Neutral Fermions and Skyrmions in the Moore-Read state at $\nu = 5/2$

Gunnar Möller
Cavendish Laboratory, University of Cambridge

Collaborators:
Arkadiusz Wójs, **Nigel R. Cooper**
Cavendish Laboratory, University of Cambridge

Steven H. Simon
Peierls Centre for Theoretical Physics, Oxford University

DaQuist, Sept 8, 2011

Overview

Introduction

- Quantum Hall effect (QHE) and the story of $\nu = 5/2$

Neutral fermion excitations in $\nu = 5/2$

- Neutral Fermions: qualitative features of pairing physics and non-abelian statistics
- Experimental detection: Photoluminescence

Role of Spin Polarization in PL

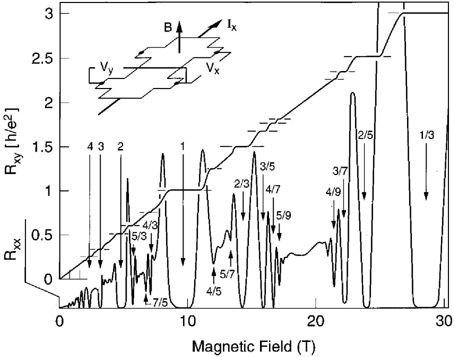
- Skyrmions: spin-wave theory and a closer look at spin-resolved spectra from exact diagonalization
- Partial spin polarization: competition of skyrmions and localized quasiparticles $\leftrightarrow$ transport experiments

- Conclusions

Quantum Hall Effect - a quick introduction

QHE: a macroscopic quantum phenomenon in low temperature magnetoresistance measurements

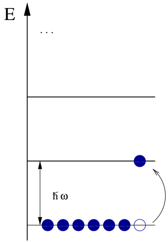
- 2D electron gas
- quantized plateaus in Hall resistance $\sigma_{xy} = \nu \frac{e^2}{h}$
- filling factor $\nu = \frac{\#\text{electrons}}{\#\text{states}}$
- $T \ll \hbar\omega_c,\ V_{\text{disorder}}$
- typically $T \sim 100mK$



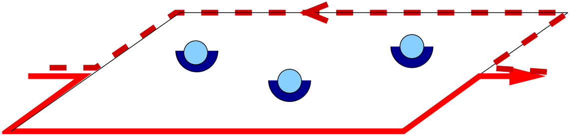
Integer quantum Hall effect

- explained by single particle physics: fillings bands

- single-particle eigenstates in magnetic field: degenerate Landau levels with spacing $\hbar\omega_c$, ($\omega_c = eB/mc$)
- degeneracy per surface area: $d_{LL} = eB/hc$
- integer filling $\nu = n/d_{LL} \Rightarrow$ gap for single particle excitations



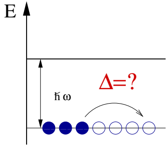
- Insulating bulk, chiral transport along edges ($\rightarrow$ topol. ins.)



Fractional QHE (FQHE)

- in transport, FQHE has same phenomenology as IQHE

- IQHE: quantized plateaus $\leftrightarrow$ gapped excitations in bulk
- partially filled Landau-level (LL) $\Rightarrow$ naïvely expect degenerate groundstate & $\Delta \to 0$

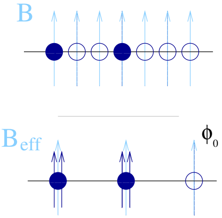


$\Rightarrow$ The nature of interactions determines the groundstate!

- Complicated many body problem in LLs

$$\mathcal{H} = \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|)$$

But: very successful trial wavefunctions exist: composite fermions with 'flux attached' [Jain 1989]
$\Rightarrow$ Effective problem in reduced magnetic field



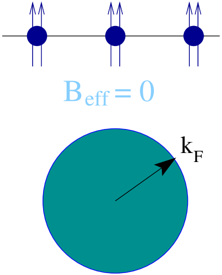
$$B_{\text{eff}} = B - 2n\Phi_0$$

FQHE – half filled Landau levels

- half filling: all flux attached to electrons in CF transformation

- CF non-interacting $\Rightarrow$ fill Fermi-sea
  $\Psi = \mathcal{P}_{\mathrm{LLL}} \prod_{i<j}(z_i - z_j)^2 \Psi_{\mathrm{FS}}^{\mathrm{CF}}$
- But CF have interactions: screened Coulomb + Chern-Simons gauge field from flux-attachment

$\Rightarrow$ If CF have net attractive interaction, CF Fermi-sea is unstable to pairing & gap opens



QHE occurs at $\nu = 5/2$ and is thought to be described by (p-wave) pairing of composite fermions (Moore-Read 1991)

$$\Psi_{MR} = \prod_{i<j}(z_i - z_j)^2 \mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]$$

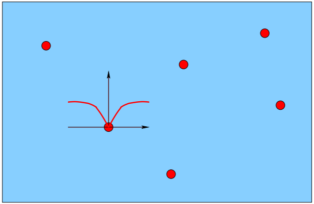
Topological quantum computation

Vortices of p-wave superconductors and the $\nu = 5/2$ state

Vortices of superconducting order parameter $\leftrightarrow$ $e/4$ quasiparticles of the $\nu = 5/2$ state: have non-abelian exchange statistics

Topologically protected groundstates:

- Multiply degenerate Hilbert-space $\mathfrak{H}_0$ of zero-modes in the presence of vortices / quasiparticles
- Braiding of vortices induces transitions within $\mathfrak{H}_0$
- Finite gap towards unprotected states



System of non-abelion anyons provides possible basis for inherently fault-tolerant topological quantum computer

Moore & Read 1991, Kitaev 2003, Ivanov 2001

The Moore Read wavefunction

NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

Gregory MOORE

Department of Physics, Yale University, New Haven, CT 06511, USA

Nicholas READ

Departments of Applied Physics and Physics, Yale University, New Haven, CT 06520, USA

Received 31 May 1990
(Revised 5 December 1990)

$$\Psi_{MR} = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right)\prod_{i<j}(z_i - z_j)^2$$ = Paired chiral p-wave composite fermions

Since the combination $\psi^{\dagger}U^{q}$ is always a fermion at $\nu = 1/q$, q even, and so these must pair if they are to have any chance to condense, and since the pfaffian state is the simplest way for them to do so, we feel that it is likely that if an incompressible state is ever observed at these filling factors with full spin polarization, it should be this state. Such a state will inevitably have neutral fermion and charged nonabelion excitations.

The story of the $\nu = 5/2$ state

A story full of Red Herrings: see talk by S.H.Simon (Nordita 2010)

Experimental of evidence so far:

- existence of FQHE [Willett *et al.* '88 + many others]
- $e/4$ charge of quasiparticles [Dolev *et al.* 2008]
- edge tunneling [Radu *et al.* 2008]
- interference expts ? [Willett '08,'10, Kang]

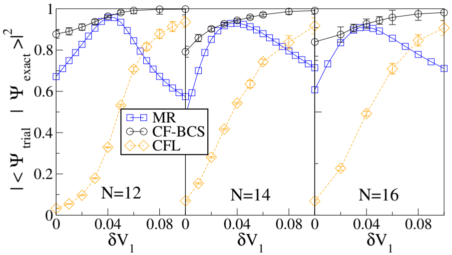
Numerical experiments give strong support of Moore-Read so far:

- spin-polarization of groundstate [Morf '98, Feiguin *et al.* '08]
- scenario for impact of tilted field [Rezayi & Haldane '00]
- non-zero gap & overlap of Ψ_{MR} with exact groundstate
- (approximate) groundstate degeneracy on torus

Strong focus on groundstate: $\Psi_{MR} = \prod_{i<j}(z_i - z_j)^2 \mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]$

Core evidence: Overlaps with the exact groundstate at $\nu = 5/2$

Model Coulomb Hamiltonian on sphere (thin 2DEG);
additionally consider varying short-distance interactions V_1



[overlaps; CF-BCS trial states with optimized parameters $\{g_n\}$ at each δV_1]

- Ψ_{MR} good trial state [N=16: $d(\mathcal{H}_{L=0}) = 2077$]
- even better: $\Psi_{CF-BCS} = \mathcal{P}_{\text{LLL}} \prod_{i<j}(z_i - z_j)^2 \langle\{\mathbf{r}_1, \dots, \mathbf{r}_N\}|\text{BCS}\rangle$

GM and S. H. Simon, Phys. Rev. B **77**, 075319 (2008).

Time to get excited: nature of quasiparticles

$e/4$ quasiparticles $\leftrightarrow$ vortices of a p-wave SC

- directly probing non-abelian statistics difficult
- considerable overlaps with trial states [e.g. works by Morf, Wójs]
- qp size large compared to system size for numerical calculations
- occur in pairs $\Rightarrow$ more finite size effects

Neutral fermion (NF) $\leftrightarrow$ Bogoliubov quasiparticles

Bogoliubov theory for p-wave SF: $|\mathbf{k}\rangle = \gamma_{\mathbf{k}}^{\dagger} |\mathrm{BCS}\rangle$,

with $E_{\mathbf{k}} = \sqrt{\frac{1}{2m^*}(k^2 - k_F^2)^2 + k^2\Delta^2}$, and $\gamma_{\mathbf{k}} = u_{\mathbf{k}}^* \hat{c}_{\mathbf{k}} + v_{\mathbf{k}} \hat{c}_{\mathbf{k}}^{\dagger}$.

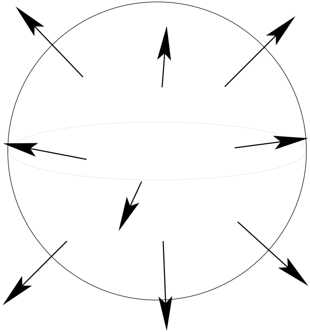
- single localized quasiparticle
- called 'neutral', as addition of $1e^-$ and 2 flux quanta conserves overall charge density ρ of ground state
- *pair-breakers* – NF gap direct evidence for pairing in the system

Numerical studies on the sphere

Our tool: exact diagonalization on the sphere

- Convenient geometry without boundaries
- Shift σ relating integer number of flux N_ϕ and number of particles N naturally separates Hilbert-spaces of competing states

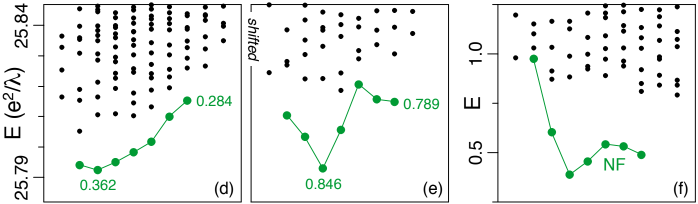
$$N_\phi = \nu^{-1} N - \sigma$$



Diagonalize Hamiltonian in subspace with fixed quantum numbers L, L_z, $[S, S_z]$, using a *projected* Lanczos algorithm.

Numerical studies of $\nu = 5/2$ on the sphere
Sample spectra

Angular-momentum resolved spectra for different Hamiltonians (Coulomb, modified Coulomb, Pfaffian model $\mathcal{H}_{\rm Pf} = \sum P_{ijk}^{(m=3)}$ at the shift of the Moore-Read state $N_\phi = 2N - 3$, with odd $N (= 15)$



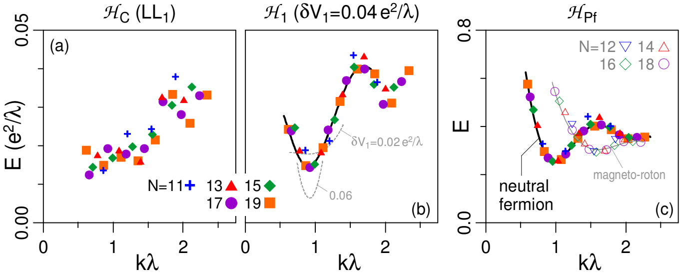
(d) Coulomb Hamiltonian $\mathcal{H}_C$, (e) $\mathcal{H}_1 = \mathcal{H}_C + 0.04\hat{V}_1$, (f) Three-body repulsion $\mathcal{H}_{\rm Pf}$

- dispersive mode well separated from the continuum
- spacing of levels $\Delta L = 1 \Rightarrow$ single particle

GM, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011)

Numerical studies of $\nu = 5/2$ on the sphere
Dispersion of the neutral fermion mode

Dispersion relation from spectra of $N = 11, \ldots, 19$
[shifted to account for finite-size scaling of $E_0(N) \simeq \Delta_{\rm NF} + \beta/N$]

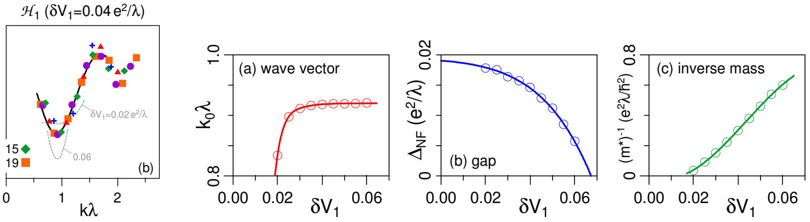


- well formed dispersion for $\delta V_1 > 0$ (or LL-mixing) has *two minima* ($\rightsquigarrow$ phase transition near $\mathcal{H}_C$, Rezayi & Haldane '00)
- second minimum sharp feature (below NF+MR threshold)
- finite gap $\Delta_{\rm NF}$ [see also Bonderson et al. PRL '11]
- qualitative features of Pfaffian-model reproduced

Numerical studies of $\nu = 5/2$ on the sphere

Evolution of NF Dispersion – parameters near minimum of dispersion

Tune interactions from 2nd LL-like ($\delta V_1 = 0$) to LLL-like ($\delta V_1 \simeq 0.08$)

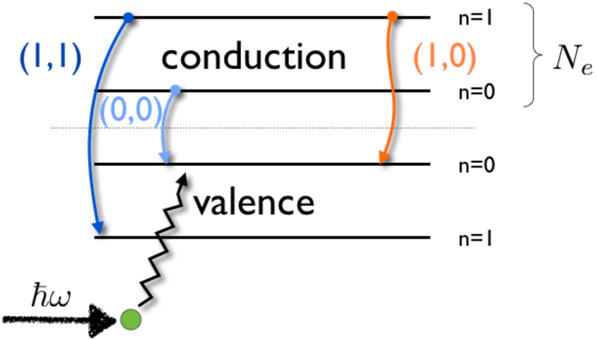


- minimum of dispersion near Fermi-momentum $k_0 \sim k_F = \lambda$
- Δ_{NF} remains finite at small δV_1 – first order transition to CDW
- Δ_{NF} collapses gradually at large δV_1, while effective NF mass diverges (BdG $\rightarrow$ kink!)

Experimental signature of NF Dispersion

How to probe NF dispersion? – Need to change (electron-) fermion #.

Photoluminescence (PL) is a suitable probe (ignoring role of spin below):

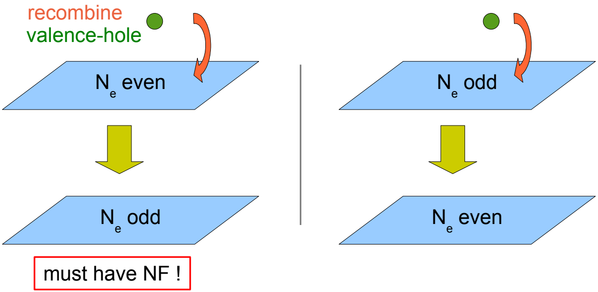


- valence hole h^+ relaxes thermally and then recombines with carriers in 2DEG
- need non-zero matrix-element with 2nd LL electrons

Experimental signature of NF Dispersion

How to probe NF dispersion? – Need to change (electron-) fermion #.

Photoluminescence (PL) is a suitable probe: 2 possible processes

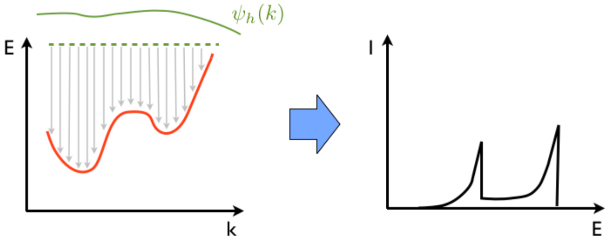


- *initial* state: even N is preferred in ground state (disorder?)
- any *final* state with odd N entails presence of a NF

Experimental signature of NF Dispersion

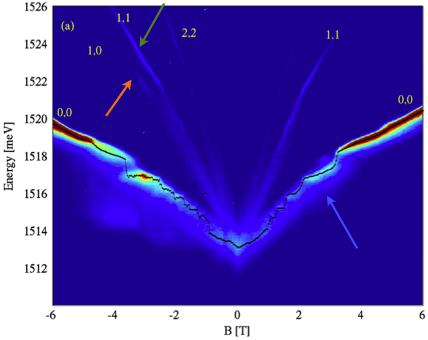
What does one see in PL experiments of the Moore-Read state?

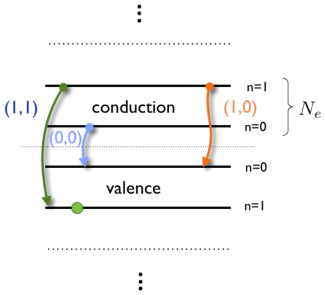
localized h^+ essentially probes DOS $\Rightarrow$ double-peak structure in PL of (1,0) or (1,1) transitions



- each of the threshold peaks may have 'shake-up' processes involving additional magnetorotons

PL experiments in practice
Signals for recombination in different channels



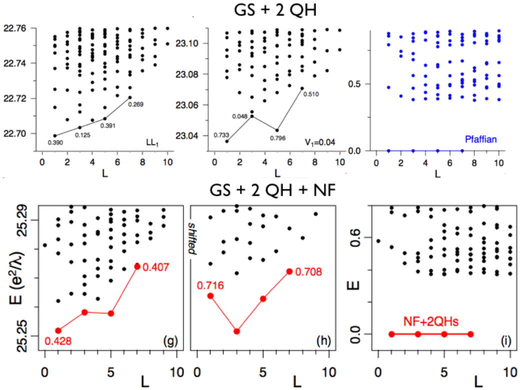


Direct recombination in 2nd LL visible experimentally
(albeit weaker than LLL⇒LLL)

M. Stern *et al.*, Phys. Rev. Lett. (2010), J. K. Jain, Physics (2010)

Energetics of the NF in presence of quasiparticles

Spectra of quasihole states (insertion of one flux quantum to the GS)



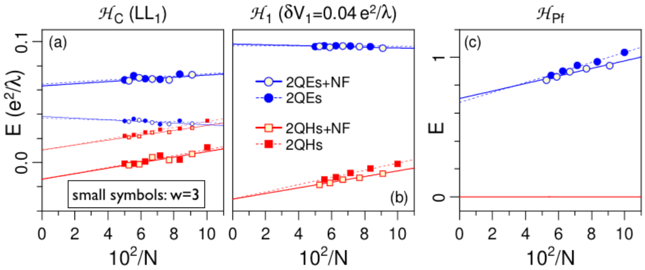
- Spacing of angular momenta $\Delta L = 2$ indicates pair of mobile quasiparticles
- Dispersive band of low-energy excitations both in presence and absence of NF $\Rightarrow$ tricky to compare energies

Energetics of the NF in presence of quasiparticles

In presence of QPs: parity of fermion # $\leftrightarrow$ fusion-channel 1 or ψ

Probe energies of excited states (2QP [+NF]) relative to homogeneous groundstate.

- case 1: arbitrary position of QHs – average within low-lying band



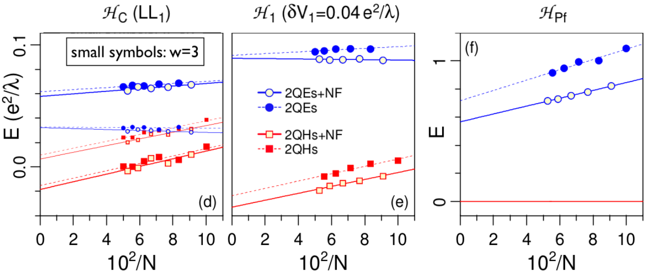
$\Rightarrow$ fusion-channels degenerate for well-separated QPs

Energetics of the NF in presence of quasiparticles

In presence of QPs: parity of fermion # $\leftrightarrow$ fusion-channel 1 or ψ

Probe energies of excited states (2QP [+NF]) relative to homogeneous groundstate.

- case 2: QP's nearby – largest angular momentum



$\Rightarrow$ ψ-channel wins at small r, for both QE and QH

Conclusions 1: Neutral Fermion

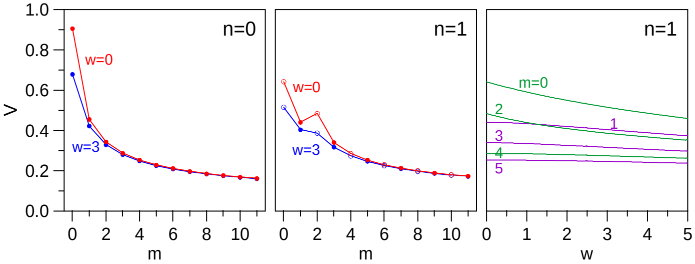
- Presence of neutral fermion excitations affirms the pairing character of the $\nu = 5/2$ state without referring to trial wavefunctions
- Characteristic structure of NF dispersion with double minimum observable both qualitatively and quantitatively in photoluminescence (PL)
- Energetics consistent with topologically degenerate fusion channels 1, ψ of QPs
- First determination of the splitting of fusion channels for both QHs and QEs

GM, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011)

Partial spin polarization at $\nu = 5/2$?

Why revisit the role of spin at $\nu = 5/2$?

- Finite width of 2DEG known to be important at $\nu = 5/2$, however, was not considered in previous work.
- Pseudopotentials in finite width $w > 0$ ease reversal of spins:



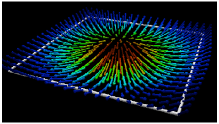
[m: relative angular momentum; w: sample width]

$$V_m = \langle m, M|V(r)|m, M\rangle$$

[$|m, M\rangle$ two-particle state with rel. and CMS angular momentum m, M]

Spontaneous ferromagnetism at $\nu = 5/2$

- Numerical analysis of the spectrum with partial spin-polarization shows wealth of low-lying states
- Analysis reveals these are spin textures of the groundstate

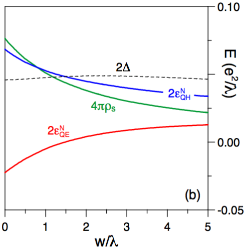


a skyrmion

a skyrmion's spin structure gives rise to Berry's phase that mimics effect of one flux quantum

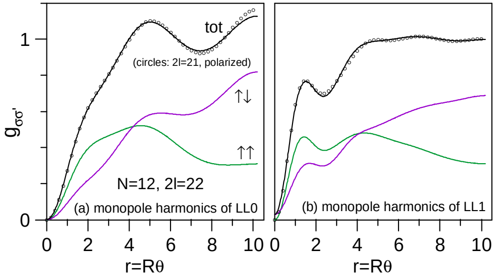
$\Rightarrow$ charge $q_{\mathrm{Sk}} = \nu e = e/2$

- Using spin-stiffness $\Rightarrow E_{\mathrm{sk}} = 4\pi\rho_s$
- From long wavelength spin waves:
  $w = 0$: $2\epsilon_{QE} < 2\epsilon_{QH} \lesssim E_{\mathrm{sk}}$
- At finite width w: $E_{\mathrm{sk}} \lesssim 2\epsilon_{QH}$



Probing for skyrmion states
Correlation functions

Characterize exact eigenstates with quantum numbers of skyrmion (spin $S = 0$, shift $\sigma = \sigma_{\text{pol}} \pm 1$, here: $\sigma = 2$)



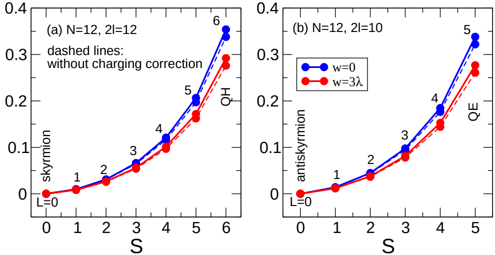
[left: correlations $g_{\uparrow\uparrow}$, $g_{\downarrow\downarrow}$ and $g_{\text{tot}} = g_{\uparrow\uparrow} + g_{\downarrow\downarrow}$ for guiding center coordinates; right: same for electrons]

- The $g_{\uparrow\uparrow}(r)$ has a dip at large r, while $g_{\uparrow\downarrow}$ becomes large
- Total correlations g_{tot} closely match those of the polarized 5/2 state at $\sigma = 3$ (length units rescaled for difference in σ)

Skyrmions at partial spin polarization - I

Generic behaviour for skyrmion state

Having identified the spin-singlet state at $N_\phi = N_\phi^{pol} + 1$, analyze sequence of states with successively higher spin: generic case



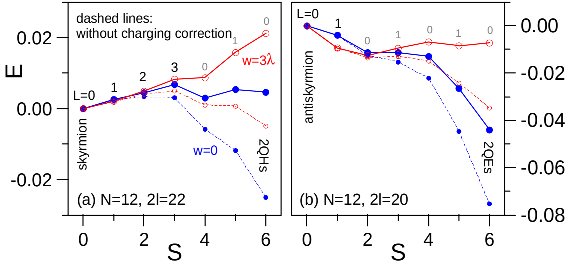
[$\nu = 1$: energy of skyrmion/quasiparticle states versus spin S]

- as polarization increases, a charging correction is required: $\delta E(S) = [S/S_{\max}]^3\, \delta E_{qp}$; $\nu = \frac{5}{2}$: $\delta E_{qp} = \frac{3}{32\sqrt{N}} \frac{e^2}{\epsilon \ell_0}$ (Morf 2002)
- roughly quadratic dispersion; the localized qp has the highest correlation energy (correction negligible at $\nu = 1$)

Skyrmions at partial spin polarization - II

Behaviour for the skyrmion states over $\nu = 5/2$

Spin dependent energy at $\nu = 5/2$



[$\nu = 5/2$: energy of skyrmion/quasiparticle states versus spin S]

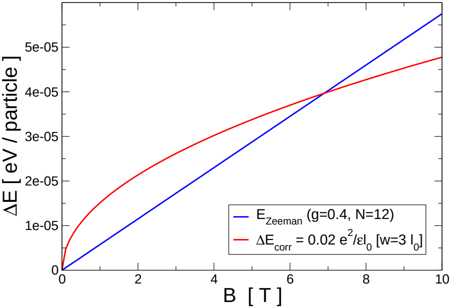
- Kink separating skyrmion-like quadratic dispersion at small S and drop-off towards fully polarized state
- $e/2$ skyrmion formed by binding two $e/4$ quasi-particles, *unlike* $\nu = 1$ or $\nu = 3$ where $q_{\text{skyrmion}} = q_{qp}$ ($\rightarrow$ low L)

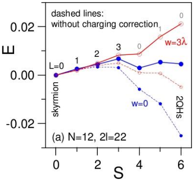
  $N = 10$: A. Feiguin *et al.*, Phys. Rev. B **79**, 115322 (2009)

Skyrmions at partial spin polarization - III

Behaviour for the skyrmion states over $\nu = 5/2$

With appropriate charging correction, Skyrmion has *lower* correlation energy than pair of qh's, especially in finite width





[quasihole vs skyrmion energy: $\Delta E = E_{qh} - E_{skyrmion}$]

- Skyrmion might be favourable up to fields $B \sim 6.5T$
- caveat: finite size effects for large skyrmions

Skyrmions at partial spin polarization - IV

Mechanisms to nucleate skyrmions

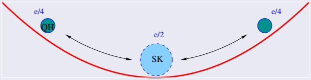
- at low field / Zeeman coupling, skyrmions are the lowest energy excitations – of *abelian* / top. trivial nature

$\Rightarrow$ will affect braiding and interference experiments!

Mechanisms to nucleate skyrmions

- non-zero density of quasiparticles: tuning magnetic field away from center of Hall plateau induces quasiparticles $\rightarrow$ could yield Wigner crystal of Skyrmions rather than WC of qh's

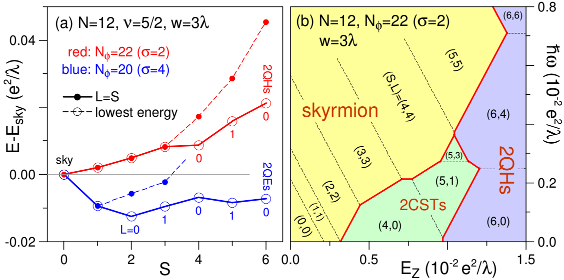
- disorder: if two pinning sites are at short separation, mutual binding and introducing a spin-texture may be the energetically most favourable way to accommodate pinned quasiparticles; also: valence-h in PL!!



Skyrmions at partial spin polarization - V

Phase diagram for skyrmions vs quasiholes

- localized $e/2$ skyrmions may be preferred over $2 \times e/4$ CST by confining disorder potential

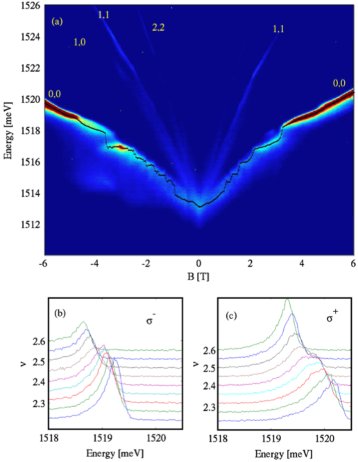


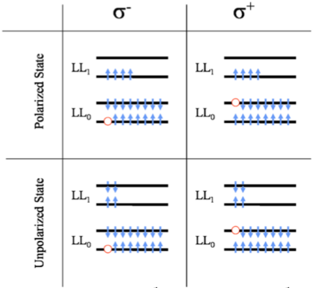
[$\nu = 5/2$: energy of skyrmion/quasiparticle states versus spin S]

A. Wójs, GM, S. H. Simon, N.R. Cooper, PRL (2010)

more on $e/4$ CST: J. Romers, L. Huijse, K. Schoutens, NJP (2011)

Spin polarization in PL experiments
Selection rules for recombination





$$S_z^e = -\frac{1}{2} \qquad S_z^e = +\frac{1}{2}$$

$$J_z^h = +\frac{3}{2} \quad (B > 0) \quad J_z^h = -\frac{3}{2}$$

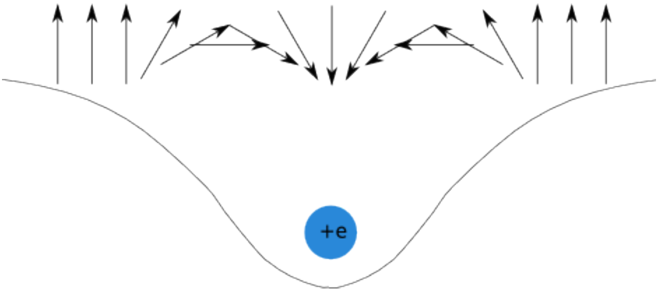
difference in interaction energies

$$\Delta E = (g_h + g_e)\mu_B B + \Delta\Sigma$$

M. Stern *et al.*, Phys. Rev. Lett. (2010), J. K. Jain, Physics (2010)

Spin polarization in PL experiments
Role of skyrmions

- valence hole acts as strong disorder potential near 2DEG



- skyrmions favoured in local environment

⇒ Expect spin polarization of GS (partially) hidden in PL

Conclusions

- neutral fermion excitations reveal qualitative features for pairing and non-abelian statistics of the Moore-Read state at $\nu = 5/2$
- identified low-lying spin-textured excitations as (anti-)skyrmions of Moore-Read (correlations, overlaps)
- $q_{\mathrm{Sk}} = 2q_{qh}$ skyrmions are promoted by disorder and cause unusual transport phenomenology

- The physics of $\nu = 5/2$ is that of a spin polarized quantum liquid. The groundstate is in the non-abelian weakly paired phase, but its quasielectrons/-holes compete with *abelian* skyrmions to be the lowest lying excitations

GM and S. H. Simon, PRB (2008)
A. Wójs, GM, S. H. Simon and N. R. Cooper, PRL (2010)
GM, A. Wójs, and N. R. Cooper, Phys. Rev. Lett. **107**, 036803 (2011)