Extracting excitations from a fractional quantum Hall groundstate

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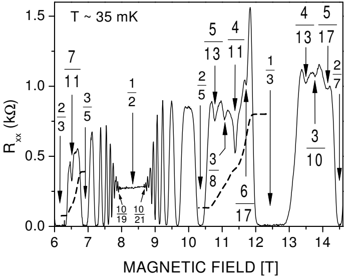
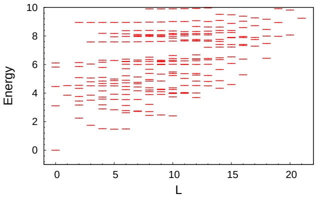
Motivations :

nu=4/11 paper, Fig.1





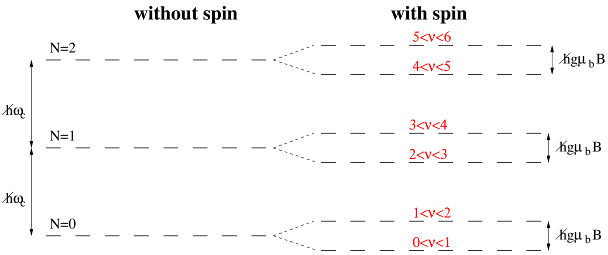
- testing candidate wavefunctions for a given fraction using numerical simulations
- overlap can be misleading. At least one known example where two different states have large overlaps : Abelian (Jain CF) vs non-abelian (Gaffnian).
- is the groundstate enough to characterize a FQH phase ?
- new tools to probe the groundstate
- how deep are encoded the excitations within the groundstate ?

Outline :

- 1. Orbital entanglement spectrum
- 2. Conformal limit
- 3. From the edge to the bulk
- 4. Probing the non-universal part of the OES
- 5. Conclusion

Orbital entanglement spectrum

Landau level



Filling factor : $\nu = \frac{hn}{eB} = \frac{N}{N_\phi}$

Cyclotron frequency : $\omega_c = \frac{eB}{m}$

Lowest Landau level ($\nu < 1$) : $z^m \exp\left(-|z|^2/4l^2\right)$

N-body wave function : $\Psi = P(z_1, ..., z_N)\exp(-\sum |z_i|^2/4)$

the Hamiltonian is just the (projected) interaction !

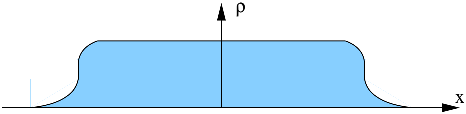
$$\mathcal{H} = \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

(including screening effect, finite width, Landau level,...)

The Laughlin wave function

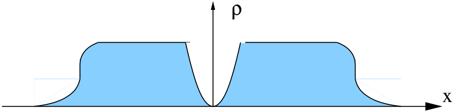
A (very) good approximation of the ground state at $\nu = \frac{1}{3}$

$$\Psi_L(z_1, \ldots z_N) = \prod_{i<j} (z_i - z_j)^3 e^{-\sum_i \frac{|z_i|^2}{4l^2}}$$



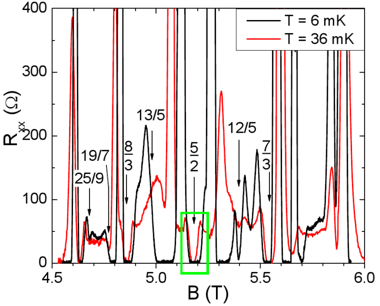
add one flux quantum at z_0 = one quasi-hole

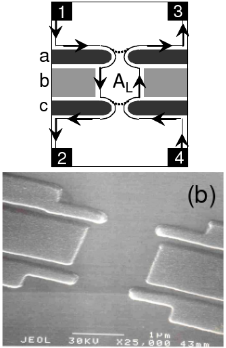
$$\Psi_{qh}(z_1, \ldots z_N) = \prod_i (z_0 - z_i) \;\; \Psi_L(z_1, \ldots z_N)$$

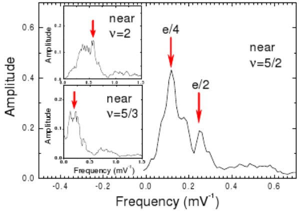


- Locally, create one quasi-hole with fractional charge $\frac{+e}{3}$

$\nu = 5/2$: the Moore-Read state







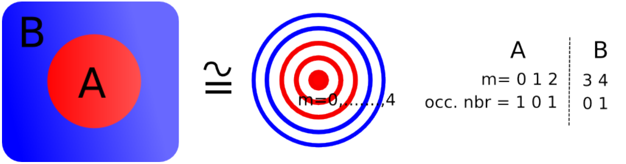
R.L. Willett, L.N. Pfeiffer, K.W. West **(PNAS 0812599106)**

$$\Psi_{pf}(z_1, \ldots, z_N) = Pf\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2$$

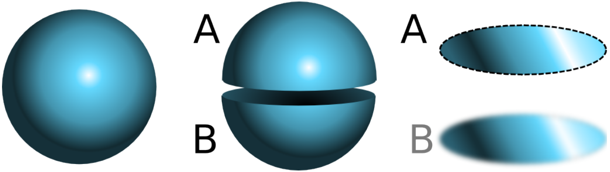
- add/remove one flux quanta $\longrightarrow$ create a pair of quasi-holes /quasi-electrons ($\pm e/4$)
- non Abelian statistics !

Entanglement entropy for the FQHE

- look at the ground state $|\Psi\rangle$
- cut the system into two parts A and B in orbital space ($\simeq$ real space, orbital partition)



- reduced density matrix $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$, block-diagonal wrt N^A and L_z^A
- compute the entanglement entropy $S_A = -\mathrm{Tr}_A(\rho_A \log \rho_A)$.

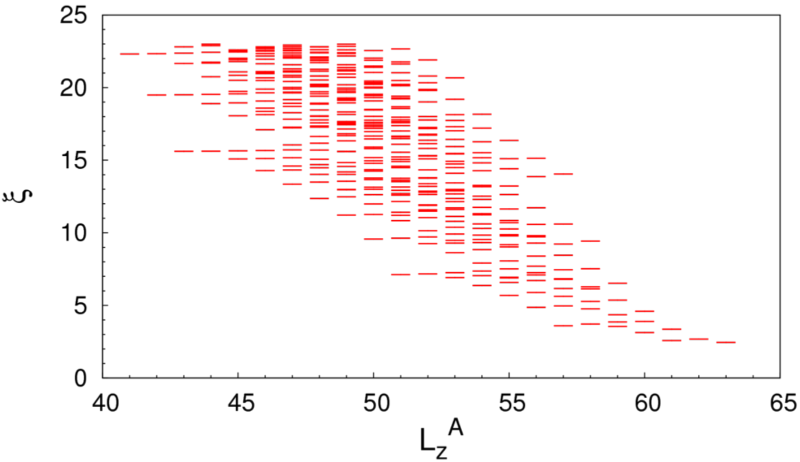


Entanglement entropy for the FQHE

- calculation directly done at the level of the Fock decomposition
- topological entanglement entropy : extract the γ from $S_A = cL - \gamma$ (Haque et al.). Only depends on the nature of the excitations.
  But : highly non-trivial
- looking at the entanglement spectrum : plot $\xi = -\log \lambda_A$ vs L_z^A for fixed cut and N^A
- Schmidt decomposition $|\Psi\rangle = \sum_p \exp(-\xi/2)\,|A, p\rangle \otimes |B, p\rangle$

key idea : think about $\exp(-\xi)$ as a Boltzmann weight, ξ as "energies" of a fictious Hamiltonian for N_A particles

Entanglement spectrum (Li and Haldane)

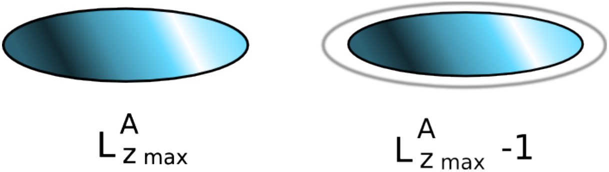


Laughlin $N = 13, l_A = 36$ (hemisphere cut), $N_A = 6$

L_z^A angular momentum of A, $\xi = -\log \lambda_A$, λ_A's are ρ_A eigenvalues.

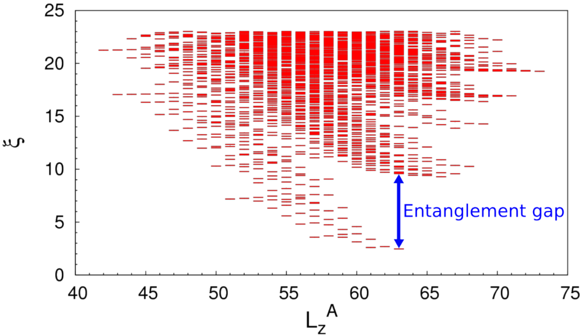
Entanglement spectrum

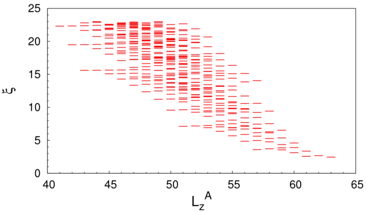
- a way to look at the Fock space decomposition
- "banana" shaped spectrum for pure CFT state (not only Jack polynomials) with a given maximum L_z^A
- "low energy" part : a signature of the state (edge mode degeneracy).
- example Laughlin (1,1,2) : Ψ_L, $\Psi_L \times \sum_i z_i$, $\Psi_L \times \sum_i z_i^2$ and $\Psi_L \times \sum_{i<j} z_i z_j$

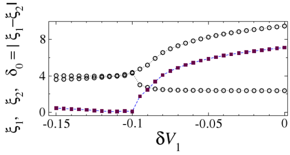


Probing physics of the edge from the ground state on a closed surface

Coulomb case and entanglement gap







Entanglement spectrum for the FQHE : some results

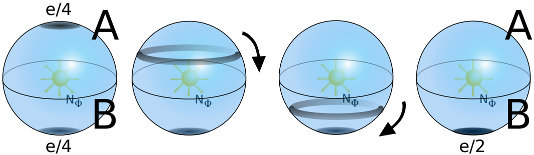
- probing non abelian statistics **(Li, Haldane 2008)**
- looking at (precursor of) phase transition through closing entanglement gap **(Zozulya, Haque, NR, 2009)**
- differentiate states with large overlap but different excitations (from the ground state only !) **(NR, Bernervig, Haldane 2009)**
- non-trivial relation between ES and edge mode **(Bernervig, NR 2009)**
- when $N \to \infty$ recover degenerate multiplets and linear (relativistic) dispersion relation for the edge mode **(Thomale, Stedyniak, NR, Bernervig 2010)**
- torus geometry, tower of edge modes (**Läuchli et al. 2010**)

Entanglement spectrum : beyond FQHE

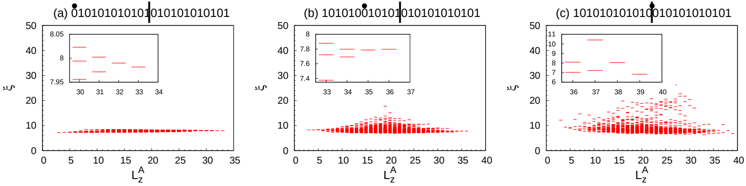
- quantum Hall bilayers
- quantum spin systems
- superconductor
- topological insulators
- Bose-Einstein condensates
- SUSY lattice models

An application : probing statitics of excitations

Write wavefunctions for localized excitations and move them !

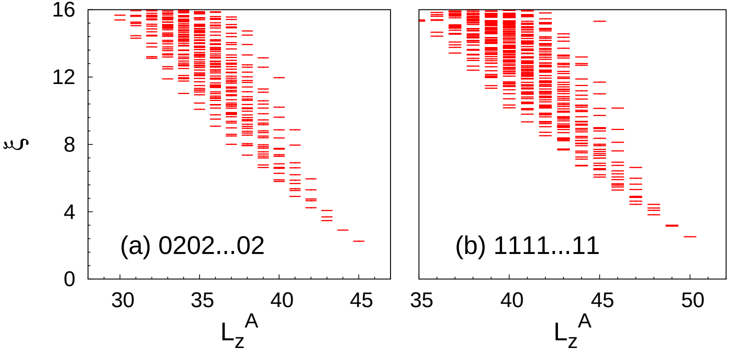


In the Laughlin case (abelian excitations), the counting stays the same (1,1,2,...)



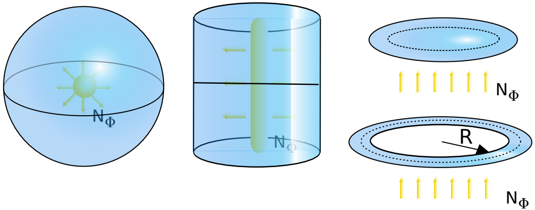
An application : probing statistics of excitations

In the Moore-Read case, the counting is able to detect if there is an even or odd number of excitations.



Conformal limit

Different geometries, similar ES

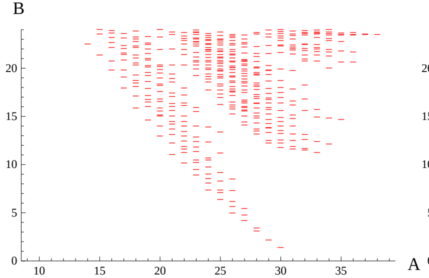


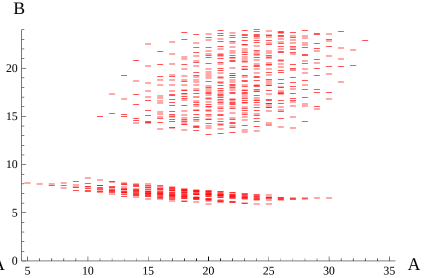
- sphere
- cylinder
- disk
- annulus

- $\Psi = \sum_\mu c_\mu sl_\mu$, c_μ will one differ by some geometrical factors
- different eigenvalues of ρ_A (shape of the ES) but the same number of non-zero eigenvalues (counting)
- The counting IS the important feature. For model states (CFT) , exponentially lower than expected

Defining a "clear" entropy gap

- entanglement gap collapses a few momenta away from the maximum one (the system "feels" the edge)
- remove the information coming from the geometry ($\simeq$ annulus with large radius)
- example : Coulomb $\nu = 1/2$ N=11 bosons





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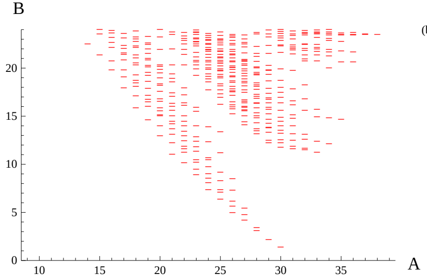
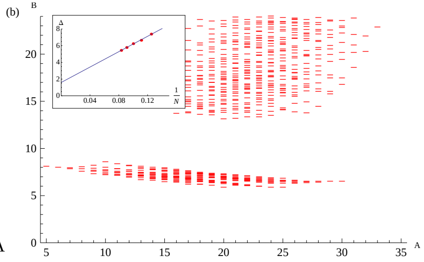
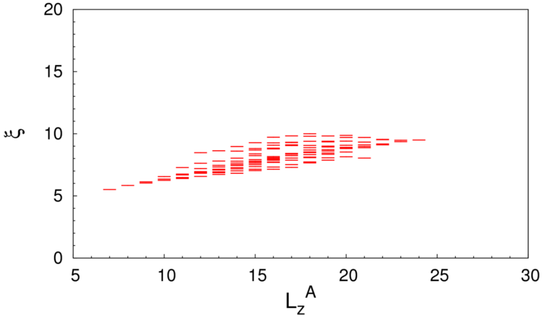




Entanglement adiabatically continuable states

from Moore-Read state to delta ground state N=14 bosons, $\nu = 1$

$$\mathcal{H}_\lambda = (1-\lambda)\sum_{i<j<k}\delta(r_i - r_j)\delta(r_j - r_k) + \lambda\sum_{i<j}\delta(r_i - r_j)$$



No gap closing despite moderate square overlap (0.887) !

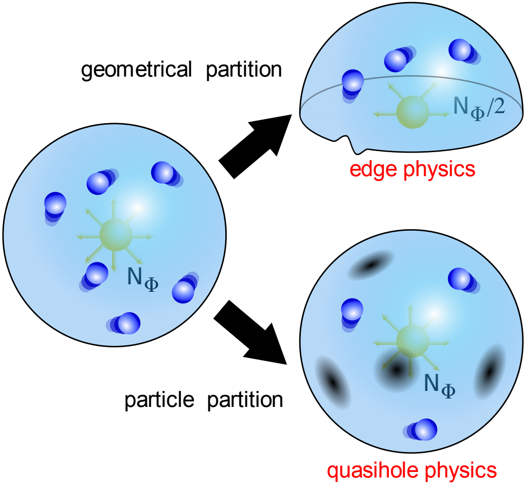
What is encoded within the OES ?

- focus on the Laughlin state $\prod_{i<j}(z_i - z_j)^m$
- **conjecture** (numerically checked) : the full counting is given by the Haldane statistics
- when finite size effects are nice :
  - thermodynamical limit : the counting is the same for any m ($U(1)$ boson)
  - finite size : depends explicitly on m, give access to the boson compactification radius
- the entanglement gap protects the state statistical properties.

From the edge to the bulk

From orbital to particle partition

Particle entanglement entropy in FQHE (Zozulya, Haque, Schoutens)

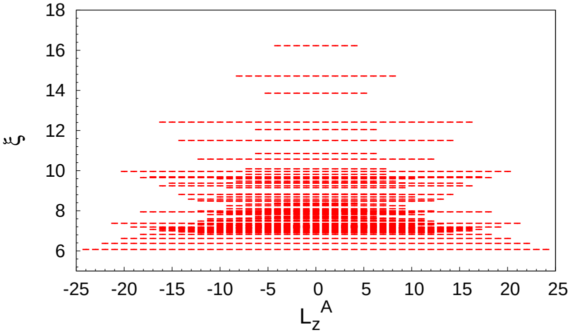


removing particles while keeping the same geometry
$\simeq$ smaller system with extra flux quanta

probing quasihole states !

Particle entanglement spectrum

- can be extended to other geometries : here we focus on the spehere
- both L_z and L^2 are good quantum numbers
- multplet structure $L_z^A \longrightarrow L^A$



Laughlin $\nu = 1/3$ state $N = 8$, $N_A = 4$

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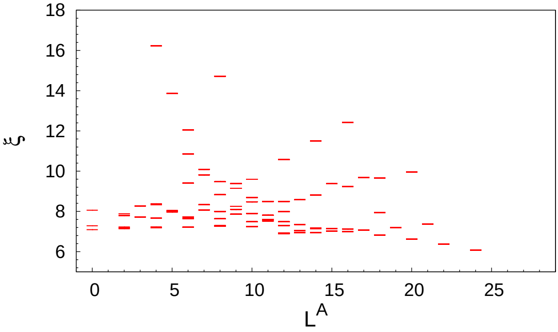

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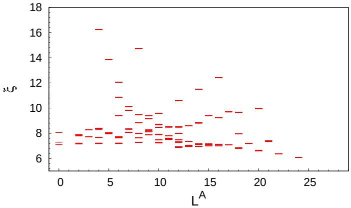
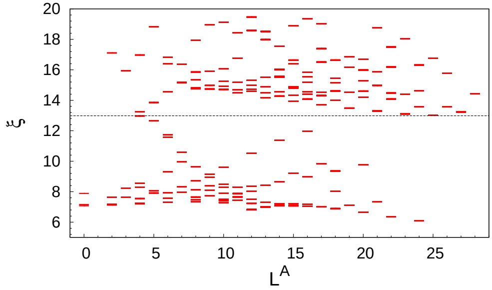


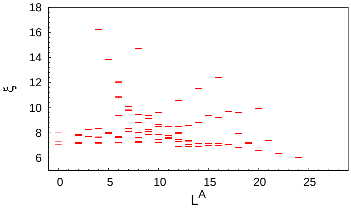
Laughlin $\nu = 1/3$ state $N = 8$, $N_A = 4$

- we are look at the Laughlin state with 4 particles and 12 quasiholes !
- the counting per L^A sector exactly matches the counting of quasihole states
- the eigenstates of reduced density matrix also exactly match the quasihole states

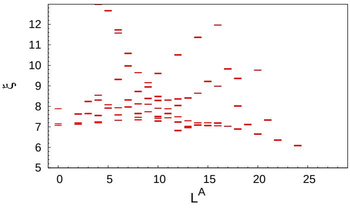
Particle entanglement spectrum : Coulomb interaction



Coulomb $\nu = 1/3$, $N = 8$ and $N_A = 4$



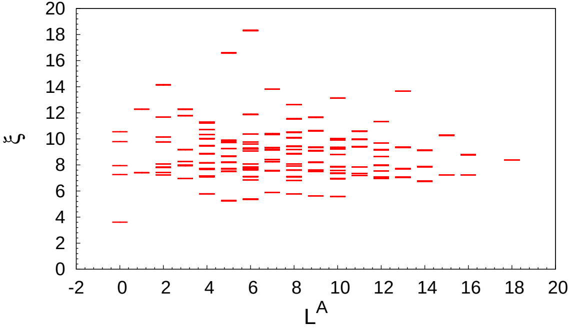
Laughlin



Coulomb (zoom)

Particle entanglement spectrum : Moore-Read state

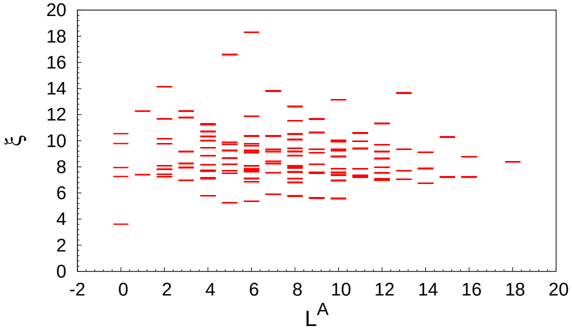
candidate for $\nu = 5/2$, exhibits non-abelian excitations
the PES has the same features !

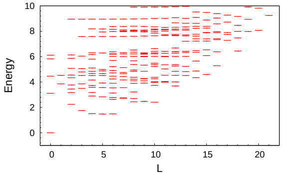


Moore-Read state $N = 12$, $N_A = 6$ (bosons)

Particle entanglement spectrum : Moore-Read state

candidate for $\nu = 5/2$, exhibits non-abelian excitations
the PES has the same features !





At half cut, looks like the spectrum of an incompressible state.
PES "groundstate" close to the Laughlin state...

Particle entanglement spectrum : other states

Is there anything special about the Laughlin and Moore-Read state ?

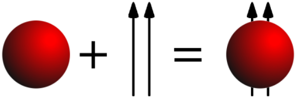
- 1. completely defined throught an exact local Hamiltonian
- 2. single Jack polynomials

Actually, PES features hold true for

- Haffnian state (satisfies 1 but not 2)
- other single Jack polynomial with no known exact Hamiltonian like the clustered state ($k = 3, r = 4$),...
- the Jain's states (neither 1 nor 2 !)

Composite fermions

Jain's model :



Map FQHE into an integer quantum Hall effect for these composite fermions.

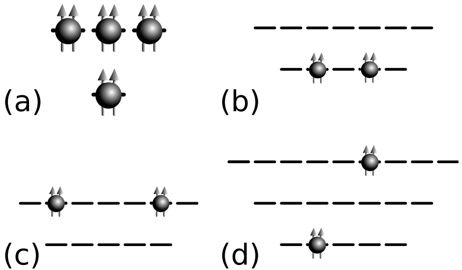
$$
\begin{aligned}
N_\phi^* &= N_\phi - 2N \\
\nu^* = N/N_\phi^* = p \quad &\longrightarrow \quad \nu = \frac{p}{2p+1}
\end{aligned}
$$

More than a nice picture, we can build test wave functions !

$$\Psi_{CF} = \mathcal{P}_{\mathrm{LLL}} \prod_{i<j} (z_i - z_j)^2 \, \Phi_p^{CF}$$

Particle entanglement spectrum : Jain's states

How the particle partition translate into the CF picture ? What does the PES tell us about the CF state ?

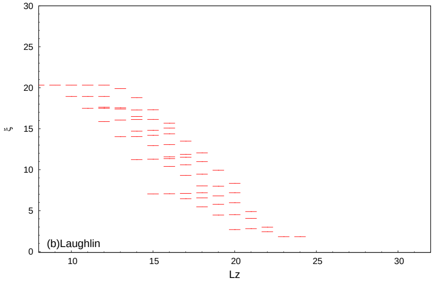


- 1. start with the CF groundstate (here $\nu = 2/5$)
- 2. removing two electrons $\rightarrow$ removing two CFs plus adding 4 flux quanta
- 3. for the qh excitations, do not sort CF states with respect to their effective kinetic energy, only consider all 2 Landau level excitations (i.e. discard d, keep b and d).

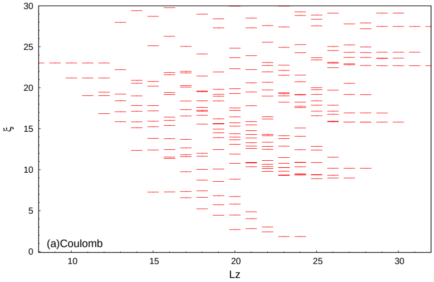
the $\nu = \frac{p}{2p+1}$ CF state is inherently related to the p Landau level physics even for the qh excitations

Probing the non-universal part of the OES

A deeper look at $\nu = 1/3$

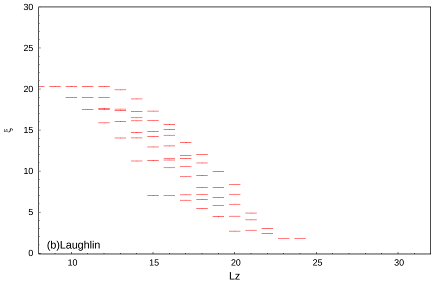


Laughlin $\nu = 1/3$, $N = 8$



Coulomb $\nu = 1/3$, $N = 8$

A deeper look at $\nu = 1/3$



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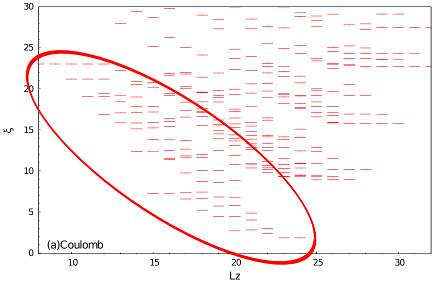
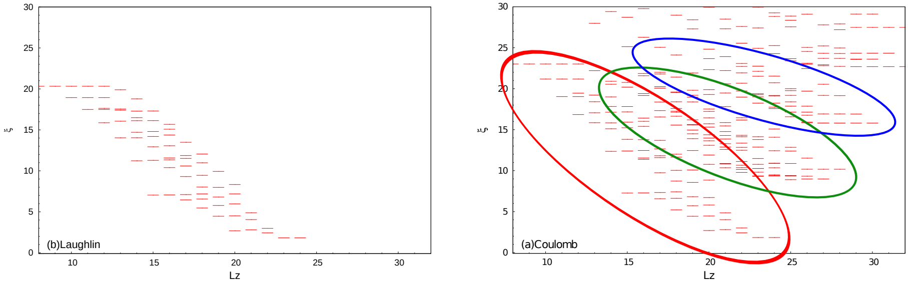


Coulomb $\nu = 1/3,\ N = 8$

"Low energy part" of the Coulomb OES $\simeq$ Laughlin OES

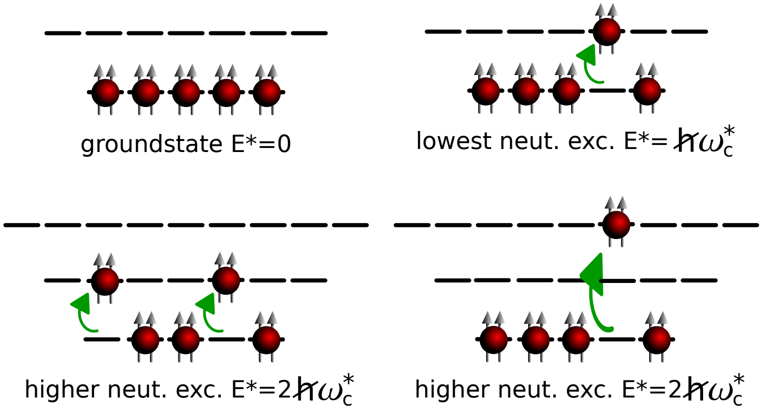
A deeper look at $\nu = 1/3$



Laughlin $\nu = 1/3$, $N = 8$ Coulomb $\nu = 1/3$, $N = 8$

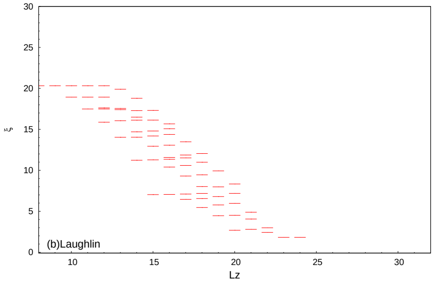
A hierarchical substructure also appears in the non-universal part of the Coulomb OES. Is there meaningful information here?

Understanding the true spectrum using CF

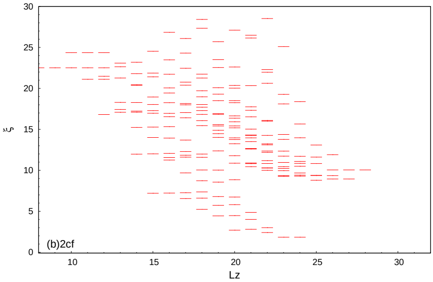


Effective energy hierarchy matches then one of the Coulomb spectrum.

from Laughlin to Coulomb, using CF excitations

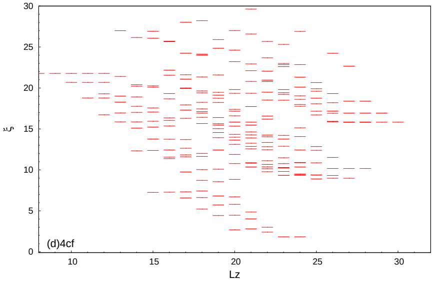


Laughlin

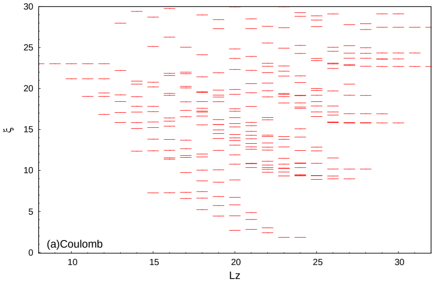


Laughlin + first CF correction

from Laughlin to Coulomb, using CF excitations



Laughlin + up two second CF correction



Coulomb

- non-universal part contains information about neutral excitations.
- the ES "energy" structure mimics the true energy structure of the system.

Conclusions

- numerical calculations are a powerful method to probe the FQHE but...
- more tools are needed to clearly identify phases
- entanglement spectra a way to investigate this problem
- extracting physics of the edge (orbital partition) and bulk (particle partition) from the ground state
- how much information is encoded within the groundstate of these phases ?

Future works :

- relation between OEM and PEM ?
- some mathematical proofs are missing !
- real space cut ?
- ES at finite temperature ? relation between ES gap and true gap
- what is specific about FQHE ? What about other topological phases ?

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- code available at http ://www.nick-ux.org/diagham
- entanglement entropy database
  http ://www.nick-ux.org/ regnault/entropy