Particles in non-Abelian gauge potentials: Insertion of non-Abelian flux

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Workshop on Quantum Information and Condensed Matter Physics, Maynooth

September 8, 2011

“Real” gauge fields

SOVIET PHYSICS JETP VOLUME 5, NUMBER 6 DECEMBER 15, 1957

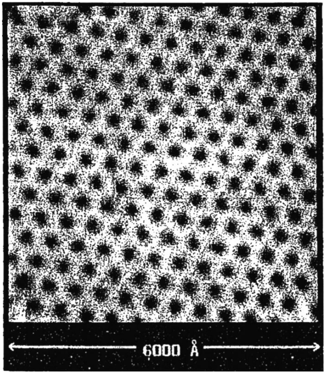
On the Magnetic Properties of Superconductors of the Second Group

A. A. ABRIKOSOV

Institute of Physical Problems, Academy of Sciences, U.S.S.R.

(Submitted to JETP editor November 15, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1442-1452 (June, 1957)

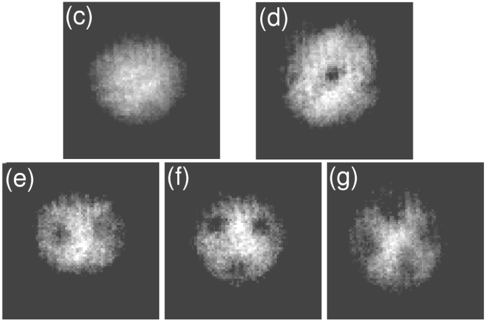


Artificial gauge fields 1: Rotating gases

VOLUME 84, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 2000

Vortex Formation in a Stirred Bose-Einstein Condensate

K. W. Madison, F. Chevy, W. Wohlleben,* and J. Dalibard
Laboratoire Kastler Brossel,† Département de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75005 Paris, France



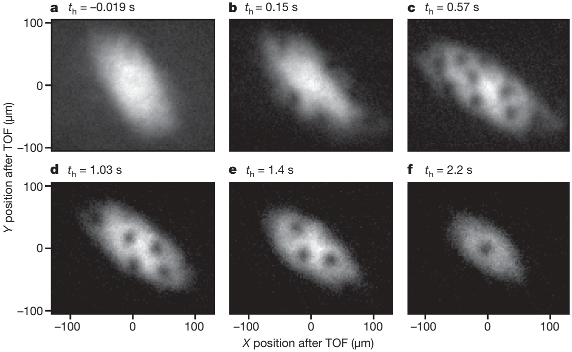
Artificial gauge fields 2: Atom-laser coupling

nature Vol 462|3 December 2009|doi:10.1038/nature08609

LETTERS

Synthetic magnetic fields for ultracold neutral atoms

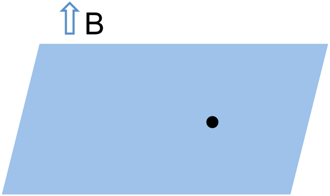
Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]



Outline

- Introduction
  - (Abelian) Landau problem
  - On non-Abelian gauge fields
- Adiabatic insertion of non-Abelian flux

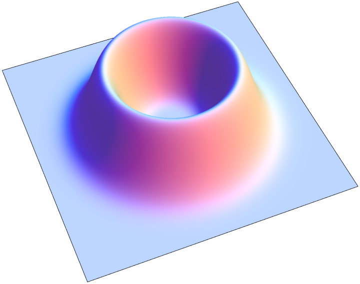
Charged 2D particle in perpendicular magnetic field



$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2$$

$$\vec{B} = B\hat{z}$$

$$\vec{A} = \frac{B}{2}\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$



$$E_n = \hbar\omega_c(n + 1/2), \quad n = 0, 1, \ldots$$

$$\psi_{\mathrm{LLL}}(z) = z^m e^{-|z|^2/4} = |0, m\rangle$$

On non-Abelian gauge fields

Components can be written as matrices: $\vec{\mathbf{A}} = \mathbf{A}_x \hat{x} + \mathbf{A}_y \hat{y} + \mathbf{A}_z \hat{z}$

Magnetic field: $\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}} - i\vec{\mathbf{A}} \times \vec{\mathbf{A}}$

Gauge transformation: $\mathbf{A}_\mu \rightarrow U\mathbf{A}_\mu U^\dagger + iU\partial_\mu U^\dagger$

Covariant quantities: $\vec{\mathbf{B}} \rightarrow U\vec{\mathbf{B}}U^\dagger$

$$\mathbf{H} = \frac{1}{2m}\left(\vec{p} - \vec{\mathbf{A}}\right)^2 \rightarrow U\mathbf{H}U^\dagger$$

Two gauge inequivalent vector potentials can produce same magnetic field, e.g.:

$$\left.\begin{array}{rcl} \vec{\mathbf{A}} & = & \sigma_z(-y, x, 0)^\mathsf{T} \\ \vec{\mathbf{A}}' & = & (-\sigma_y, \sigma_x, 0)^\mathsf{T} \end{array}\right\} \Rightarrow \vec{\mathbf{B}} = 2\sigma_z \hat{z}$$

Adiabatic insertion of non-Abelian flux

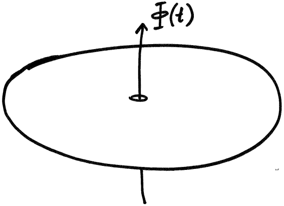
Laughlin's flux insertion argument

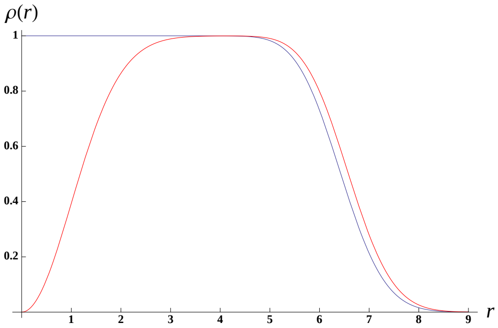
$\Phi = \oint \vec{A} \cdot d\vec{\ell}$

$\vec{A}(t) = \frac{1}{2} Br\hat{\phi} + \frac{\Phi(t)}{2\pi r}\hat{\phi}$

Away from origin $\vec{B}(t) = B\hat{z}$

$|m\rangle \rightarrow |m+1\rangle$





Spin-1/2 particles

$\mathbf{H} = \frac{1}{2m}(\vec{p} - \vec{\mathbf{A}})^2$

$\vec{\mathbf{A}} = \frac{B}{2} r \hat{\phi} \mathbb{I} \quad \Rightarrow \quad \vec{\mathbf{B}} = B \hat{z} \mathbb{I}$

Eigenstates: $|n, m, \epsilon\rangle$, where $\epsilon \in \{\uparrow, \downarrow\}$

How to insert non-Abelian flux?

$\vec{\mathbf{A}} \rightarrow \vec{\mathbf{A}} + \delta \vec{\mathbf{A}}(t)$

Adiabatic insertion of non-Abelian flux

Initial state:
$|\psi_i\rangle = \bigotimes_{m=0}^{m_f} |m \uparrow\rangle$

$$\delta \mathbf{A}_r(\lambda) = \quad \frac{-\lambda}{1+(\lambda r)^2}\sigma_\phi$$
$$\delta \mathbf{A}_\phi(\lambda) = \quad \frac{-\lambda^2 r}{1+(\lambda r)^2}\sigma_z + \frac{\lambda}{1+(\lambda r)^2}\sigma_r$$

$H(0) \to H(\lambda)$

$H(\lambda)|\psi(\lambda)\rangle = E(\lambda)|\psi(\lambda)\rangle$

Conservation of angular momentum: $J_z = L_z + \frac{1}{2}\sigma_z$

$$|\psi_f(\lambda)\rangle = \bigotimes_{m=0}^{m_f} \left[u_m(\lambda)|m \uparrow\rangle - v_m(\lambda)|m+1 \downarrow\rangle\right]$$

B. Estienne, S.M. Haaker and K. Schoutens, New J. Phys. (2011)

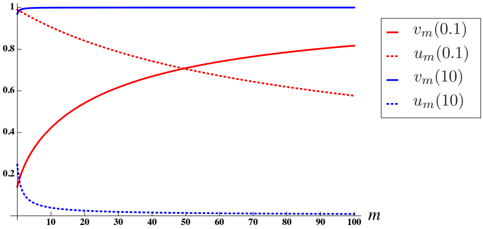
Mixing angle

$$|\psi_f(\lambda)\rangle = \bigotimes_{m=0}^{m_f} \left[u_m(\lambda)|m\uparrow\rangle - v_m(\lambda)|m+1\downarrow\rangle\right]$$

$$u_m(\lambda) = \cos(\theta_m(\lambda))$$

$$v_m(\lambda) = \sin(\theta_m(\lambda))$$

$$\theta_m(\lambda) = \int_0^\infty dr \arctan(\lambda r) \frac{r^{2m+2} e^{-r^2/2}}{2^m \sqrt{2m!(m+1)!}}$$

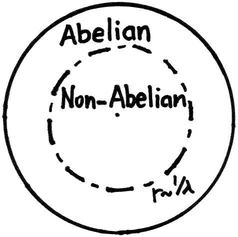


B. Estienne, S.M. Haaker and K. Schoutens, New J. Phys. (2011)

Quantum Hall Skyrmion

Initial state:

$|\psi_i\rangle = \bigotimes_{m=0}^{m_f} |m\uparrow\rangle$

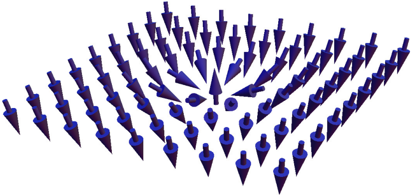


$$\delta \mathbf{A}_r(\lambda) = \frac{-\lambda}{1+(\lambda r)^2}\sigma_\phi$$

$$\delta \mathbf{A}_\phi(\lambda) = \frac{-\lambda^2 r}{1+(\lambda r)^2}\sigma_z + \frac{\lambda}{1+(\lambda r)^2}\sigma_r$$

$r \ll \frac{1}{\lambda}$: $\quad \delta\vec{\mathbf{A}} \sim \lambda \begin{pmatrix} -\sigma_y \\ \sigma_x \\ 0 \end{pmatrix}$

$r \gg \frac{1}{\lambda}$: $\quad \delta\vec{\mathbf{A}} \sim \frac{-1}{r}\sigma_z\hat{\phi}$



$$|\psi_f(\lambda)\rangle = \bigotimes_{m=0}^{m_f} \left[u_m(\lambda)|m\uparrow\rangle - v_m(\lambda)|m+1\downarrow\rangle\right]$$

B. Estienne, S.M. Haaker and K. Schoutens, New J. Phys. (2011)

More results

- Spectrum on a sphere
- Non-Abelian flux insertion in spin-unpolarized $\nu = 2$ quantum Hall state

Outlook

- Connection to TI
  - $\vec{\mathbf{A}} = \frac{B}{2}(-y\hat{x} + x\hat{y})\sigma_z$: quantum spin Hall effect
  - Li and Wu (arXiv:1103.5422): 3D TI's w/ Landau levels
  - Domain wall between two phases with different non-Abelian magnetic fields

Conclusion

- Artificial non-Abelian gauge fields can in principal be created in cold atomic gases
- Way to simulate charged spin-1/2 particles in a perpendicular magnetic field
- Adiabatic insertion of non-Abelian flux in a spin-polarized background $\Rightarrow$ quantum Hall Skyrmion