^{-1}$$

$$T^*(L) = \frac{1}{2\pi L}\left(\frac{g_c}{v_c} + \frac{g_n}{v_n}\right)^{-1}$$

Neutral and charge velocities appear

Note: $v_c >> v_n$

So g_n determines the coherence length/temperature (need small g_n)

W. Bishara and C. Nayak, Physical Review B **77**, 165302 (2008)

Also: Ardonne/Kim, Fidkowski

Scaling exponents for various filling 5/2 candidate states

$\nu = \frac{5}{2}$	e^*	n-A?	θ	g_c	g_n	g
MR:	$e/4$	yes	$e^{i\pi/4}$	$1/8$	$1/8$	$1/4$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
$\overline{\text{Pf}}$:	$e/4$	yes	$e^{-i\pi/4}$	$1/8$	$3/8$	$1/2$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
$\text{SU}(2)_2$:	$e/4$	yes	$e^{i\pi/2}$	$1/8$	$3/8$	$1/2$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
K=8:	$e/4$	no	$e^{i\pi/8}$	$1/8$	0	$1/8$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$
(3,3,1):	$e/4$	no	$e^{i3\pi/8}$	$1/8$	$1/4$	$3/8$
	$e/2$	no	$e^{i\pi/2}$	$1/2$	0	$1/2$

Note 1: e/2 is always relevant for tunneling ($g<1$), but usually disfavoured
Note 2: e/2 has $g_n=0$; could dominate at "high" T or with "large" devices

X. Wan, Z.-X. Hu, E. H. Rezayi, and K. Yang, PRB 77, 165316 (2008), arXiv:0712.2095.

Estimated coherence lengths/temperatures

$e/4$	MR	$\overline{\text{Pf}}/SU(2)_2$	K=8	(3,3,1)	e/2
L_ϕ in μm	1.4	0.5	19	0.7	4.8
T^* in mK	36	13	484	19	121

Used numerically obtained values for edge velocities

Lengths for the experimentally relevant temperature of 25 mK
Temperatures for the experimentally relevant size of 1μm

The e/2 Lengths/temperatures are the same for all candidate descriptions

Notes:
- Persistence of e/2 oscillations at higher T fits well with T*
- Sample size also seems consistent with significant e/2 contribution

Discussion/Conclusions?

- Data is not conclusive (but promising) – no real conclusions
- Extra checks are needed (varying B, checking for phase slips etc.)
- Other possible explanations discussed, (and dismissed) in recent PRB
- Many theoretical questions remain...
- How about 7/3 and 8/3 ? (or more wishfully, 12/5 ?)