Adiabatic control of many-particle states in coupled quantum dots

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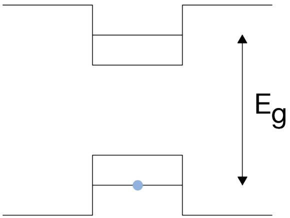
Outline

1. **Introduction**
   - Excitons in quantum dots as qubits
   - State preparation by resonant excitation
   - Adiabatic rapid passage
2. **Adiabatic control in many-particle systems**
   - Theoretical models and approaches
   - Pairwise-coupled dots
   - 1D chains
   - Mean-field limit
3. **Conclusions**

Excitons in quantum dots as qubits?

Island of reduced bandgap in optically active semiconductor, e.g. InGaAs in GaAs.



$$H = E_g s^z + g\left[s^+ E(t) + E^*(t) s^-\right]$$

Why not?

Decoherence?

Lifetimes typically $\lesssim 1\text{ns}$

… but E(t) fast – $\lesssim 1\text{ps}$

Inhomogeneity

$1/(\Delta E_g) \sim 0.01$ ps (best 0.3 ps?)

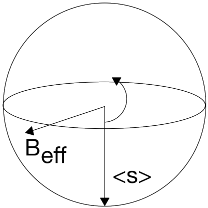
State preparation by resonant excitation

$$H = E_g s^z + g \left[s^+ E(t) + E^*(t) s^- \right]$$

How to prepare an initial state $|\uparrow\rangle$?

Resonant excitation $E(t) = e^{iE_g(t)t}|E(t)|$,

$$\begin{aligned} H &\to UHU^\dagger - iU^\dagger \frac{dU}{dt} \\ &= g|E(t)|(s^+ + s^-), \\ \frac{d\vec{s}}{dt} &= (g|E(t)|, 0, 0) \times \vec{s} \end{aligned}$$



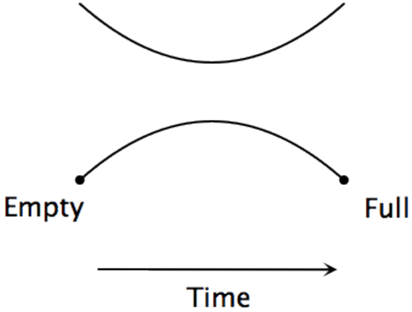
✓ $|\uparrow\rangle$ after pulse when $\int g|E(t)|dt = \pi, 3\pi, 5\pi, \ldots$

Chirped adiabatic rapid passage

Inhomogeneous ensemble: dot-to-dot fluctuations in E_g, g
$\Rightarrow$ resonant excitation unusable.

Use chirped pulse

$$E(t) = e^{i\omega(t)t}|E(t)|$$
$$\omega(t) = E_g + \alpha t$$



$$H = [E_g - \omega(t)]\, s_z + 2g|E(t)|s^x$$

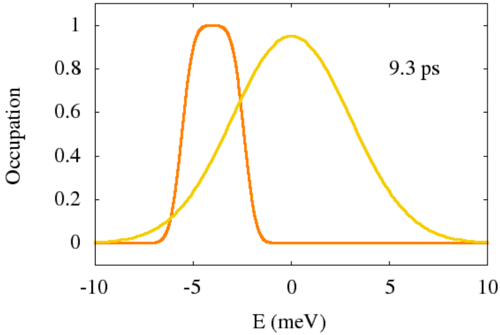
$$1 - P_\uparrow \sim e^{-g^2|E|^2/\alpha}.$$

PRE and R. T. Phillips, Phys. Rev. B **79** 165303 (2009);

E. R. Schmigdall, PRE and R. T. Phillips, Phys. Rev. B **81** 195306 (2010)

Chirped adiabatic rapid passage in ensembles

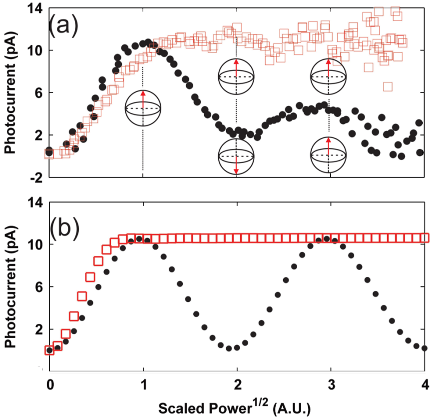
✓ Works in ensembles despite variation in E_g, g, for all those dots satisfying adiabatic criterion



$\sim$ ps pulse creates a population equivalent to thermal equilibrium at 0.6 K

Experimental implemetations

Single quantum dot in photodiode, pulsed laser excitation



$\longleftarrow$ 1 exciton/pulse

Chirped excitation

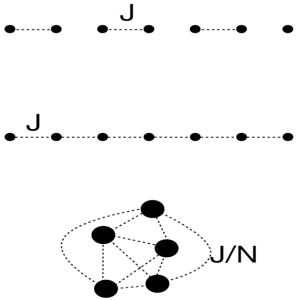
Resonant excitation

[Wu et al., Phys. Rev. Lett. **106** 067401 (2011);

Simon et al., Phys. Rev. Lett. **106** 166801 (2011).]

Theoretical models

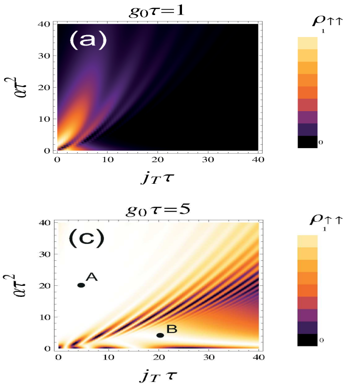
$$H = \sum_i E_{g,i} s_i^z + g_i \left[s_i^+ E(t) + E^*(t) s_i^- \right] - \sum_{\langle ij \rangle} J_{ij} (s_i^+ s_j^-)$$



1. Pairwise coupling
   – Stacked quantum dots + Förster coupling/wavefunction overlap
2. 1D chain
   – Coupled cavity-QED?
3. Mean-field limit
   – Many quantum dots + optical cavity?

ARP to populate pairwise-coupled dots

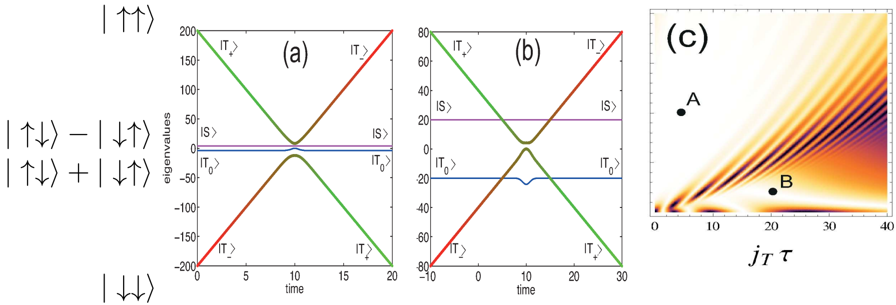
Solve equations of motion for pair w/coupling j_T, with model pulse,
– duration τ, chirp rate α, centre frequency E_g, peak Rabi frequency g_0.



- Large g_0: fully occupied regions, separated by lines of fringes
- Moderate g_0: finite j_T improves adiabaticity

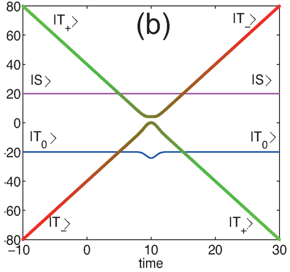
Interpretation: Pairwise-coupled dots

$$H = -\alpha(t - t_0)s^z + 2g|E(t)|s^x + j_T s^+ s^-$$



- A: all crossings inside pulse and adiabiatic. $|T_-\rangle \to |T_0\rangle \to |T_+\rangle$.
- B: $|T_-\rangle$ crosses $|T_0\rangle$ outside pulse $\therefore |T_0\rangle$ unoccupied, but perturbatively couples $|T_\pm\rangle$, recovering adiabaticity.
- Diagonal fringes: $|T_-\rangle, |T_0\rangle$ crossing becoming non-adiabatic.

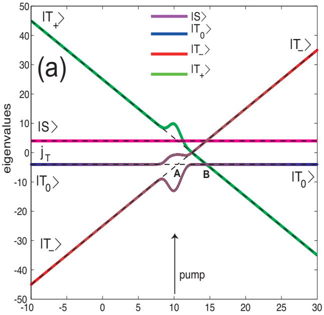
Pairwise-coupled dots: creating entangled states

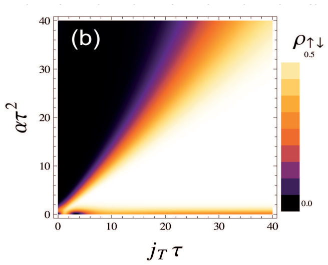


Could populate (entangled) state $|T_0\rangle$

– centre pulse on $|T_-\rangle, |T_0\rangle$ crossing, pulse off before $|T_0\rangle|T_+\rangle$ crossing

Pairwise-coupled dots: creating entangled states





[R. G. Unanyan, N. V. Vitanov and K. Bergmann, Phys. Rev. Lett. **87** 137902 (2001)]

1D chains

$$H = \sum_i -\alpha t s_i^z + 2g|E(t)| s_i^x + 4J(s_i^+ s_{i+1}^- + \text{h.c.})$$

Diagonalize with Jordan-Wigner transform

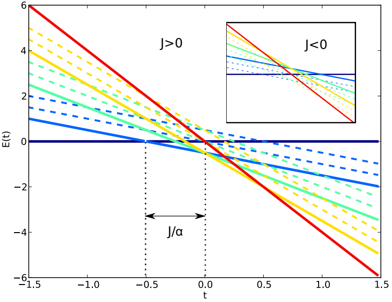
$$s_i^z = c_i^\dagger c_i - \frac{1}{2}$$
$$s_i^- = \frac{1}{2} e^{i\pi \sum_{j<i} c_j^\dagger c_j} c_i = T_i c_i$$

$$H = -\sum_k [\frac{\alpha t}{2} + J \cos k] c_k^\dagger c_k + 2g|E(t)| \sum_i s_i^x$$

1D chains

$$H = -\sum_k [\frac{\alpha t}{2} + J\cos k] c_k^\dagger c_k + 2g|E(t)|\sum_i s_i^x$$

Energy levels for $N = 4$ sites



Colors–

$N+1$ "bands" labelled with $n = \sum c^\dagger c$ (S^z/population)

In each band, set of levels from n fermions in N k-states (S^2)

Uniform field conserves S^2.

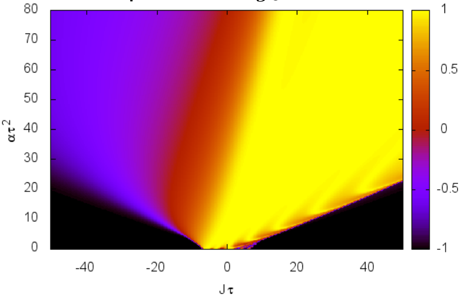
Mean-field limit

Numerically solve equations of motion in mean-field approx :

$$H = \sum_i [-\alpha t] s_i^z + g|E(t)| \left[s_i^+ + s_i^-\right] - \sum_{i \neq j} J_{ij}(s_i^+ s_j^-),$$
$$\rightarrow - \sum_{i \neq j} J_{\text{eff}}[s_i^+ \langle s_j^- \rangle + \text{h.c.}]$$

– Exact for $J_{ij} = J/N^2, N \rightarrow \infty$; LMG model for finite N.

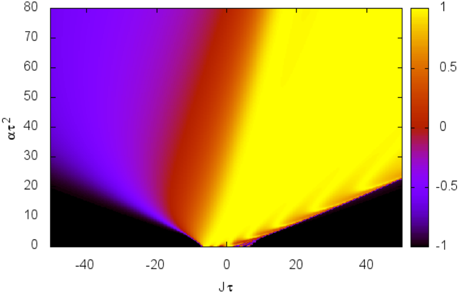
Final occupation for $g_0\tau = 3$

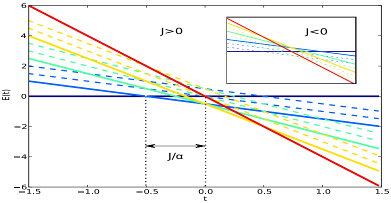


- Loss of adiabaticity for fast chirp
- Fan of finite occupation with sharp boundaries
- $J \gtrless 0$ increases (reduces) occupation/adiabaticity

Mean-field limit: interpretation

Final occupation for $g_0\tau = 3$





- Loss of adiabaticity for fast chirp
- Fan of finite occupation with sharp boundaries
- $J \gtrless 0$ increases (reduces) occupation/adiabaticity

Conclusions

- Can adiabatically populate a single quantum dot by driving with chirped laser pulses
- In models (anti-)ferromagnetic x-y coupling initially enhances (suppresses) populations
- ... but too strong coupling $J \sim \alpha\tau \to$ no mixing at critical level crossing $\to$ scheme fails
- Virtual transitions allow population even for large J in small systems
- Straightforward extensions to generate entangled states

Future directions

- Experimental implementations of entanglement generation, non-equilibrium condensation
- Theoretical modelling of tolerance to fluctuations in $E_{g,i}, g_i, J$ (random-field models)
- Decoherence due to acoustic phonons, Johnson-Nyquist noise
- Approaches to probing decoherence, interaction strengths (cf. NMR!)