Correlated Phases of Atomic Bose Gases on a Rotating Lattice

Gunnar Möller
T.C.M. Group, Cavendish Laboratory,
University of Cambridge

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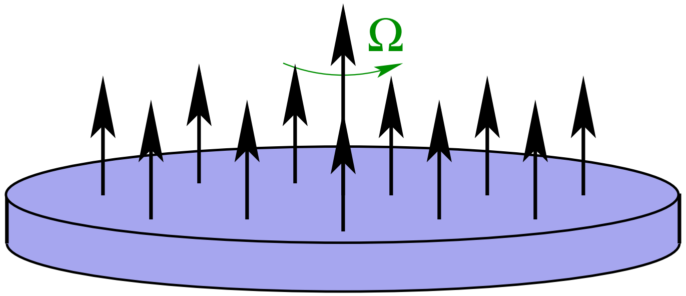
Overview

- Rotating Atomic Bose Gases: Continuum vs Lattice
- Condensed Phases
- Novel FQH States
- Summary

Rotating Atomic Bose Gases: Continuum

Rotation frequency, Ω



density of quantized vortices $$n_{\rm v} = \frac{2M\Omega}{h}$$

Rapidly rotating gas is characterized by the "filling factor"

$$\nu \equiv \frac{n_{\rm 2d}}{n_{\rm v}}$$

Strong correlation regime: $\nu \leq \nu_{\rm c} \simeq 6$

$\Rightarrow$Bosonic versions of fractional quantum Hall states, conventional (Laughlin, composite fermion) and exotic ("non-abelian") + ...? [Many refs...]

The challenge: the interaction scale at $\nu \sim 1$ is small $\sim \frac{\hbar^2 a_{\rm s}}{M} n_{\rm 3d} \sim \frac{a_{\rm s}}{a_\parallel} \hbar\omega_\perp$.

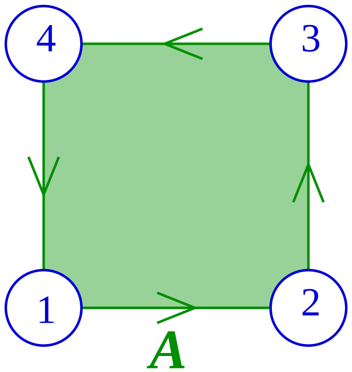
Rotating Atomic Gases: Lattice

Bose-Hubbard model with "magnetic field" (square lattice)

$$H = -J \sum_{\langle i,j \rangle} \left[\hat{a}_i^\dagger \hat{a}_j e^{iA_{ij}} + h.c. \right] + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

Particle density, n

Vortex density, n_v



$\sum_{\mathrm{plaquette}} A_{ij} = 2\pi n_\mathrm{v}$

$(0 \leq n_\mathrm{v} \leq 1)$

Time-dependent modulation of tunneling/site energies [Jaksch & Zoller, NJP **5**, 56 (2003); Mueller, PRA **70**, 041603 (2004); Sørensen, Demler & Lukin, PRL **94**, 086803 (2005)]

Rotating lattice [Tung, Schweikhard, Cornell, PRL **97**, 240402 (2006); Hafezi, Sørensen, Demler & Lukin, PRA 76, 023613 (2007)]

$n, n_\mathrm{v} \ll 1 \Rightarrow$continuum limit [Sørensen, Demler & Lukin, PRL **94**, 086803 (2005); Palmer & Jaksch, PRL 96, 180407 (2006), PRA 78, 013609 (2008); Hafezi, Sørensen, Demler & Lukin, PRA 76, 023613 (2007)]

What are the new features/phases on the lattices?

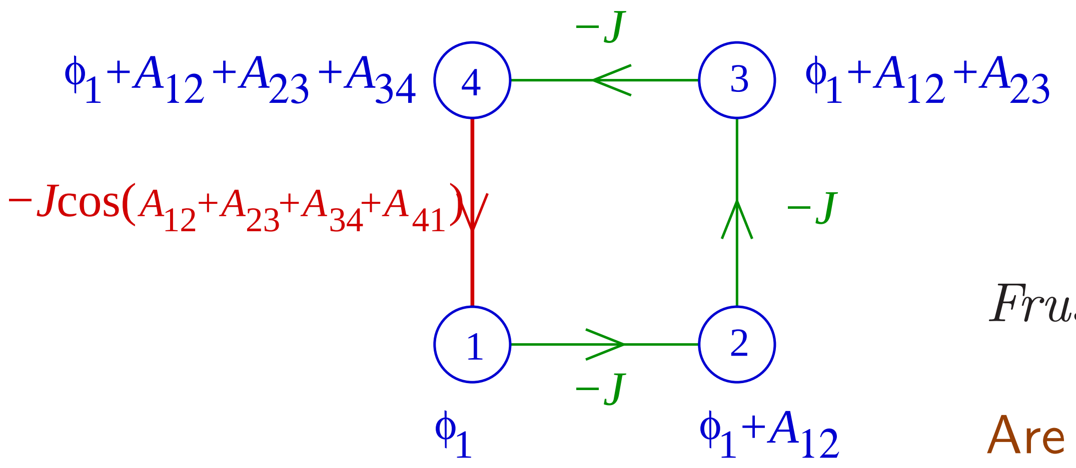
Hard-core limit, $U \gg J$

Spin-1/2 system: $\hat{s}_i^z = \hat{n}_i - \frac{1}{2}$, $\hat{s}_i^+ = \hat{a}^\dagger$, $\hat{s}_i^- = \hat{a}$

$$H = -J \sum_{\langle i,j \rangle} \left[\hat{s}_i^+ \hat{s}_j^- e^{iA_{ij}} + h.c. \right] - \mu \sum_i \hat{s}_i^z + \text{const.}$$

Mean-field theory: $\vec{s} = S(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

$H = -JS^2 \sum_{\langle i,j \rangle} \sin\theta_i \sin\theta_j \cos(\phi_i - \phi_j + A_{ij}) - \mu S \sum_i \cos\theta_i$



Frustrated quantum spin system.

Are there any "spin-liquid" phases?

Overview of Results

Exact-diagonalization studies reveal:

(I) "Fully-frustrated" limit, $n_\mathrm{v} = \frac{1}{2}$:

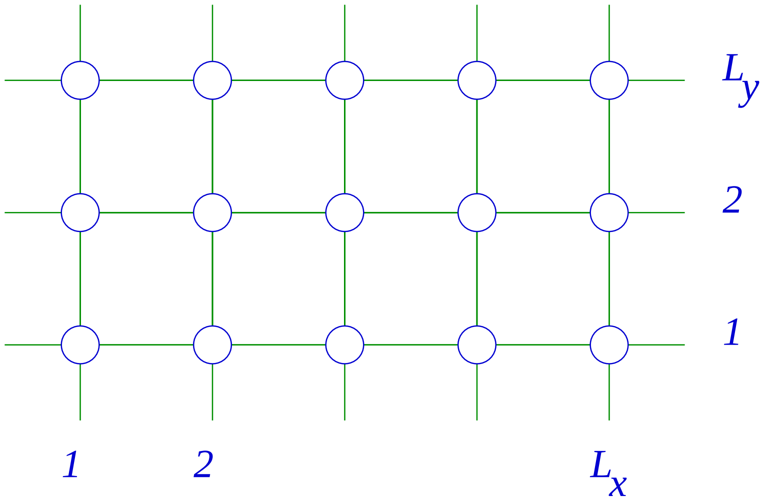
– groundstate is a BEC (vortex lattice) for all n;
– ordered magnetic groundstate.

(II) Novel strongly-correlated states at certain (n, n_v).

– Fractional quantum Hall states that exist only on the lattice;
– "spin-liquid" phases.

Numerical Methods

$L_x \times L_y$ square lattice, with periodic boundary conditions.



Number of particles, $N = nL_xL_y$
Number of vortices, $N_\mathrm{v} = n_\mathrm{v}L_xL_y$

Translational symmetry ⇒energy eigenstates characterized by a conserved momentum $\boldsymbol{K}$.

Single-particle density matrix of the groundstate(s)

$$\rho_{ij} = \langle\Psi_0|\hat{a}_i^\dagger \hat{a}_j|\Psi_0\rangle$$

Simple BEC ⇒one eigenvalue of order N.

(I) Condensed states at $n_{\mathrm{v}} = \frac{1}{2}$

$n = 1/4, n_{\mathrm{v}} = 1/2$ ($N = 2, 4, 6, 8$)

- *Two* quasi-degenerate groundstates, e.g. $N = 4$

$K_0 = (0,0), K_1 = (1,1)$, with: $(E_1 - E_0) \sim 0.15(E_2 - E_0)$

- Each has *two* large eigenvalues of the single particle density matrix.

e.g., $N = 8$:
$$\rho_{ij}^{(0)} \Rightarrow 2.309 \times 2, 0.416 \times 4, 0.283 \times 2 \ldots$$
$$\rho_{ij}^{(1)} \Rightarrow 2.617 \times 2, 0.290 \times 4, 0.192 \times 2 \ldots$$

"Fragmented" BEC?

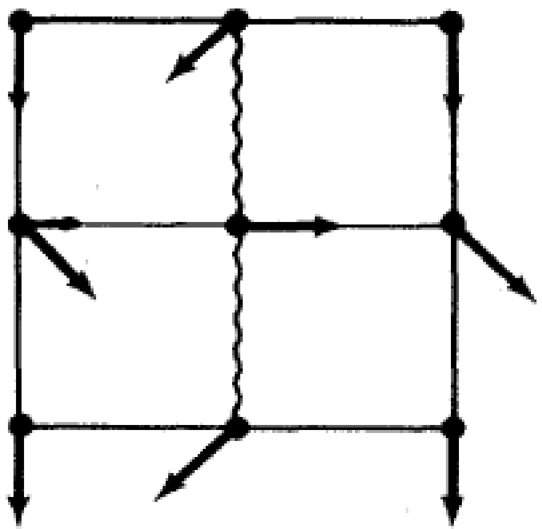
Translational symmetry breaking

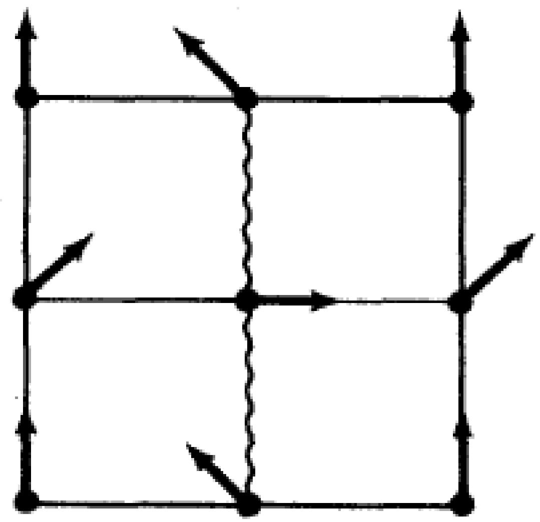
$$|\Psi\rangle = \alpha|\Psi_0\rangle + \beta|\Psi_1\rangle$$

Single-particle density matrix has a *one* large eigenvalue.

⇒simple BEC with translational symmetry breaking.

Two degenerate condensate wavefunctions:





Consistent with mean field theory.

MFT for Condensed States

For hardcore bosons, a suitable mean-field-state can be parametrized

$$|\Psi_{\rm mft}\rangle = \prod_j \left[\sin(\theta_j/2) + \cos(\theta_j/2) e^{i\chi_j} a_j^\dagger\right] |0\rangle$$

with particle number conservation $N = \sum_j \cos(\theta_j/2)^2$

For this Ansatz minimise $\text{K.E.} = -J/2 \sum_{<i,j>} \sin\theta_i \sin\theta_j \cos(\chi_i - \chi_j - A_{ij})$

$n = 1/4, n_{\rm v} = 1/2$

Projected onto fixed particle number:

e.g., $N = 4$: $|\langle \Psi_{\rm mft} | \Psi \rangle|^2 = 0.83041844$

$N = 6$: $|\langle \Psi_{\rm mft} | \Psi \rangle|^2 = 0.65978873$

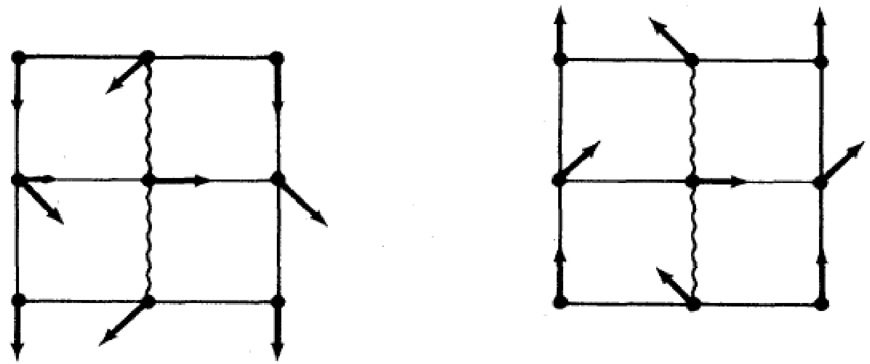
$n = 1/2, n_{\rm v} = 1/2$ [$N = 8$, $(L_x, L_y) = (4, 4)$]

- Again, *two* quasi-degenerate groundstates, at $K_0 = (0,0), K_1 = (1,1)$

e.g. $N = 8$: $(E_1 - E_0) \sim 0.11(E_2 - E_0)$

- Again, each has *two* large eigenvalues of the single particle density matrix.

- MF-GS described by the *same* condensate phases as $n = 1/4$

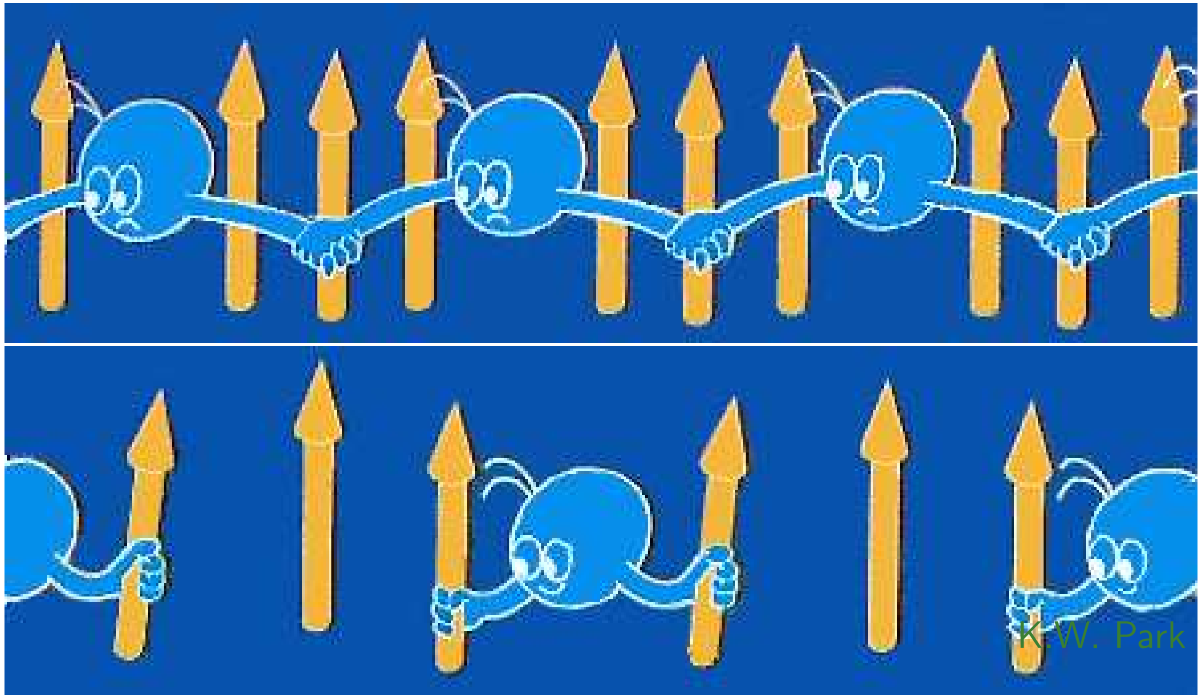


Similarly large overlap, e.g., $N = 8$: $|\langle \Psi_{\rm mft} | \Psi \rangle|^2 = 0.81853181$

$\Rightarrow$ Condensed state insensitive to particle density (as expected).

(II) Correlated States

Composite Fermions



Composite fermion = bound state of an electron with two "flux quanta"

$$n_\phi^{CF} = n_\phi - 2n$$

Interacting electrons ⇒non-interacting composite fermions.

Filled band for (n, n_ϕ^{CF}) ⇒trial incompressible state.

Rotating bosons

Composite fermion = a bound state of a boson with *one vortex* of the many-body wavefunction. [NRC & Wilkin, PRB **80**, 16279 (1999)]

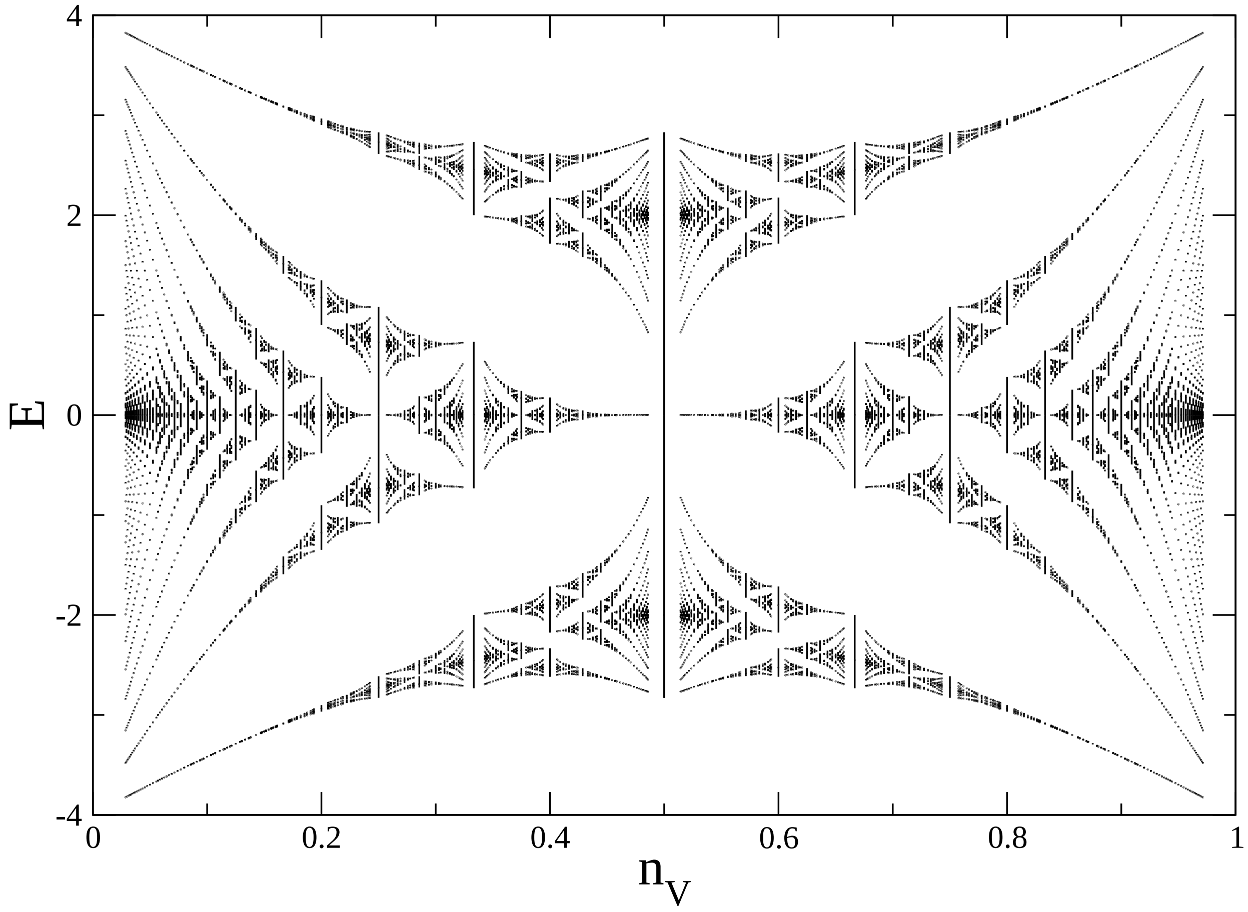
$$n_{\mathrm{v}}^{CF} = n_{\mathrm{v}} - n$$

Filled band (n, n_{v}^{CF}) $\Rightarrow$ trial incompressible state.

Continuum:

$$n/n_{\mathrm{v}}^{CF} = \pm p \qquad \Rightarrow \qquad \nu = \frac{n}{n_{\mathrm{v}}} = \frac{p}{p \pm 1}$$

Lattice: band gaps of the “Hofstadter butterfly”

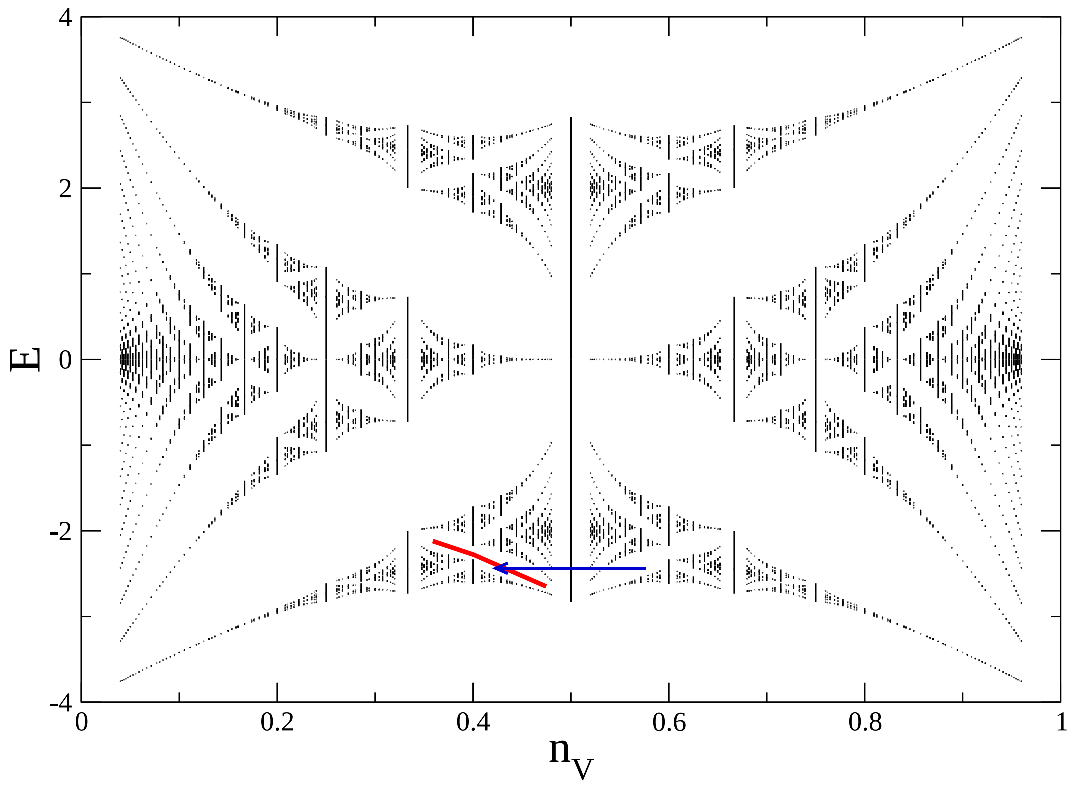


There can exist incompressible states with no counterpart in the continuum

Example CF series

- Take bosons at $\frac{1}{2} < n_{\rm v} < \frac{2}{3}$
- Make CF transformation:

$$n_{\rm v}^* = n_{\rm v} - n$$



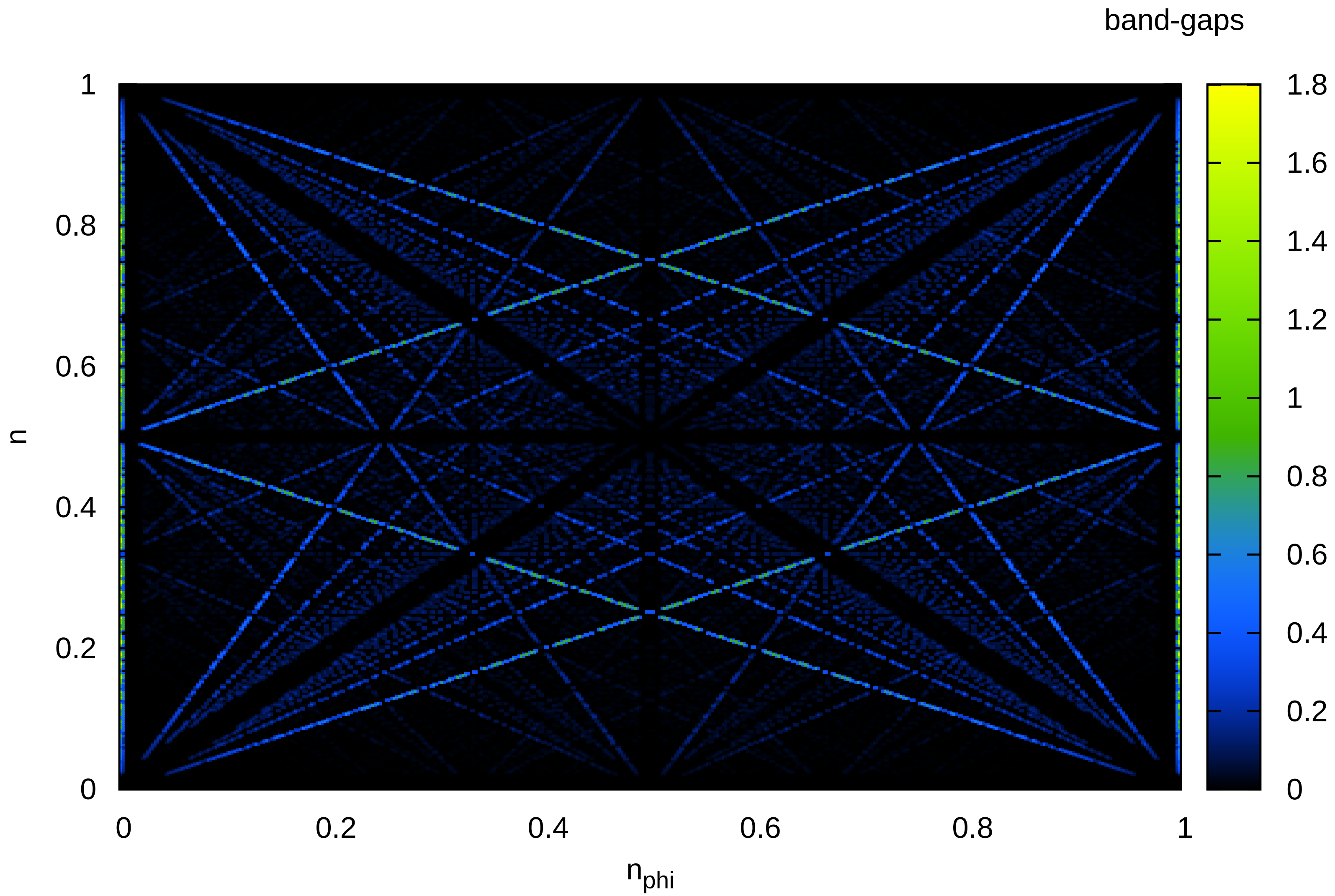
- Read off particle density [linear dependence on $n_{\rm v}$, going through $(0, \frac{1}{2})$, $(1, 1)$]:

$$n = 2(n_{\rm v} - 1/2)$$

$\Rightarrow$CF state predicted for $n_{\rm v} = \frac{1}{2} - \frac{1}{2}n$

- filling factor $\nu = n/n_{\rm v}$ varies continuously for such states!

Gaps for CFs on the square lattice



Numerical Evidence

Exact diagonalisation results show evidence of strongly-correlated many-body states at a series of these new cases:

$n = 1/7, n_{\rm v} = 3/7$ ($N = 3, 4, 5, 6$)
$n = 1/6, n_{\rm v} = 5/12$ ($N = 2, 4$)
$n = 1/9, n_{\rm v} = 4/9$ ($N = 3, 4, 5$)

Many-body spectrum has properties similar to those of the Laughlin state.

- Single-particle density matrix has N eigenvalues of order 1.

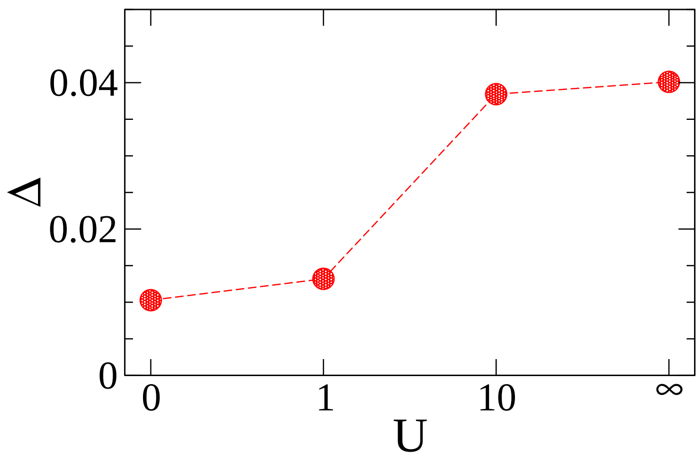
- Groundstate has very small overlap with condensed (mft) states.

- Substantial excitation gap above the groundstate $\Rightarrow$incompressibility.

Sample data: ($n = 1/7,\ n_{\mathrm{v}} = 3/7,\ N = 5$)

- gap $\Delta \approx 0.04t$ (compare to $\Delta_{\mathrm{Laughlin}} \approx 0.18t$ – naïvely expect $1/4$)
- gap increasing with on-site repulsion U
- five large eigenvalues of ρ_{ij}: $2 \times 0.85426,\ 0.85325,\ 2 \times 0.83706, \ldots$
- low overlap of MF condensed state and exact low-lying states:

n	E_n	$\lvert\langle \Psi_{\mathrm{MF}} \vert n \rangle\rvert^2$
0	-12.152984	0.009608
1	-12.112878	0.046826
2	-12.084556	0.044106
3	-12.078881	0.001082
4	-12.074798	0.001507
5	-12.059628	0.054467



$\Rightarrow$Uncondensed, incompressible fluid $\longrightarrow$ strongly correlated state

- Compatible with existence of QH state, but no proof

Summary

• We have studied the phases of rotating bosons on the lattice (the Bose-Hubbard model in a magnetic field).

• Taking account of broken translational symmetry, we find evidence for simple BEC at $n_{\mathrm{v}} = 1/2$ with a two-fold degeneracy. (Condensed, vortex lattice, phase.)

• A generalized composite fermion construction leads to the prediction of incompressible phases at certain (n, n_{v}) stabilized by the lattice.

• We find numerical evidence for uncondensed, incompressible fluids for several of these predicted cases. These are strongly-correlated phases which have no counterpart in the continuum.