Anomalous charge tunnelling in fractional quantum Hall edge states

Dario Ferraro

Università di Genova

A. Braggio, M. Carrega, N. Magnoli, M. Sassetti

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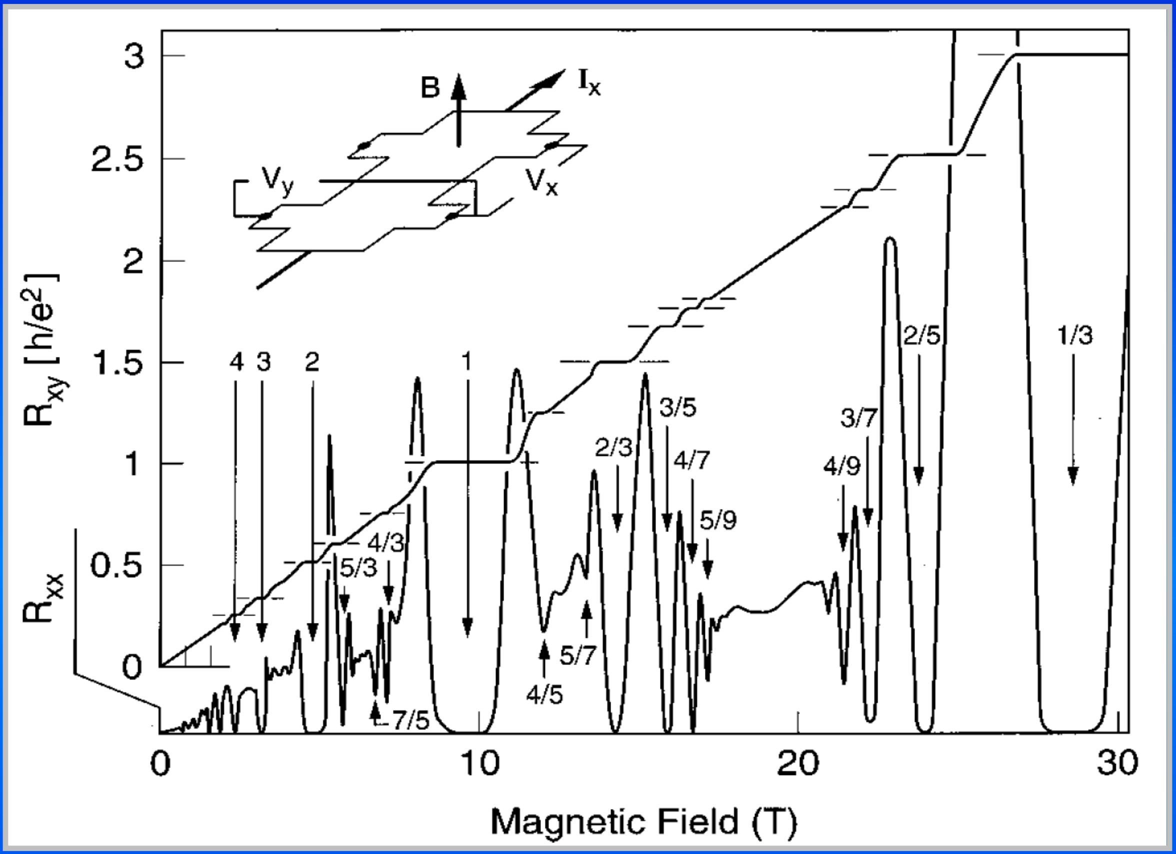
Outline

- Edge states tunnelling in Quantum Point Contact geometry
- Neutral mode dynamics: effects on transport properties
- Relevance of agglomerate tunnelling

FQHE: bulk



Stormer et al., RMP 98

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\nu = \frac{N}{N_\varphi}$$

Incompressibility

Gapped excitations with fractional charge and fractional statistics (Abelian and non-Abelian)

FQHE: filling factors

Laughlin sequence $$\nu = \frac{1}{2n+1}$$

Laughlin, PRL83

Jain sequence $$\nu = \frac{p}{2np+1}$$

Jain, PRL89

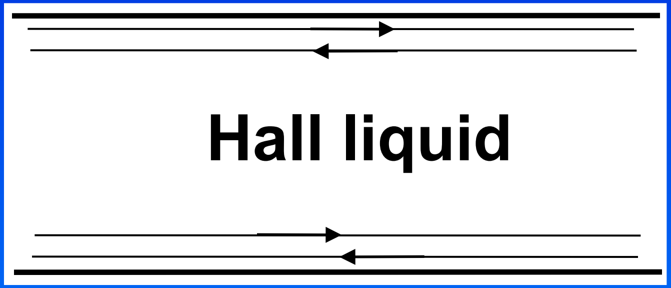
$$\nu = \frac{5}{2}$$

Moore & Read, NPB91; Halperin et al., PRB93; Fendley et al., PRB07; Lee et al. PRL07; Levin et al., PRL07; Boyarsky et al., PRB09

$$\nu = \frac{7}{3}, \frac{8}{3}, \frac{12}{5} \dots$$

Read & Rezayi, PRB99; Bishara et al., PRB08; Bonderson & Slingerland, PRB08…

FQHE: edge states



Low energy sector of the incompressible fluid

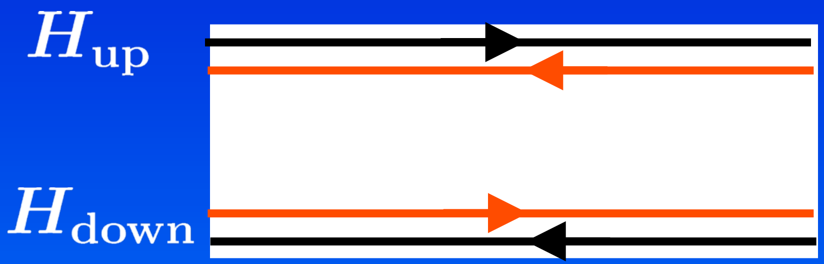
Quantization of the conductance

Halperin, PRB82; Buttiker, PRB88; Beenakker, PRL90

Gapless excitations

Same charge and statistics of bulk quasiparticles

Edge dynamics in the Jain sequence



$$H_{\rm up} = \frac{v_c}{4\pi\nu}\int dx\,(\partial_x\varphi_c)^2 + \frac{v_n}{4\pi}\int dx\,(\partial_x\varphi_n)^2$$

charged mode　　　　neutral mode

$$\rho = \frac{1}{2\pi}\partial_x\varphi_c$$

$$[\varphi_c(x),\varphi_c(y)] = i\pi\nu\,\mathrm{sign}(x-y) \quad [\varphi_n(x),\varphi_n(y)] = -i\pi\,\mathrm{sign}(x-y)$$

$$v_c \gg v_n$$

Wen & Lee, arXiv:9809160; Levkivsky & Sukhorukov, PRL08;

D. F., A. Braggio, M. Merlo, N. Magnoli, M. Sassetti, PRL 101, 166805 (2008)

Multiple-quasiparticle excitations

$m = 1$	$m > 1$
quasiparticle	m-agglomerate

Charge

$$e_m^* = m e_1^* = m \frac{\nu}{|p|} e$$

Statistics

$$\Psi^{(m)}(x)\Psi^{(m)}(y) = \Psi^{(m)}(y)\Psi^{(m)}(x)e^{-i\theta_m \mathrm{sign}(x-y)}$$

$$\theta_m = m^2\pi\left(\frac{\nu}{p^2} - \frac{1}{|p|} - 1\right) \qquad \mathrm{mod}(2\pi)$$

Monodromy: quasiparticles acquire no phase in a loop around electrons

Ino, PRL 81; D. F., A. Braggio, N. Magnoli, M. Sassetti, Physica E 42, 580 (2010)

Multiple-quasiparticle operators

Bosonization

$$\Psi^{(m)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i[\alpha_m \varphi_c(x) + \beta_m \varphi_n(x)]}$$

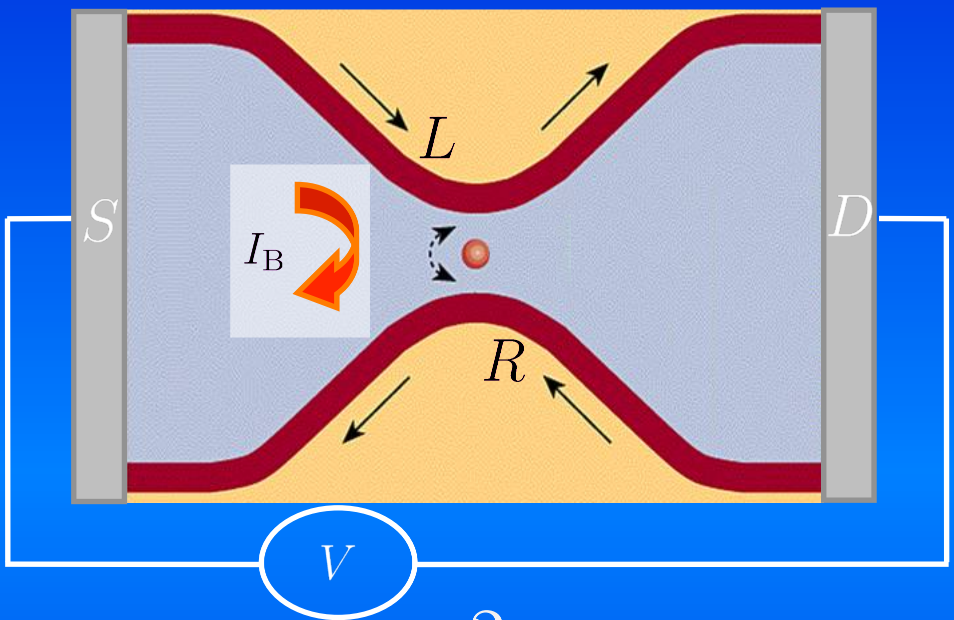
Single quasiparticle

$$\Psi^{(1)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\left[\frac{1}{|p|}\varphi_c(x) + \sqrt{1+\frac{1}{|p|}}\varphi_n(x)\right]}$$

|p|-agglomerate

$$\Psi^{(|p|)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\varphi_c(x)}$$

Quantum Point Contact geometry



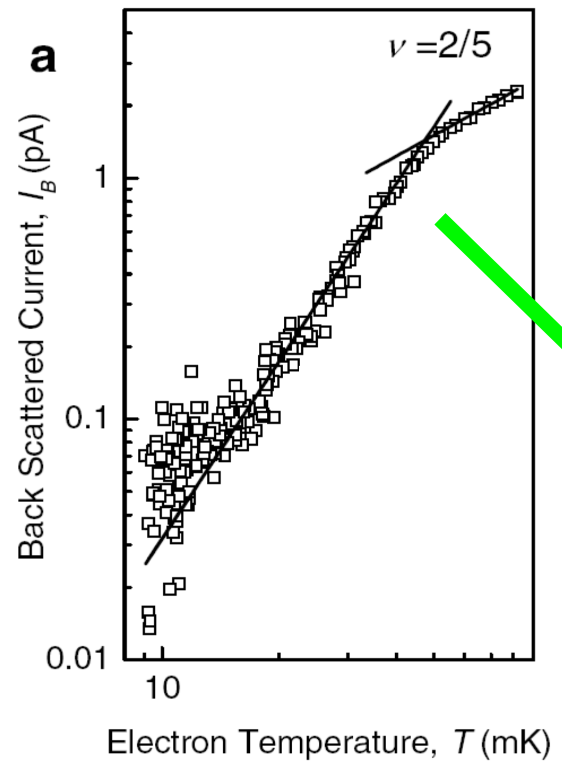
$$I = \nu \frac{e^2}{h} V - I_B$$

Out of equilibrium system

Schwinger-Keldysh contour formalism

Schwinger, J. Math. Phys 61; Keldysh, Zk. Ekso. Teor. Fiz. 64

Extremely weak backscattering: current



Chung et al. PRL03

$\nu = 2/5 \qquad p = 2, n = 1$

change in the slope

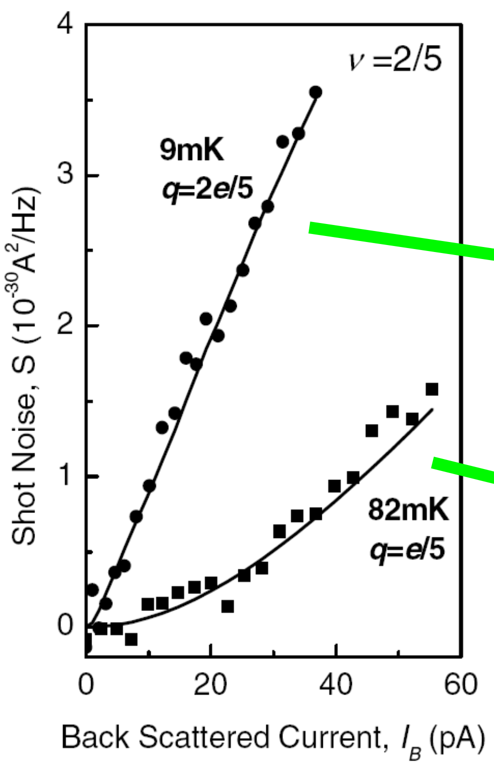
Extremely weak backscattering: noise

$$S_B = \int_{-\infty}^{+\infty} dt \langle \{ \delta I_B(t), \delta I_B(0) \} \rangle$$

$$S_B = 2e^* \coth\left(\frac{e^* V}{2k_B T}\right) I_B$$

$$S_B \approx 2e^* I_B$$
$$k_B T \ll e^* V$$

$$S_B \approx 4k_B T G_B$$
$$k_B T \gg e^* V$$



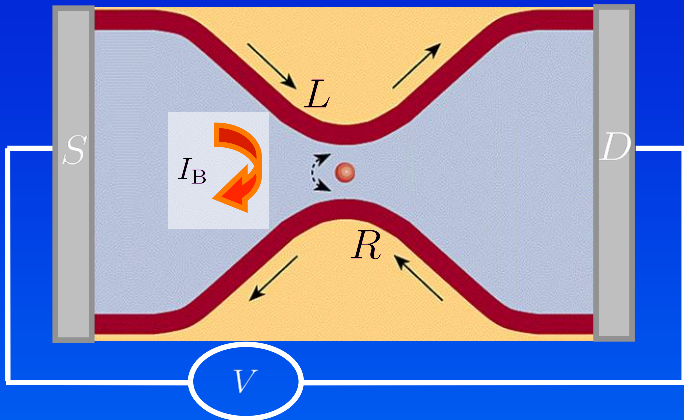
Chung et al PRL03

$$q = e_p^* = p e_1^* !!$$

bunching of qp

$$q = e_1^* = \frac{\nu}{p} e$$

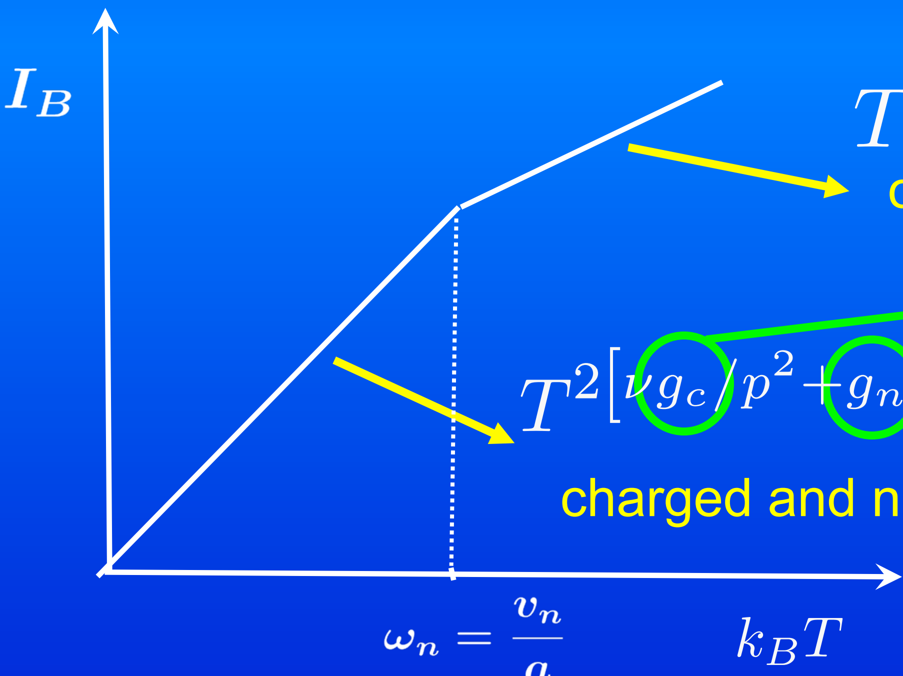
single-qp



$$H_B = t_1 \Psi_R^{(1)}(0) \Psi_L^{(1)\dagger}(0) + h.c.$$

$$\Psi^{(1)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\left[\frac{1}{|p|}\varphi_c(x) + \sqrt{1+\frac{1}{|p|}}\varphi_n(x)\right]}$$

weak backscattering and $V \to 0$



$T^{2[\nu g_c/p^2 - 1]}$

only charged modes

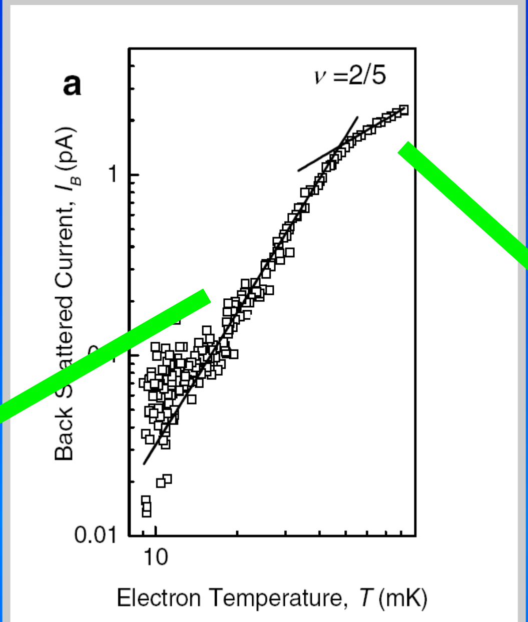
$T^{2[\nu g_c/p^2 + g_n(1+1/|p|) - 1]}$

charged and neutral modes

Renormalization parameters

Dalla Torre et al. Nat. Phys.10;
Rosenow & Halperin, PRL02;
Papa & MacDonald, PRL04;
Yang, PRL03....

Comparison with experiments



Chung et al. PRL03

$$T^{2\left[\nu g_c/p^2+g_n(1+1/|p|)-1\right]}$$

charged and neutral modes

$$T^{2\left[\nu g_c/p^2-1\right]}$$

only charged modes

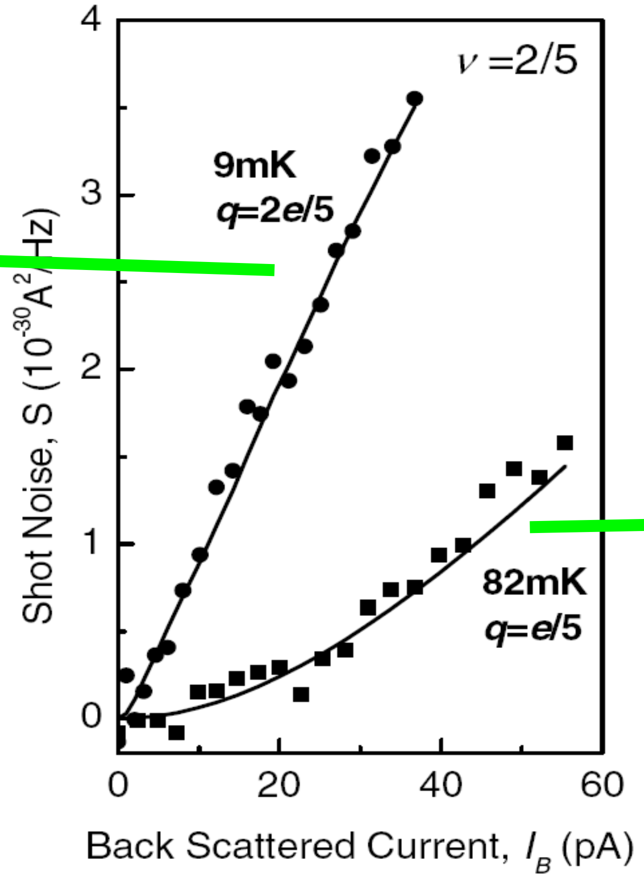
$$\omega_n \approx 50\mathrm{mK}$$

Finite velocity of neutral modes

Change in slope

Noise

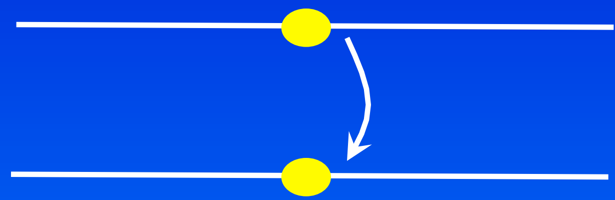
$$q = e_p^* = p e_1^* !!$$

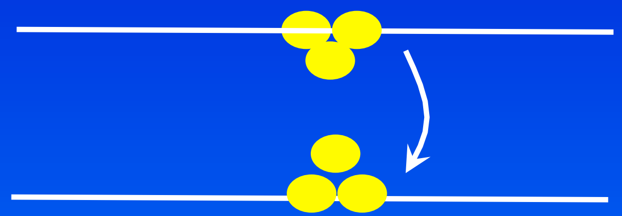


Chung et al PRL03

$$q = e_1^* = \frac{\nu}{p} e$$

Multiple-quasiparticle tunnelling





$$t_1 \Psi_R^{(1)\dagger}(0)\Psi_L^{(1)}(0) + h.c. \qquad t_m \Psi_R^{(m)\dagger}(0)\Psi_L^{(m)}(0) + h.c.$$

More relavant operators have minimal scaling dimension

Kane & Fisher, PRL92

Imaginary time Green’s function

$$\mathcal{G}_m(\tau) = \langle T_\tau \Psi^{(m)}(\tau)\Psi^{(m)}(0)\rangle$$

$$\mathcal{G}_m(\tau) = \frac{1}{2\pi a}\left(\frac{1}{1+\omega_c|\tau|}\right)^{\nu g_c \alpha_m^2}\left(\frac{1}{1+\omega_n|\tau|}\right)^{g_n \beta_m^2}$$

$$\mathcal{G}_m(\tau) \approx \tau^{-2\Delta_m}$$

Relevance

Different energy scales create two regimes

$$E \ll \omega_c, \omega_n \qquad\qquad \omega_n \ll E \ll \omega_c$$

m=p-agglomerate dominates

Kane & Fisher, PRL92;
Wen Adv. Phys. 95

single-qp dominates

$$\Psi^{(|p|)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\varphi_c(x)} \qquad \Psi^{(1)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\left[\frac{1}{|p|}\varphi_c(x) + \sqrt{1+\frac{1}{|p|}}\varphi_n(x)\right]}$$

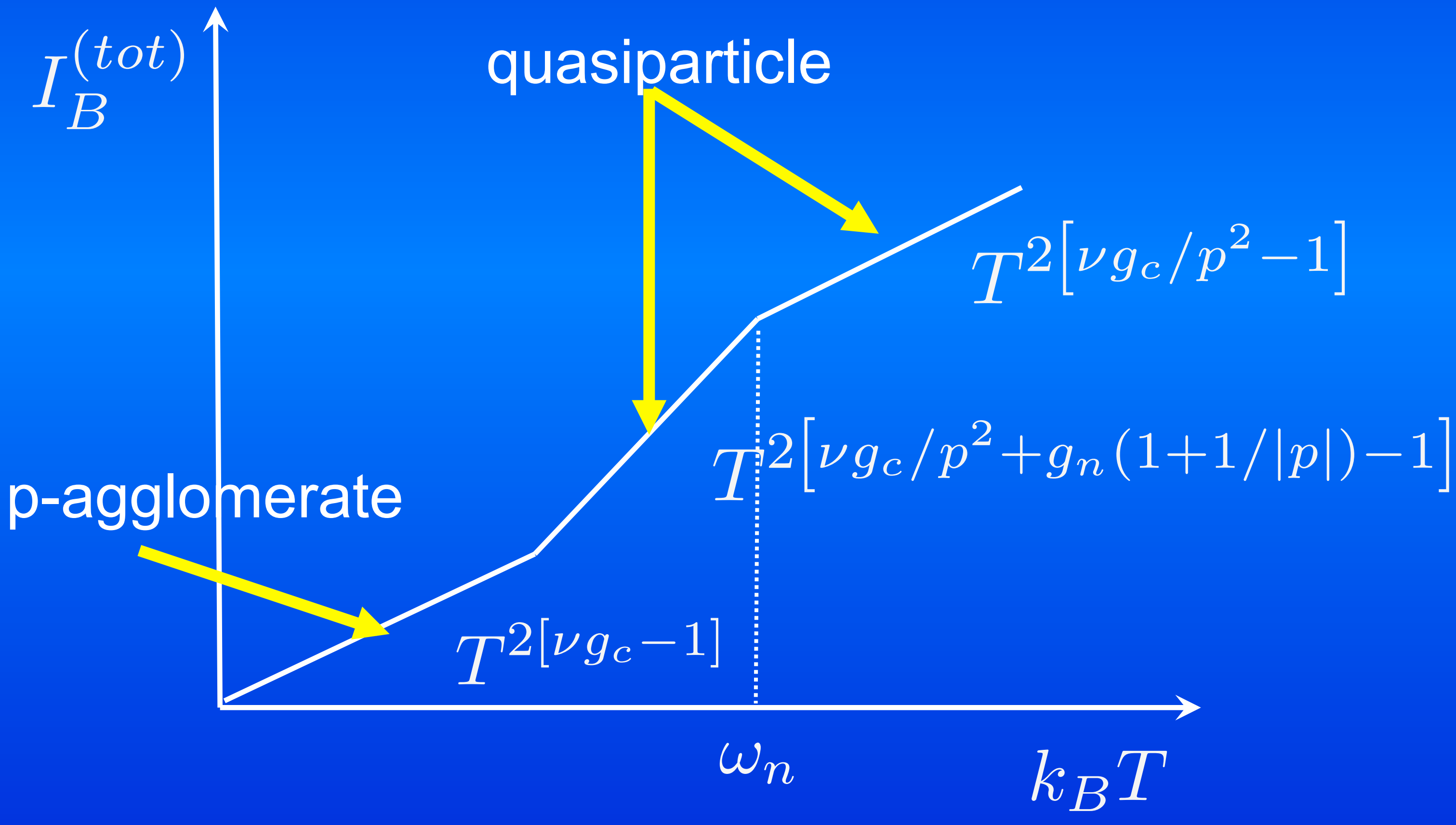
Possible crossover between the two excitations

D. F., A. Braggio, M. Merlo, N. Magnoli, M. Sassetti PRL08

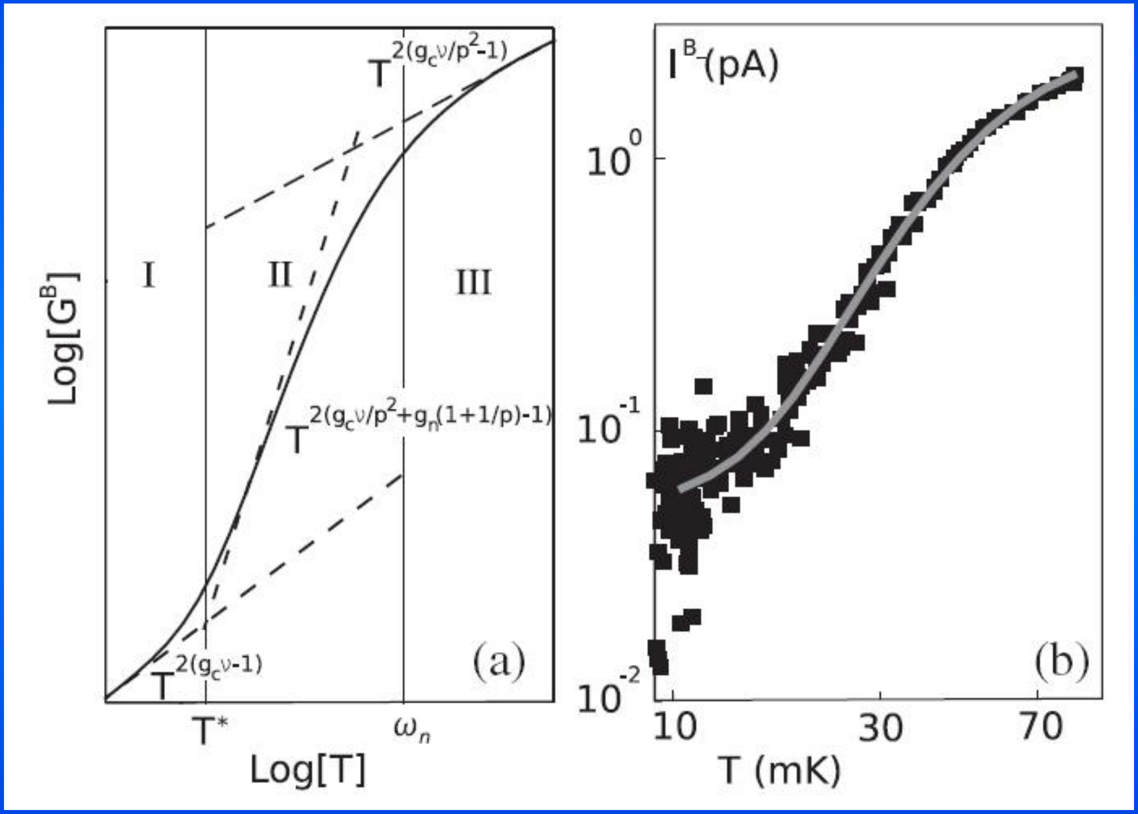
Current

$$I_B^{(tot)} = I_B^{(1)} + I_B^{(p)}$$

$$k_B T \gg e^* V$$



Fitting of the experimental data



$$\nu = 2/5, p = 2$$

$$\omega_n = 50mK$$

$$\omega_n/\omega_c = 10^{-2}$$

$$g_c = 3, g_n = 4$$

$$|t_1|^2/|t_2|^2 = 1.66$$

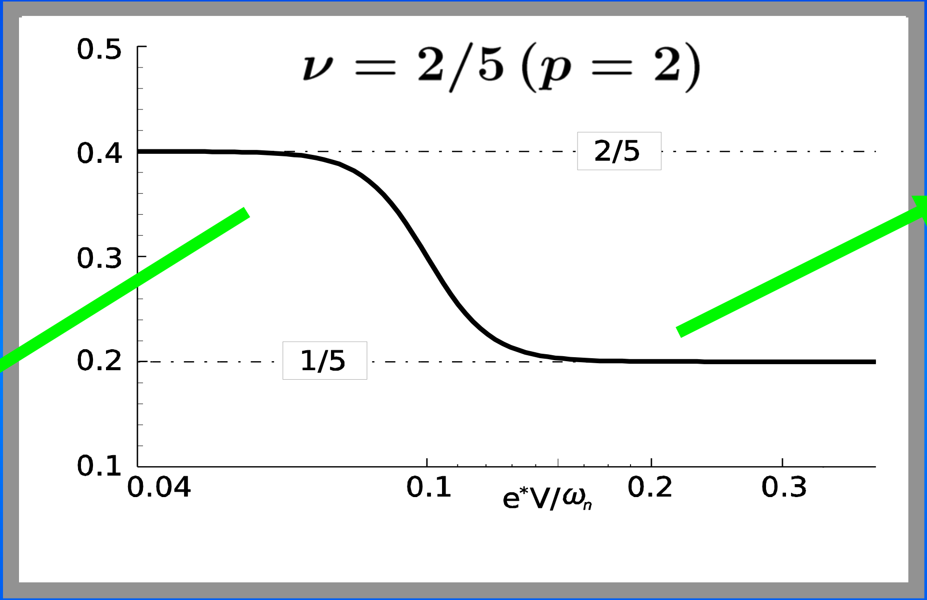
$$e^* V = 1.16mK$$

D. F., A. Braggio, M. Merlo, N. Magnoli, M. Sassetti PRL08

Shot noise

$$S^{(tot)} = 2e_1^*\left(I_B^{(1)} + pI_B^{(p)}\right) \qquad k_B T \ll e^* V$$

$$F = \frac{S^{(tot)}}{2eI_B^{(tot)}}$$



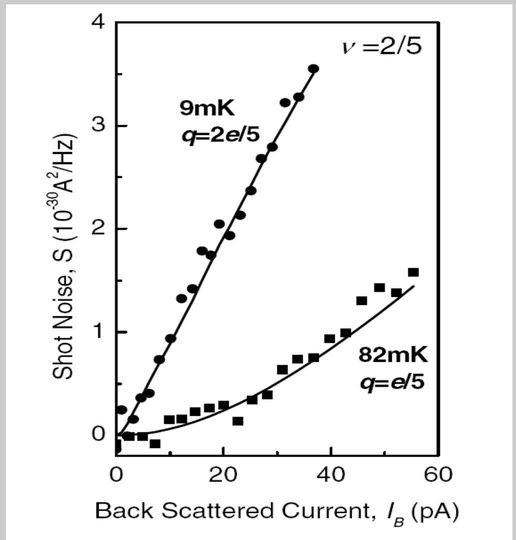
D. F. et al. PRL 08

$$e_p^* = pe_1^*$$

p-agglomerate

$$e_1^* = \frac{\nu}{p} e$$

single qp



Chung et al PRL 03

$$T = 0 \qquad g_c = 3, g_n = 4 \qquad e^* V^* = 0.1\omega_n$$

$$\omega_n = 50mK \qquad \omega_n/\omega_c = 10^{-2}$$

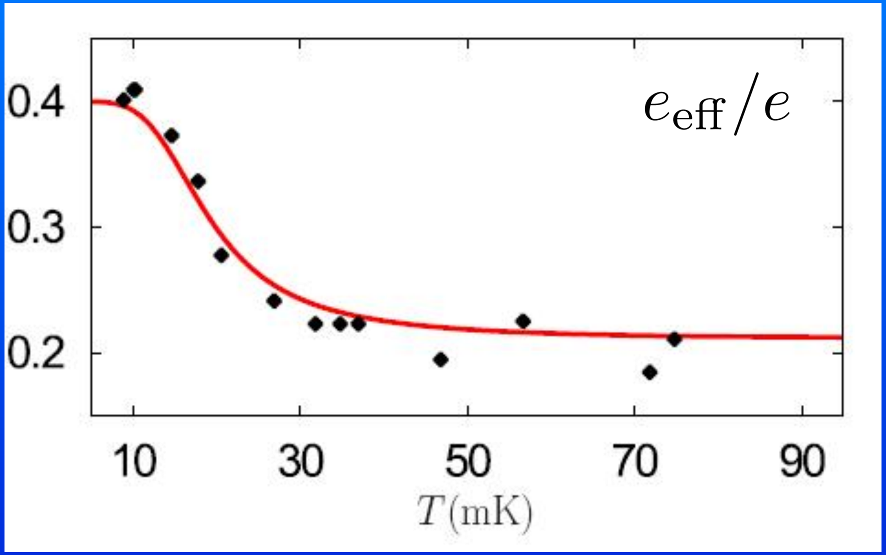
Effective charge

Excess noise

$$S_{B,ex}^{(tot)} = S_B^{(tot)} - 4k_B T G_B^{(tot)}$$

Effective charge

$$S_{B,ex}^{(tot)} = 2e_{\text{eff}} \coth\left(\frac{e_{\text{eff}} V}{2k_B T}\right) I_B^{(tot)} - 4k_B T G_B^{(tot)}$$



$$\nu = 2/5, p = 2$$

$$\omega_n = 50mK$$

$$\omega_n/\omega_c = 10^{-2}$$

$$g_c = 3, g_n = 4$$

D. F., A. Braggio, N. Magnoli, M. Sassetti, PRB 82, 085323 (2010)

Edge dynamics for $\nu = \frac{5}{2}$

Anti-Pfaffian model

Lee et al. PRL07; Levin et al. PRL07

$$\mathcal{L} = -\frac{1}{2\pi}\partial_x\varphi_c\left(\partial_t + v_c\partial_x\right)\varphi_c$$

$$-\frac{1}{4\pi}\partial_x\varphi_n\left(-\partial_t + v_n\partial_x\right)\varphi_n$$

$$-i\psi\left(-\partial_t + v_n\partial_x\right)\psi$$

$$[\varphi_c(x), \varphi_c(y)] = i\frac{\pi}{2}\mathrm{sign}(x-y)\,[\varphi_n(x), \varphi_n(y)] = -i\pi\mathrm{sign}(x-y)$$

ψ Majorana fermion

$$v_c \gg v_n$$

Hu et al. PRB09….

Multiple-quasiparticle operators

$$\Psi^{(m)}(x) \propto \chi(x) e^{i\left[\left(\frac{m}{2}\right)\varphi_c(x) + \left(\frac{n}{2}\right)\varphi_n(x)\right]}$$

$$\chi = I, \psi, \sigma$$

m, n even $\qquad$ m, n odd

$$\sigma \times \sigma = I + \psi$$

Non-Abalian statistics

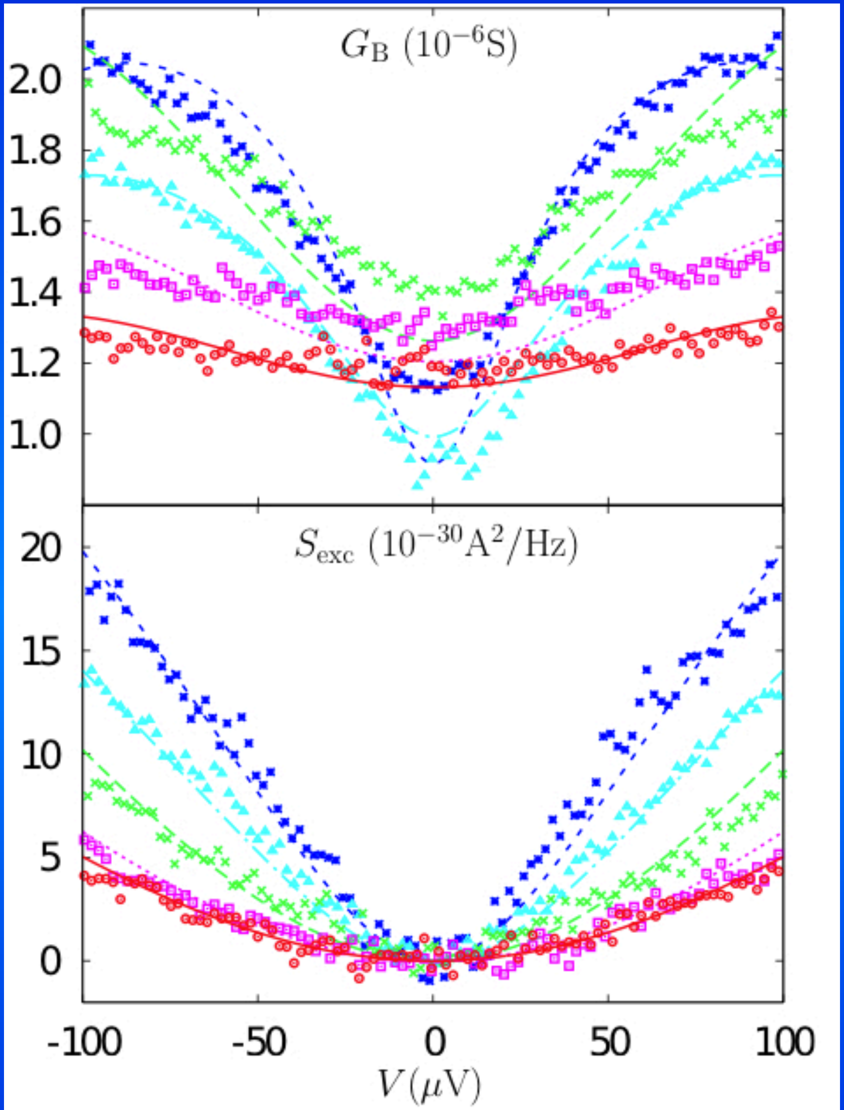
single-qp $\quad q = \frac{e}{4}$

$$\Psi^{(1)}(x) \propto \sigma(x) e^{i\left[\left(\frac{1}{2}\right)\varphi_c(x) + \left(\frac{1}{2}\right)\varphi_n(x)\right]}$$

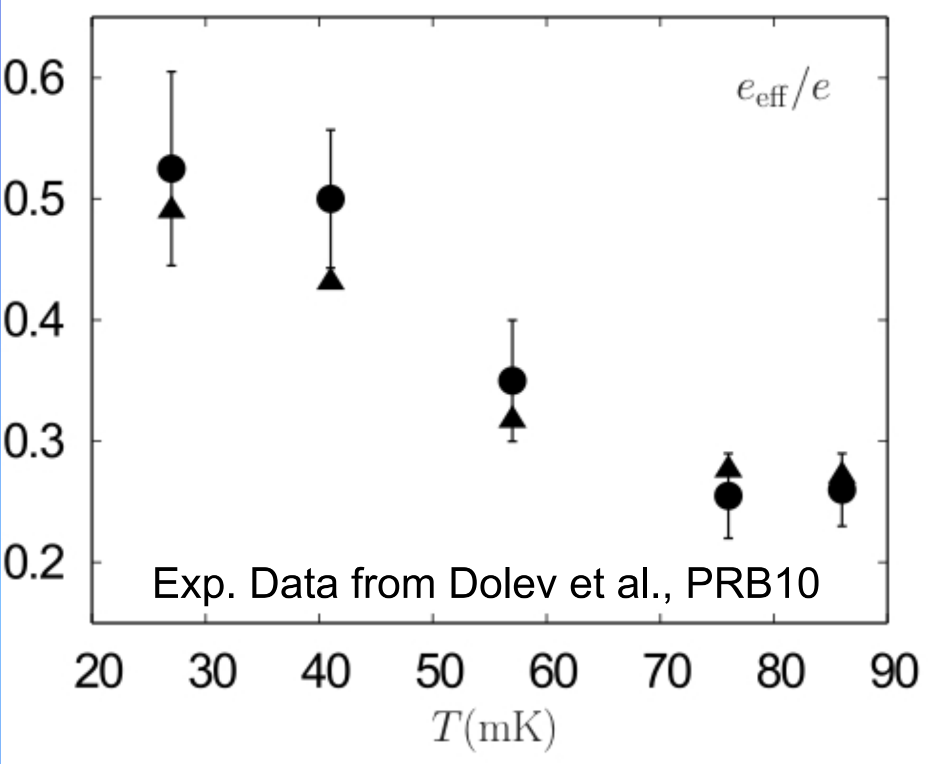
2-agglomerate $\quad q = \frac{e}{2}$

$$\Psi^{(2)}(x) \propto e^{i\varphi_c(x)}$$

Comparison with experimental data



Raw data with the courtesy of M. Heiblum



$$g_c = 2.8 \quad g_n = 8.5$$

$$\omega_n = 150mK$$

$$\omega_c = 500mK$$

M. Carrega, D. F., A. Braggio, N. Magnoli, M. Sassetti, arXiv:1102.5666
(to appear on PRL)

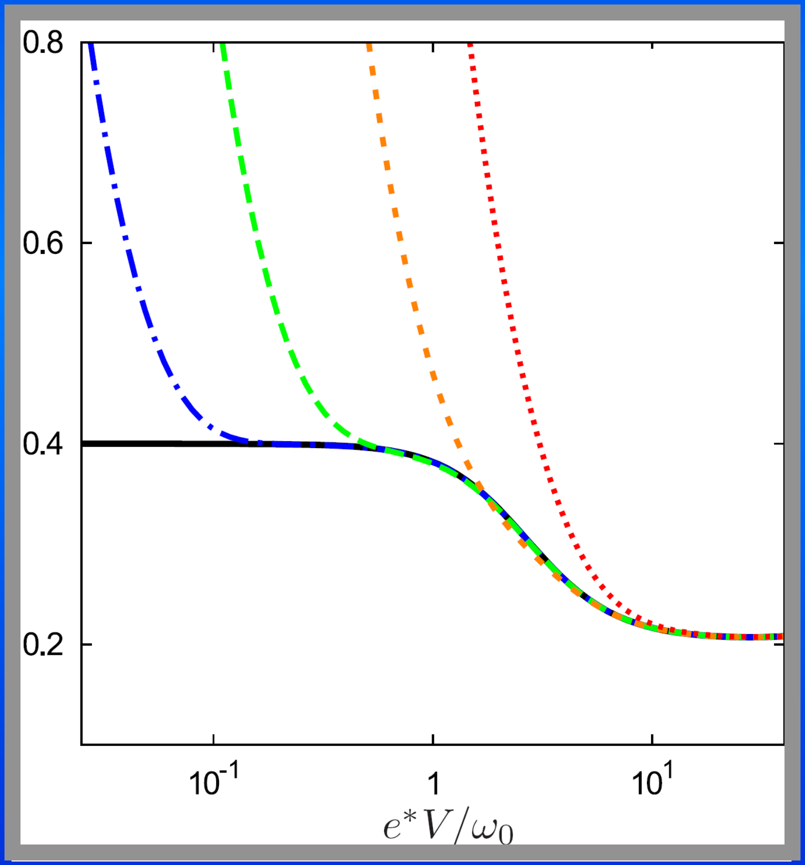
Conclusions

- Tunnelling in the composite edge states of the FQHE
- Effects of the neutral modes dynamics
- Relevance of agglomerate tunnelling

D. F., A. Braggio, M. Merlo, N. Magnoli, M. Sassetti, PRL 101 166805 (2008);
D. F., A. Braggio, N. Magnoli, M. Sassetti, NJP 12 013012 (2010);
D. F., A. Braggio, N. Magnoli, M. Sassetti, Physica E 42 580 (2010);
D. F., A. Braggio, N. Magnoli, M. Sassetti, PRB 82, 085323 (2010);
M. Carrega, D. F., A. Braggio, N. Magnoli, M. Sassetti, arXiv:1102.5666
(to appear on PRL)

Fano: temperature effect

$$S^{(m)} = 2me^* \coth\left(\frac{me^*V}{2k_BT}\right) I_B^{(m)}$$



$\nu = 2/5, p = 2$

$\omega_n/\omega_c = 10^{-2}$

$g_c = 1, g_n = 1$

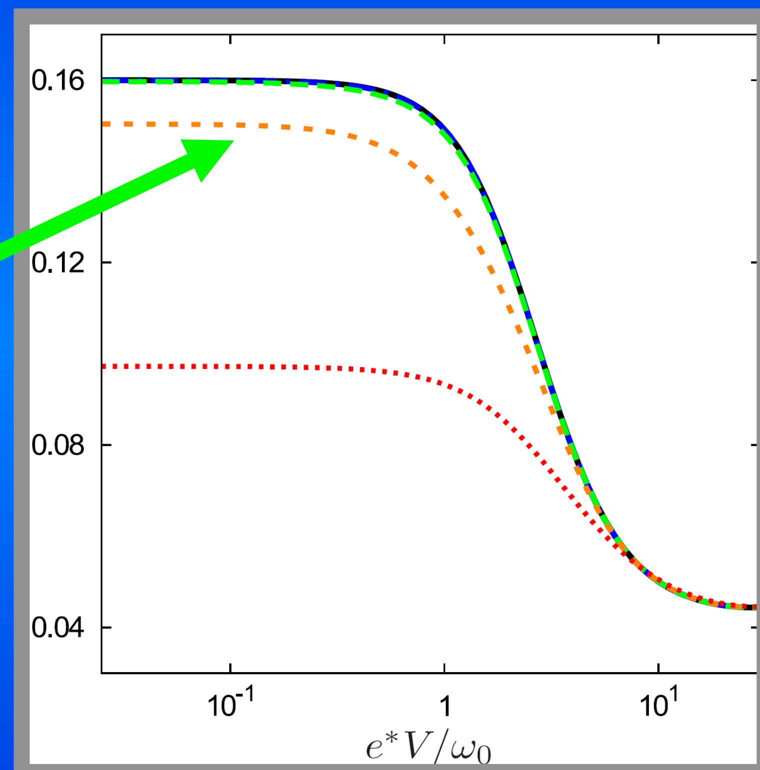
$(|t_2|/|t_1|) = 10$

D. F., A. Braggio, N. Magnoli, M. Sassetti, NJP 12 013012 (2010)

Skewness

$$F_3 = \frac{(e^*)^2 I_B^{(1)} + (pe^*)^2 I_B^{(p)}}{e I_B^{(tot)}}$$

More stable against thermal effect



D. F. et al. NJP 10

$$\nu = 2/5, p = 2 \qquad g_c = 1, g_n = 1$$

$$\omega_n/\omega_c = 10^{-2} \qquad (|t_2|/|t_1|) = 10$$

Edge-"phonon" interaction

Chiral Luttinger Liquid coupled with 1D phonons

Rosenow & Halperin, PRL02

$$S_{\chi LL} = \frac{1}{4\pi\nu}\int_0^\beta d\tau \int_{-\infty}^{+\infty} dx \partial_x \varphi \left(i\partial_\tau + v\partial_x\right)\varphi$$

$$S_{\rm ph} = \frac{1}{8\pi\tilde{\nu}u}\int_0^\beta d\tau \int_{-\infty}^{+\infty} dx \xi \left(-\partial_\tau^2 - u^2\partial_x^2\right)\xi$$

$$S_{\rm int} = \lambda \int_0^\beta d\tau \int_{-\infty}^{+\infty} dx \partial_x \xi \partial_x \varphi$$

Imaginary time Green's function

$$\mathcal{D}(0,0,\Omega_n) = g(\rho,\eta)\frac{2\pi\nu}{\Omega_n}$$

Renormalization

$$g(\rho,\eta) = \int_{-\infty}^{+\infty}\frac{dx}{\pi}\frac{(1+\rho^2x^2)(1+x^2\rho(\rho-\eta^2))}{1+x^2(2\rho^2+1)+x^4\rho(\rho^3+2(\rho-\eta^2))+x^6\rho^2(\rho-\eta^2)^2}$$

1/f noise and dissipation (1)

Dalla Torre et al., Nature Physics 10

$$S = \frac{1}{4\pi\nu} \int_{-\infty}^{+\infty} dx \int_0^{\beta} d\tau \partial_x \varphi (i\partial_\tau + v\partial_x)\varphi$$

$$S_{1/f} = i \int_{-\infty}^{+\infty} dx \int_0^{\beta} d\tau \rho(x,\tau) f(x,\tau)$$

Out of equilibrium noise with spectral density

$$\langle \tilde{f}(\omega,q) \tilde{f}^*(\omega,q) \rangle = \frac{F}{|\omega|}$$

Absorbed energy dissipated by the cooling setup

$$S_{\mathrm{diss}} = \frac{\gamma}{4\pi} \int_{-\infty}^{+\infty} dx \int_0^{\beta} d\tau d\tau' \frac{\pi^2 \left(\varphi(x,\tau) - \varphi(x,\tau')\right)^2}{\beta^2 \sin^2 \left[\pi(\tau - \tau')/\beta\right]}$$

1/f noise and dissipation (2)

Green's functions in the Keldysh picture

$$\mathbf{G} = \begin{pmatrix} \dfrac{2i\gamma|\omega| + 2i\frac{q^2}{(2\pi\nu)^2}\frac{F}{\gamma}}{\frac{1}{(2\pi\nu)^2}(\omega+vq)^2q^2+\gamma^2\omega^2} & \dfrac{1}{\frac{1}{2\pi\nu}(\omega+vq)q+i\gamma\omega} \\ \dfrac{1}{\frac{1}{2\pi\nu}(\omega+vq)q-i\gamma\omega} & 0 \end{pmatrix}$$

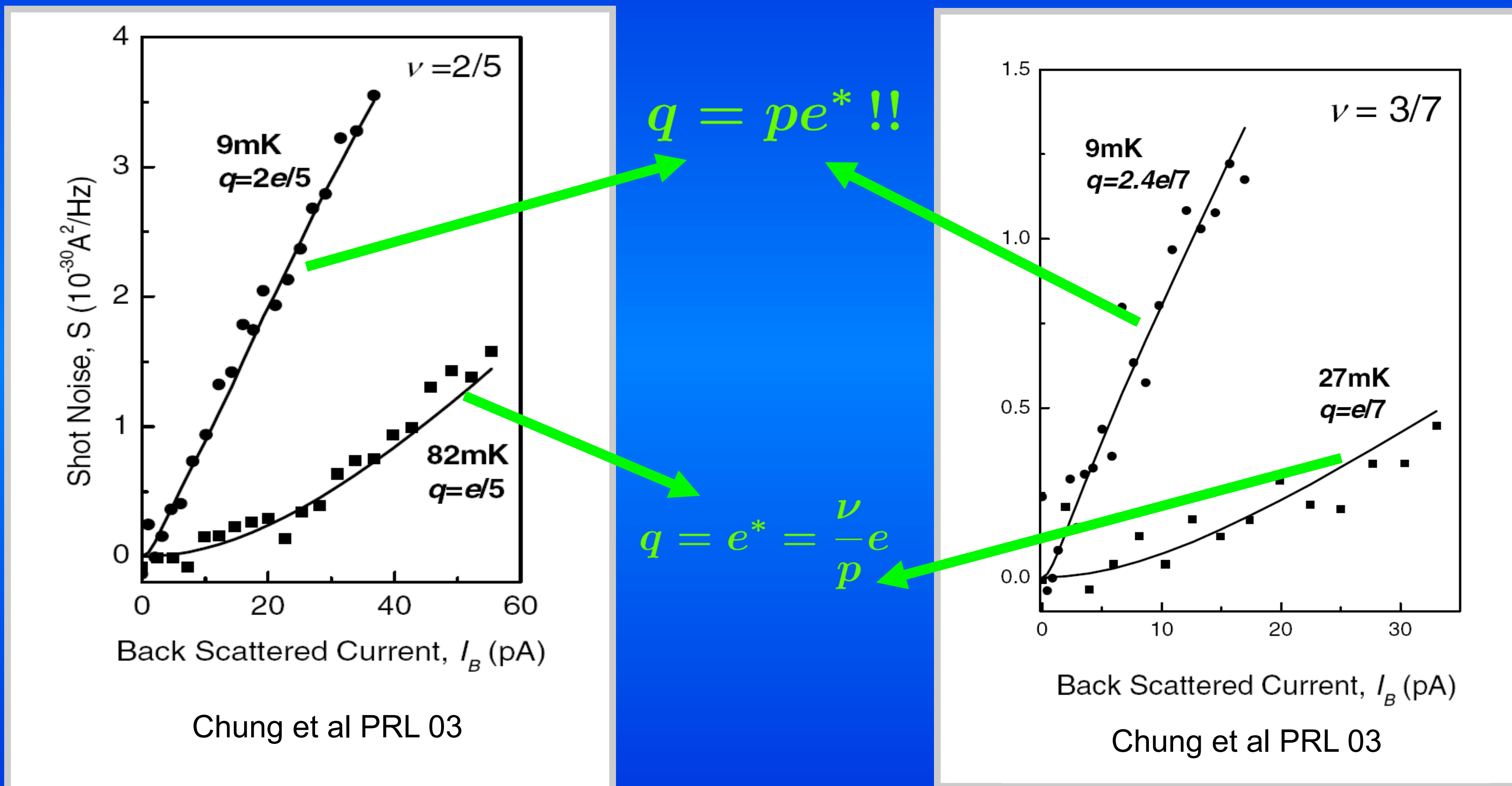
$$G^K = \langle\varphi^{\mathrm{cl}}(0,t)\varphi^{\mathrm{cl}}(0,0)\rangle = -\nu g \ln\left(1 + i\omega_\rho t\right)$$

Renormalization

$$g = 1 + \frac{1}{(2\pi)^2}\frac{F}{\gamma}$$

1/f and dissipation are relevant perturbations with massive coupling constants, it is possible to extend this approach to the disorder-dominated phase of the composite edge states

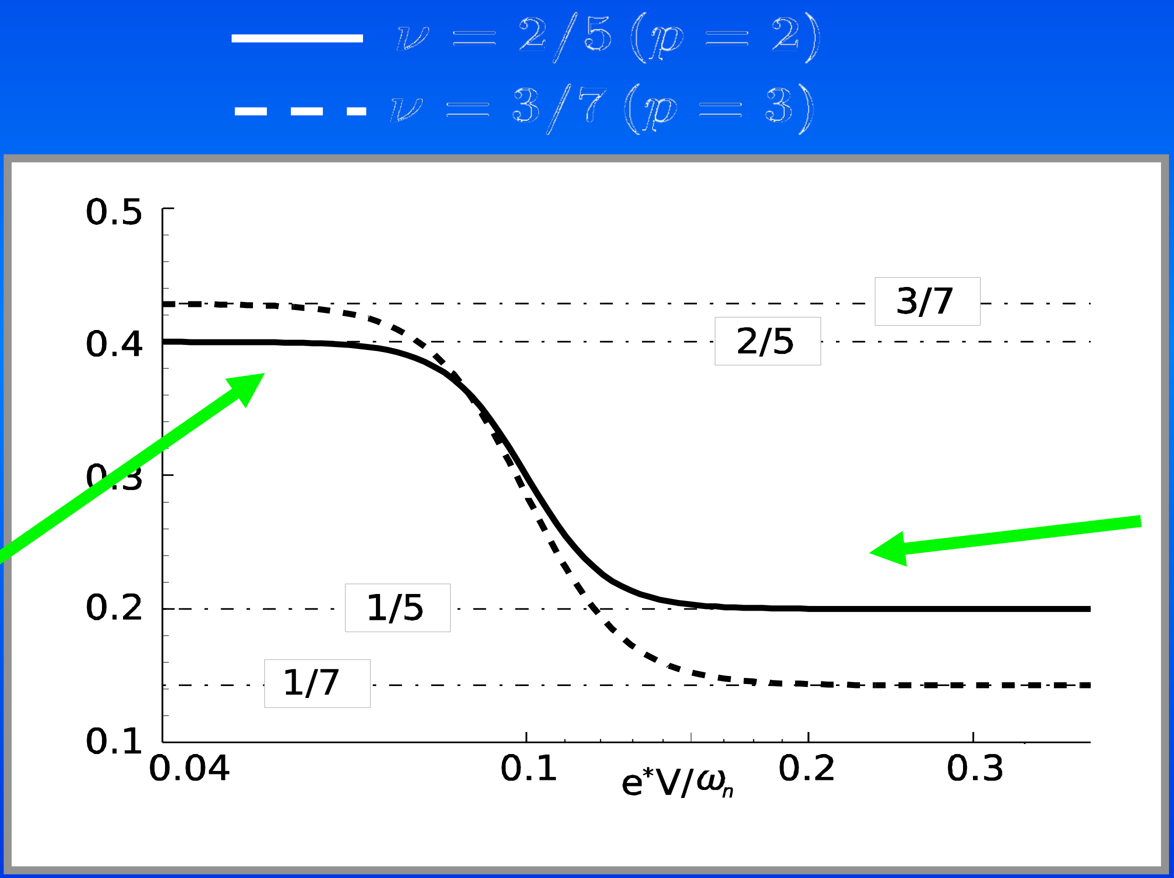
Noise



Shot Noise, S (10^{-30}A^2/Hz)
ν =2/5
9mK
q=2e/5
82mK
q=e/5
Back Scattered Current, I_B (pA)
Chung et al PRL 03
$q = pe^*$!!
$q = e^* = \frac{\nu}{p} e$
ν = 3/7
9mK
q=2.4e/7
27mK
q=e/7
Back Scattered Current, I_B (pA)
Chung et al PRL 03

$$S = 2(e_1^* I_B^{(1)} + e_p^* I_B^{(p)}) = 2e_1^*(I_B^{(1)} + p I_B^{(p)})$$

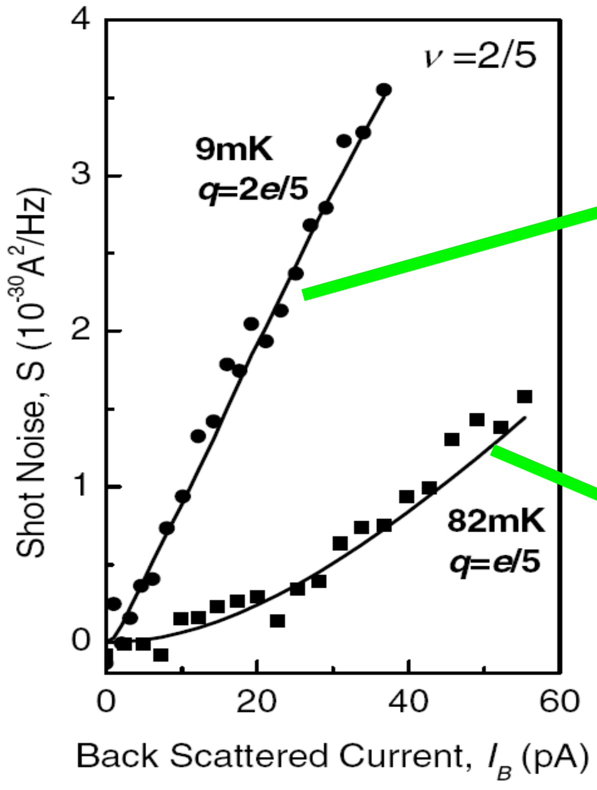
$$F = \frac{S}{2eI_B^{(tot)}}$$



$$e_p^* = \nu e$$

$$e_1^* = \frac{\nu}{p} e$$

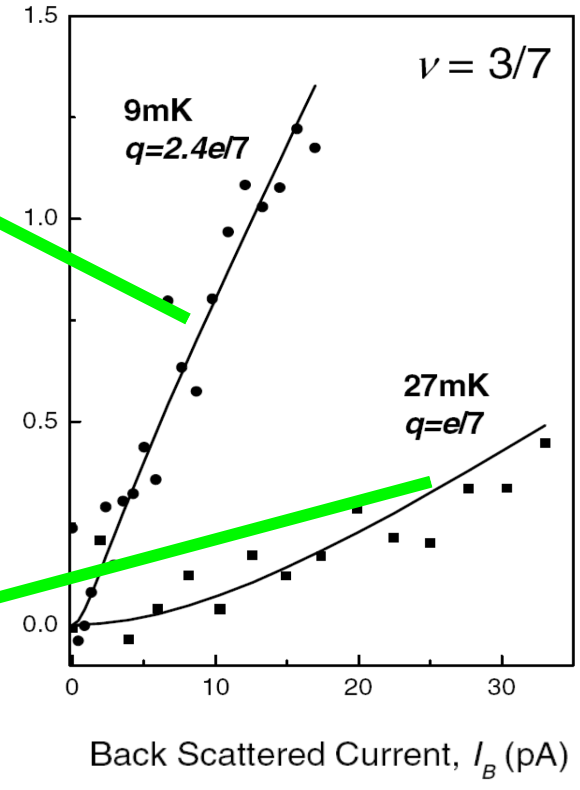
Extremely weak backscattering: noise



Chung et al PRL 03

$$q = pe^* \text{ !!}$$

$$q = e^* = \frac{\nu}{p} e$$



Chung et al PRL 03

extremely weak backscattering $(t \to 0)$

$I_B \propto t^2 V T^{2\nu-2}$ $\quad eV \ll k_B T$

$I_B \propto t^2 V^{2\nu-1}$ $\quad eV \gg k_B T$

→ mode dynamics

general solutions at any order in t

Moon, Yi, Kane, Fisher PRL 93 (MC simulations)

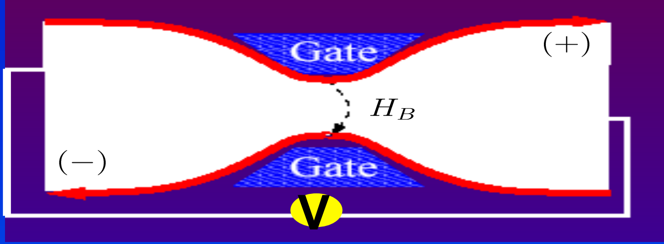
Yue, Matveev, Glazman PRB 94 (weak interaction expansion $\nu = 1 - \epsilon$)

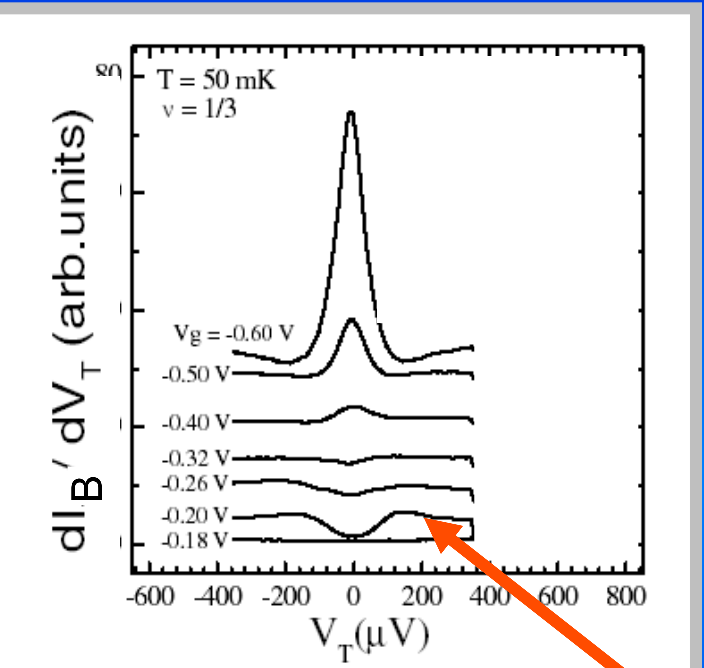
Fendley, Ludwig, Saleur PRL 95, 96 (thermodinamic Bethe Ansatz)

Weiss, Egger, Sassetti PRB 95 (real time P.I. $\nu = 1/2 + \epsilon$)

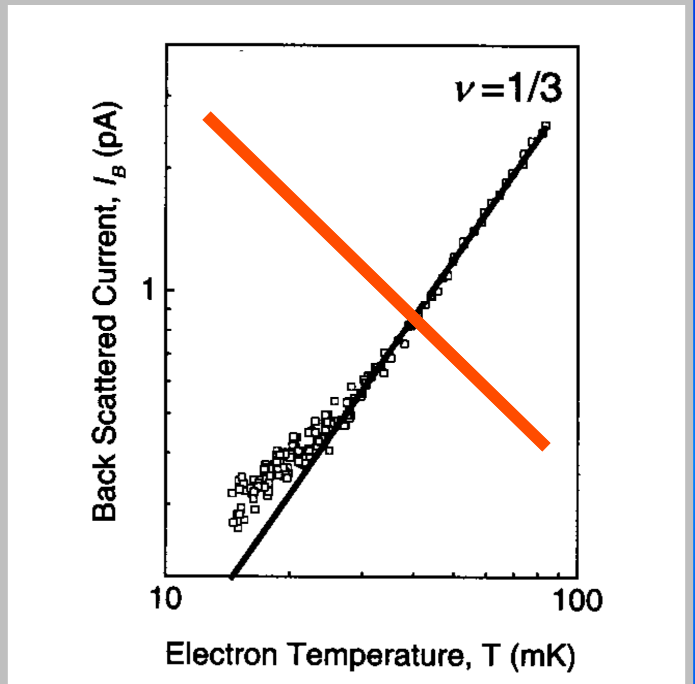
Aristov, Woelfle EPL 08 (fermionic representation, RG equation)

extremely weak backscattering





Roddaro et al. PRL 04



Chung, Heiblum, Umansky PRL 03

theory

$$G_B \propto T^{2/3-2}$$

negative slope !

non-universal power law exponent !

minimum !

Other experimental deviations

Chang et al., PRL 96; Grayson et al. PRL 98; Glattli et al. Physica E 00;
Chang et al. PRL 01; Grayson et al. PRL 01; Hilke PRL 01....

most relevant operators for tunneling processes

$$\mathcal{G}_m^k(\tau) = \langle T_\tau [\Psi_k^{(m)}(0,\tau)\Psi_k^{(m)\dagger}(0,0)]\rangle$$

$$\mathcal{G}_m^k(\tau) = \frac{1}{2\pi a}\left(\frac{1}{1+\omega_c|\tau|}\right)^{\nu\alpha_m^2}\left(\frac{1}{1+\omega_n|\tau|}\right)^{(\beta_m^k)^2}$$

$$\approx \tau^{-2\Delta_m^k}$$

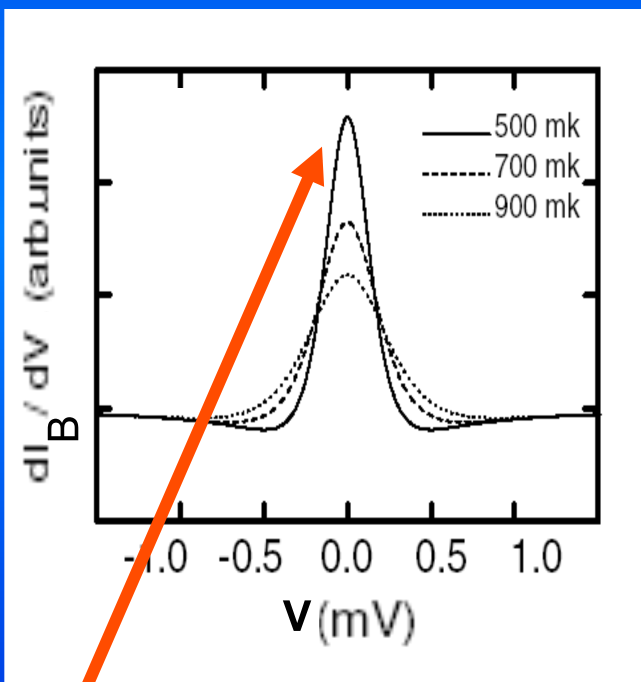
extremely weak backscattering $(t \to 0)$

$$I_B \propto t^2 V T^{2\nu - 2}[1 - c(\nu)(V/T)^2] \qquad (eV \ll k_B T)$$

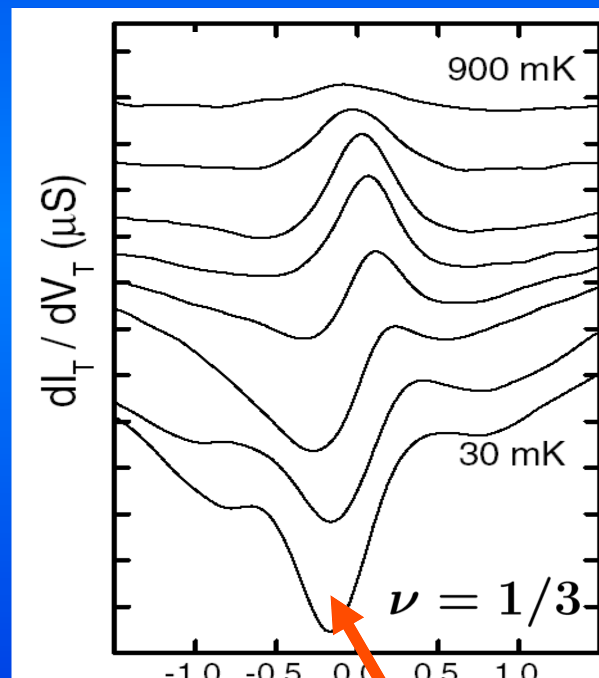
$c(\nu) > 0$

theory

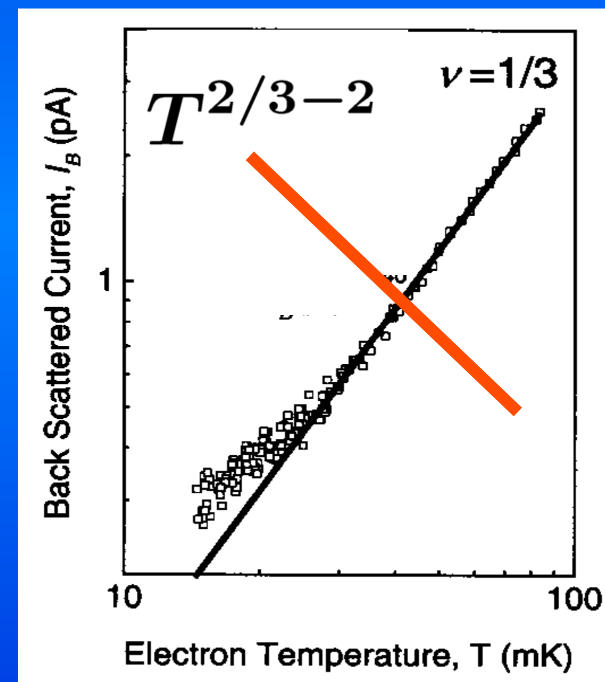
experiments



$\nu = 1/3$



Roddaro et al. PRL 03, PRL 04

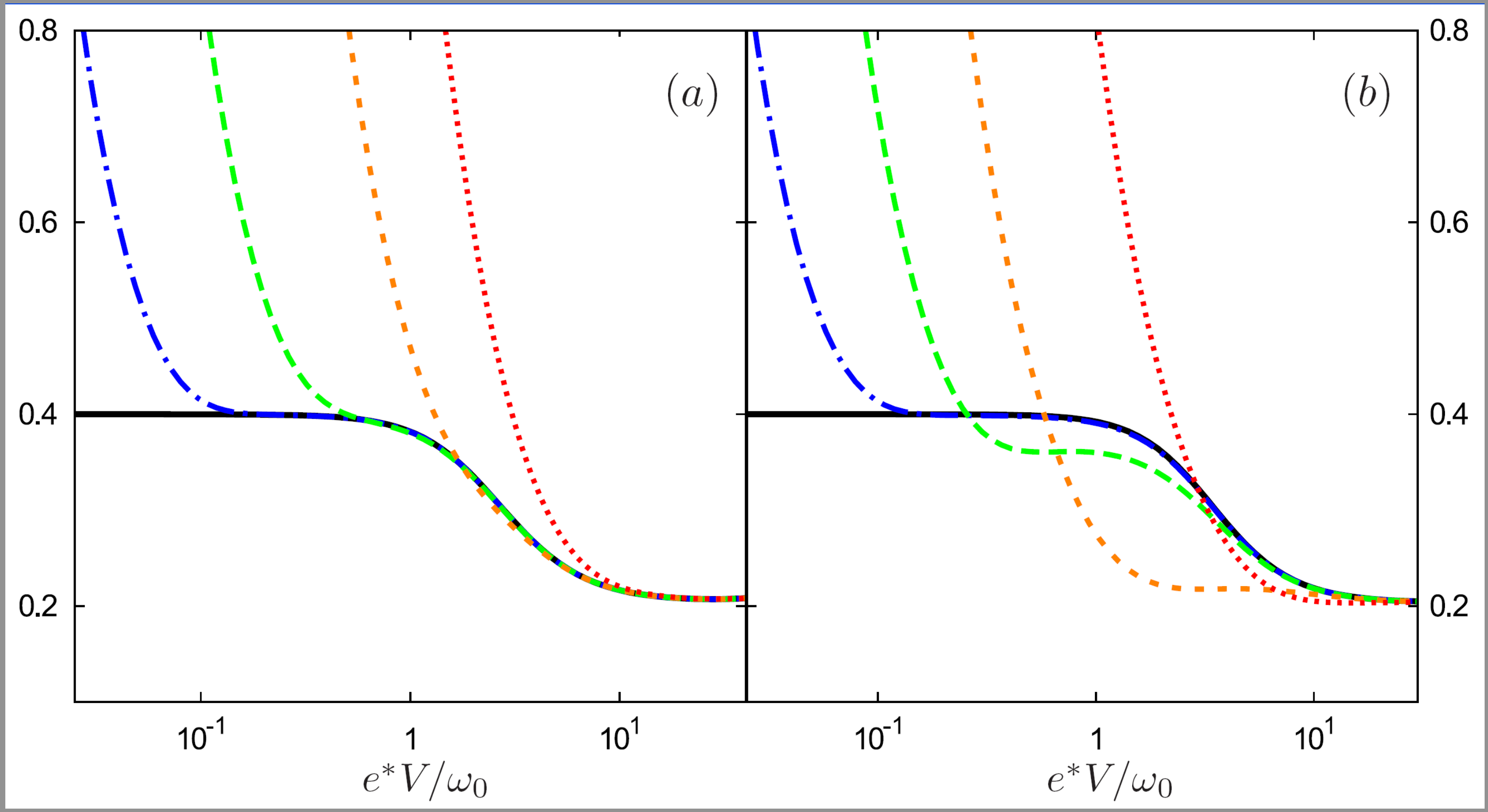


Chung et al. PRL 03

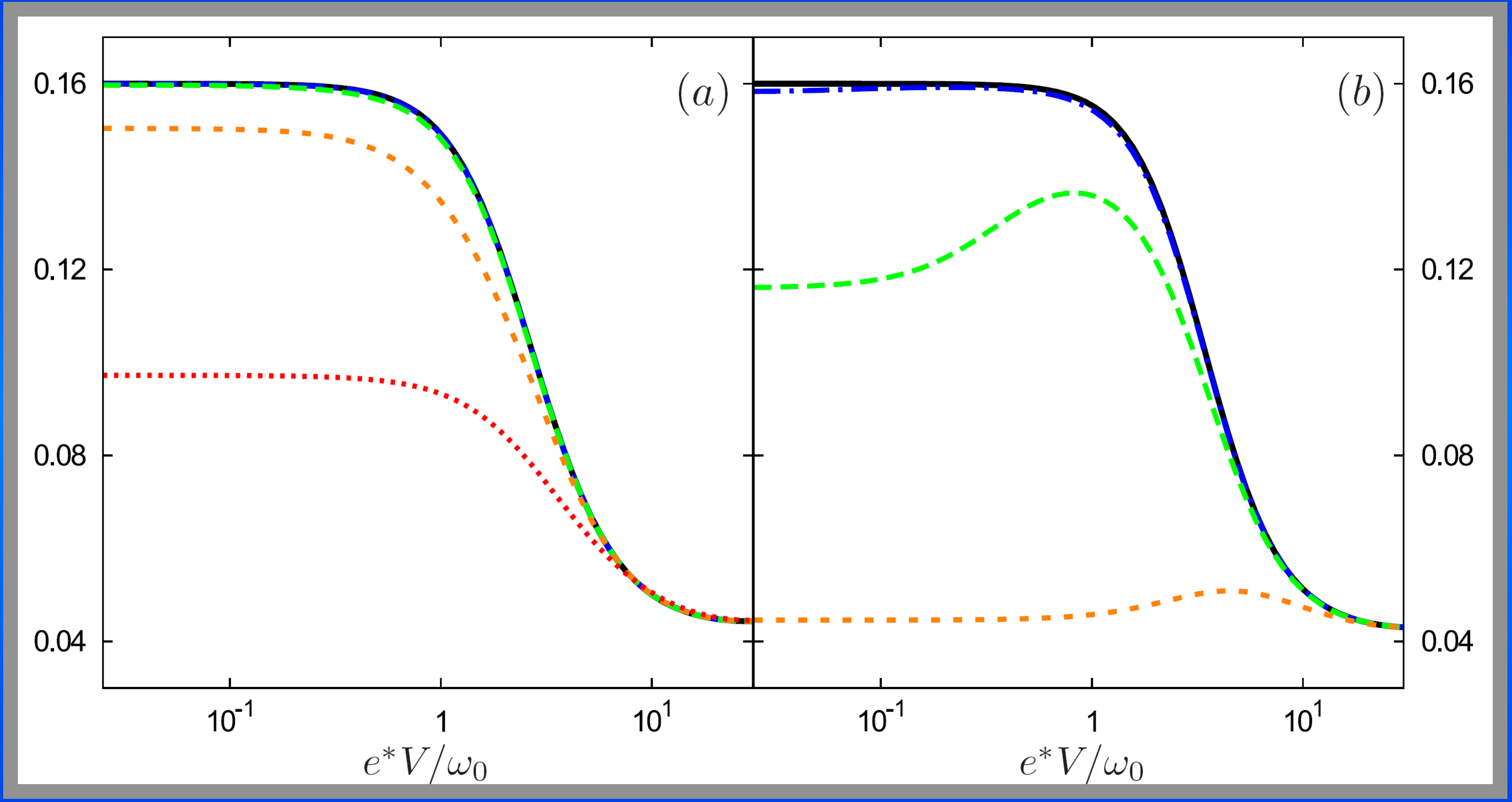
maximum

minimum

theory:
negative slope


0.8
0.6
0.4
0.2
(a)
(b)
10^-1
1
10^1
e^*V/ω_0
e^*V/ω_0

Skewness



D. F., A. Braggio, N. Magnoli, M. Sassetti, NJP 12 013012 (2010)

Models for the Jain sequence

A single mode model can only describe states of the Laughlin sequence

We need to introduce additional neutral modes

Hierarchical model
Wen, Zee, PRB 92

p-1 neutral fields

Fradkin-Lopez model
Lopez, Fradkin, PRB 99

two neutral fields

We consider a minimal model with only one neutral mode

Neutral mode

$$\phi_n(x)$$

dynamical

$$0 \neq v_n \ll v_c$$

Wen, Zee, PRB 92;
Kane, Fisher, Polchinski,PRL 94;
Lee, Wen, cond-mat 9809160;
Levkivskyi, Sukhorukov,PRB 08;
D. F. et al. PRL 08;
D. F. et al. NJP 10

topological

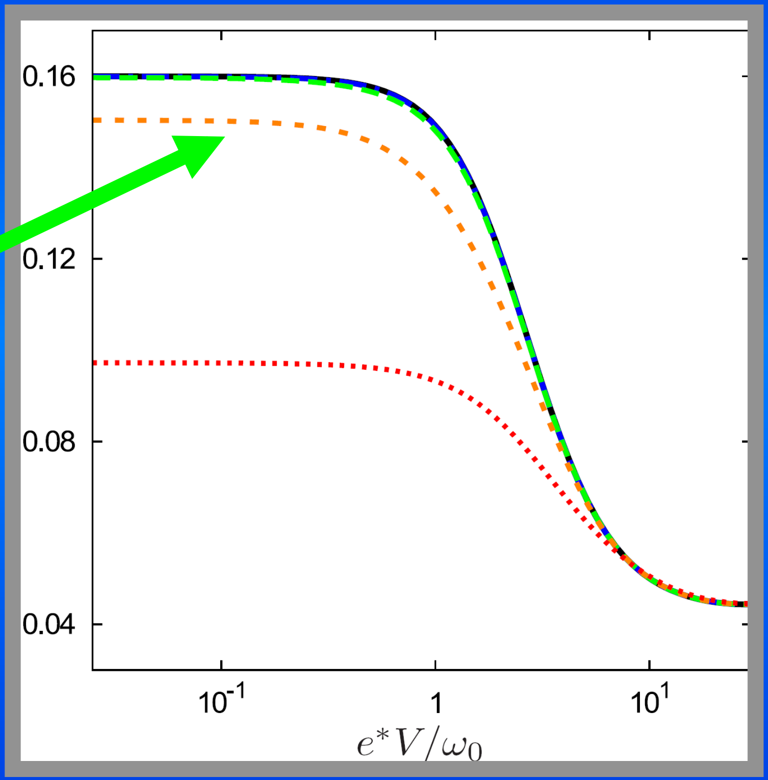
$$v_n \rightarrow 0$$

Lopez, Fradkin, PRB 99, PRB 01;
Chamon, Fradkin, Lopez, PRL 07

Skewness

$$F_3 = \frac{(e^*)^2 I_B^{(1)} + (pe^*)^2 I_B^{(p)}}{e I_B^{(tot)}}$$

More stable against thermal effect

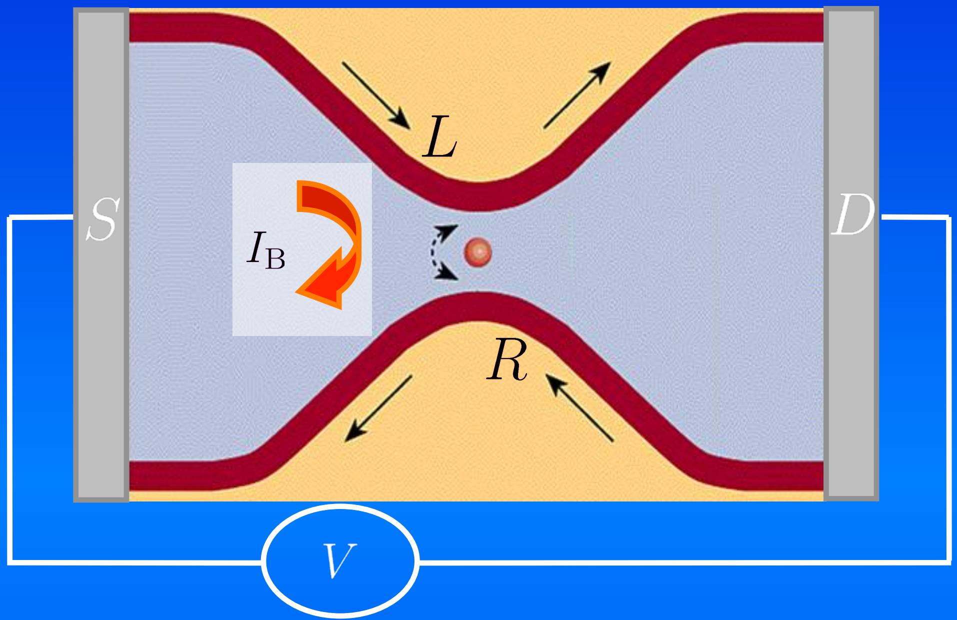


D. F. et al. NJP 10

$\nu = 2/5, p = 2$ $g_c = 1, g_n = 1$

$\omega_n/\omega_c = 10^{-2}$ $(|t_2|/|t_1|) = 10$

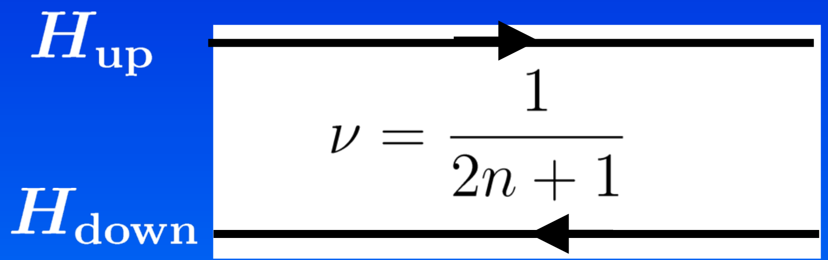
Quantum Point Contact geometry



$$H_B = t\psi_{qp}^{(+)}(x=0)\psi_{qp}^{(-)\dagger}(x=0) + \text{h.c.}$$

$$I = \nu \frac{e^2}{h} V - I_B$$

Edge states in the Laughlin sequence



chiral Luttinger liquids

$$H_{\mathrm{up}} = \frac{v_c}{4\pi\nu}\int dx\,(\partial_x \phi_c(x))^2$$

$$[\phi_c(x), \phi_c(x')] = i\pi\nu\,\mathrm{sgn}(x - x') \qquad \rho(x) = \frac{\partial_x \phi_c(x)}{2\pi}$$

Halperin PRB 82; Wen PRL 90, PRB 90,91; MacDonald PRL 90; Lopez, Fradkin PRB 99 ...

Quasiparticles excitations

Fractional charge

$$e^* = \nu e$$

Fractional statistics

$$\Psi_{qp}(x)\Psi_{qp}(x') = \Psi_{qp}(x')\Psi_{qp}(x)e^{-i\theta\,\mathrm{sgn}(x-x')}$$

$$\theta = \pi\nu$$

Laughlin PRL 83; Arovas, Schrieffer, Wilczek PRL 84.

Bosonization

$$\Psi_{qp}(x) = \frac{1}{\sqrt{2\pi a}}e^{i\sigma}$$

Extremely weak backscattering

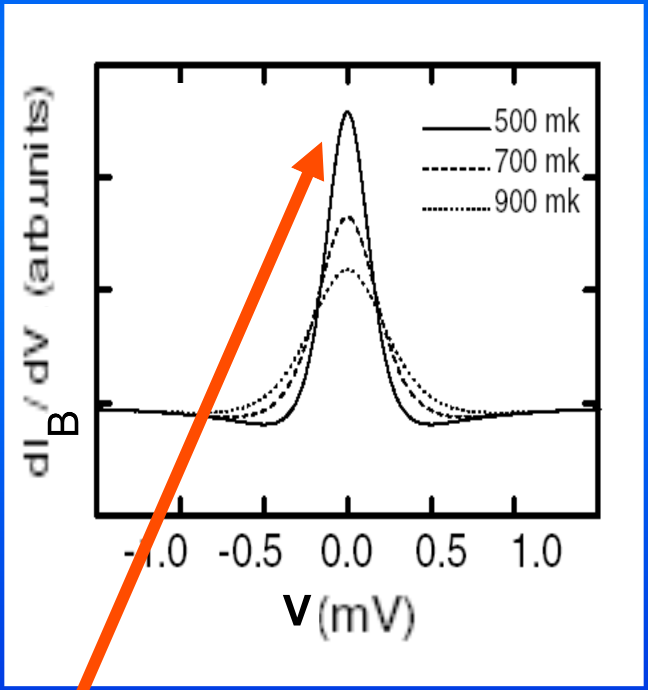
$$(t \to 0)$$

$$I_B \propto t^2 V T^{2\nu - 2}[1 - c(\nu)(V/T)^2] \quad (eV \ll k_B T)$$

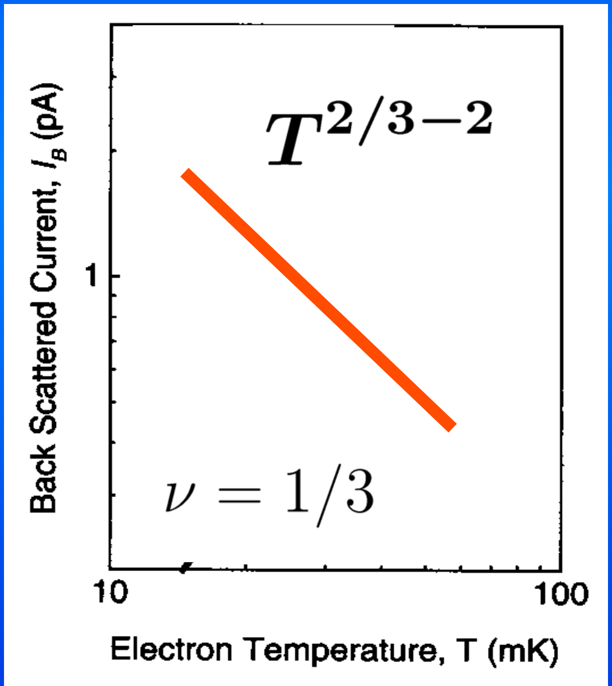
$c(\nu) > 0$

theory

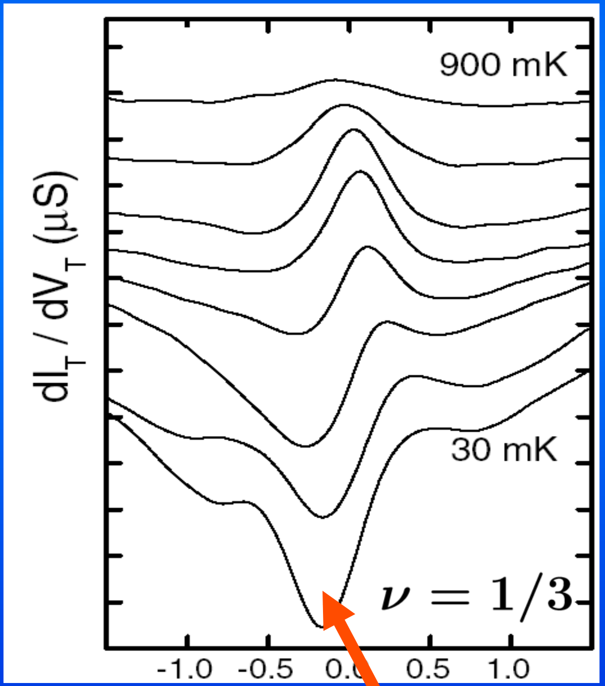
experiments



$\nu = 1/3$



Chung et al. PRL 03



Roddaro et al. PRL 03, PRL 04

maximum

minimum

Non-universal power law exponents!

deviations also for electron tunneling between edge-metal and edge-edge:
Chang et al., PRL 96; Grayson et al. PRL 98; Glattli et al. Physica E 00; Chang et al. PRL 01; Grayson et al. PRL 01; Hilke PRL 01, Grayson SSC 06 ....

Several proposals

e-ph coupling (Heinonen & Eggert PRL 96, Rosenow & Halperin PRL 02, Klhlebnikov PRB 06)

e-e interaction (Imura EPL 99, Mandal & Jain PRL 02, Papa & MacDonald PRL 05, D'Agosta et al. PRB 03)

edge reconstruction (MacDonald et al. J. Phys 93, Chamon & Wen PRB 94, Wan et al. PRL 02, Yang PRL 03)

local filling factor (Sandler et al PRB 98, Roddaro et al. PRL 04, 05, Lal EPL 07, Rosenow & Halperin cond-mat 0806.0869)

Renormalization of the Luttinger parameter

$$\nu \longrightarrow g\nu$$

Zero frequency noise

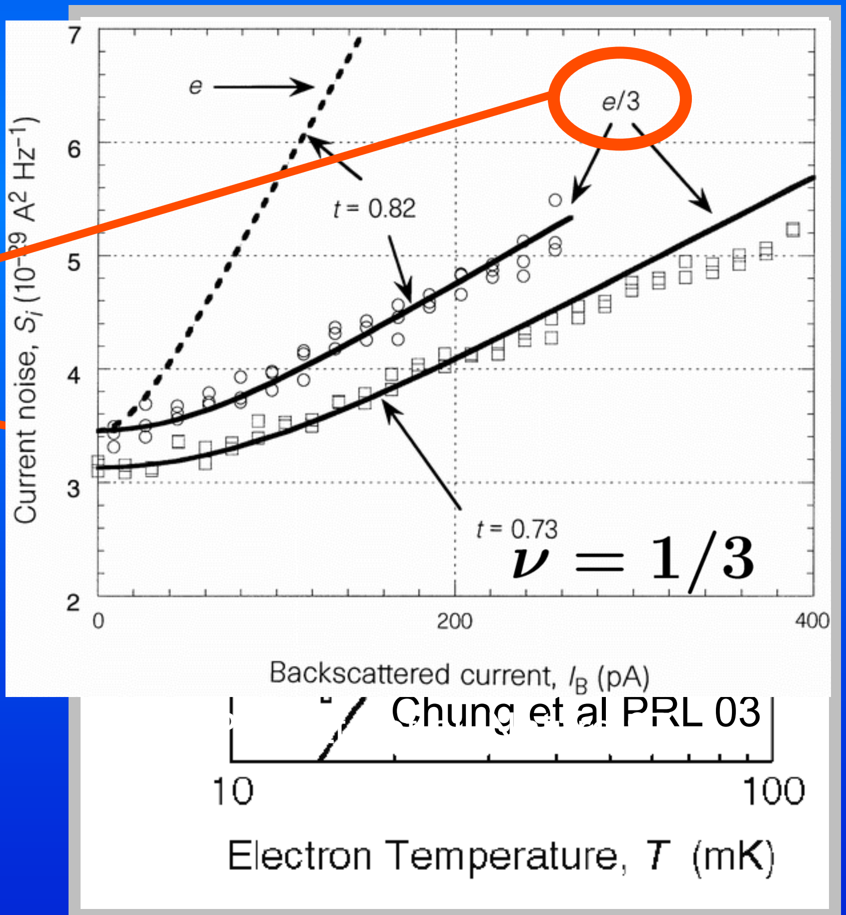
$$S(\omega = 0) = \int_{-\infty}^{\infty} dt \langle \{ \delta I_B(t), \delta I_B(0) \} \rangle$$

weak backscattering → Poissonian process

$$S = 2e^* I_{\rm B} \quad \text{for} \quad k_{\rm B}T \ll e^* V$$

Kane, Fisher PRL 94;
Fendley, Ludwig, Saleur PRL 95

$$e^* = \nu e$$

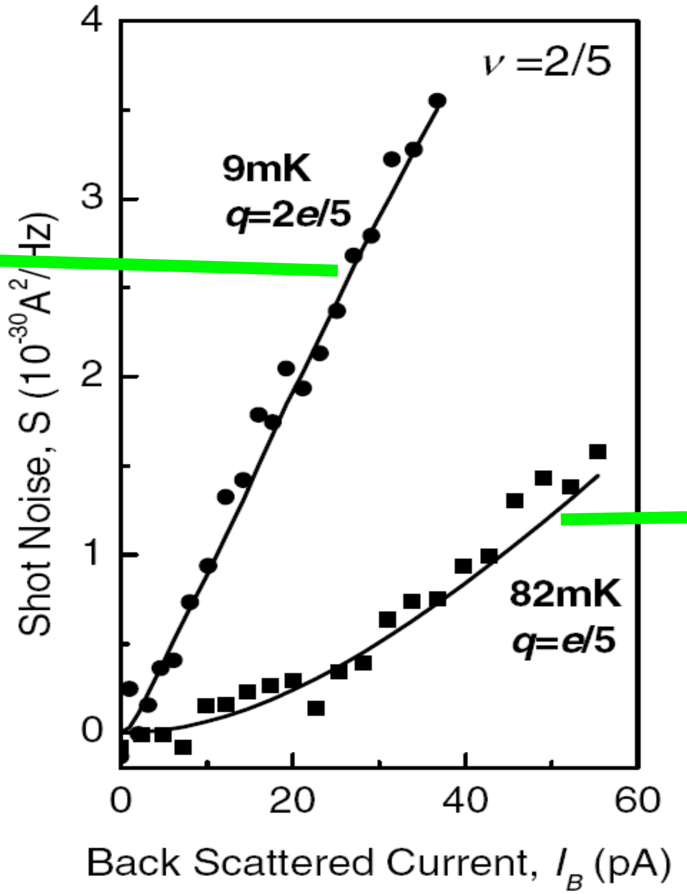


De-Picciotto et al. Nature 97
Saminadayar et al. PRL 97
Reznikov et al. Nature 99 ...

Noise

$q = pe^*$!!

bunching of qp



Chung et al PRL 03

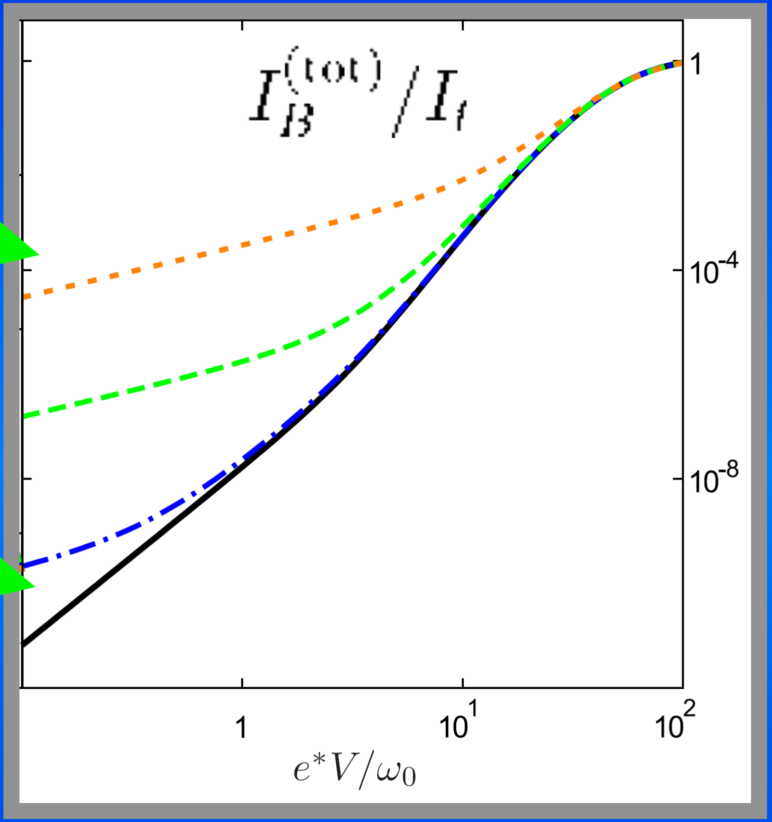
$$q = e^* = \frac{\nu}{p} e$$

single qp

Current: temperature effects

Finite temperature correction

p-agglomerates contribution



D. F. et al. NJP 10

$\nu = 2/5, p = 2$ $\quad g_c = 5.5, g_n = 2$

$\omega_n/\omega_c = 10^{-2}$ $\quad (|t_2|/|t_1|) = 10$

$$I_t = \frac{e|t_1|^2}{4\pi^2 a^2 \omega_c}$$

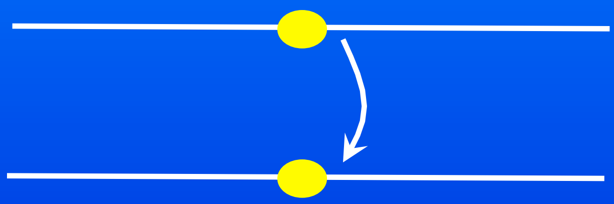
Multiple-quasiparticle tunnelling

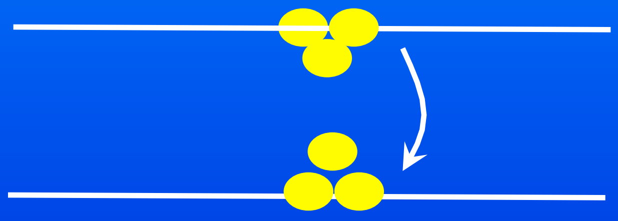
More relavant operators

single quasiparticle $(m=1)$ p-agglomerates $(m=p)$

$$\Psi^{(1)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i[\frac{1}{p}\phi_c(x)+\sqrt{1+\frac{1}{p}}\phi_n(x)]} \qquad \Psi^{(p)}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\phi_c(x)}$$

Tunnelling hamiltonian





$$t_1 \Psi^{(1)}_{\rm up}(0)\Psi^{(1)\dagger}_{\rm down}(0) + h.c. \qquad t_p \Psi^{(p)}_{\rm up}(0)\Psi^{(p)\dagger}_{\rm down}(0) + h.c.$$

Tunneling of p-agglomerates: most relevant process at low energy for

$$g_n > \nu(1 - 1/p)g_c$$