Integrability of anyonic chains with competing interactions

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Outline

- Introduction to anyonic chains
- Competing interactions
- Construction of a new, integrable 2-d model
- The corner transfer matrix method
- Analyzing the model

Anyonic Heisenberg model

Feiguin et.al., PRL (2007)

SU(2)$_3$ fusion rules

$$\frac{1}{2} \times \frac{1}{2} = 0 + 1$$
$$1 \times 1 = 0 + 1$$

energetically split multiple fusion outcomes

"Heisenberg" Hamiltonian

$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

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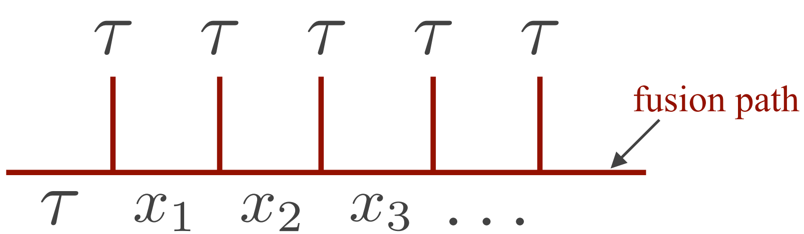
Example: chains of anyons

$(\tau = 1)$

Hilbert space

$$|x_1, x_2, x_3, \ldots \rangle$$

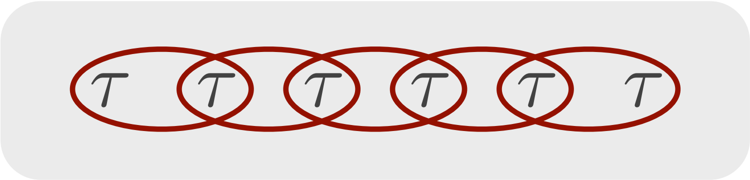


Hamiltonian

$$H = J \sum_{i} F_i \, \Pi_i^0 \, F_i$$

F-matrix = 6j-symbol

The Hilbert space of the Golden Chain



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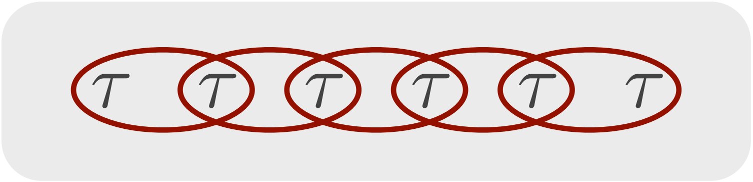

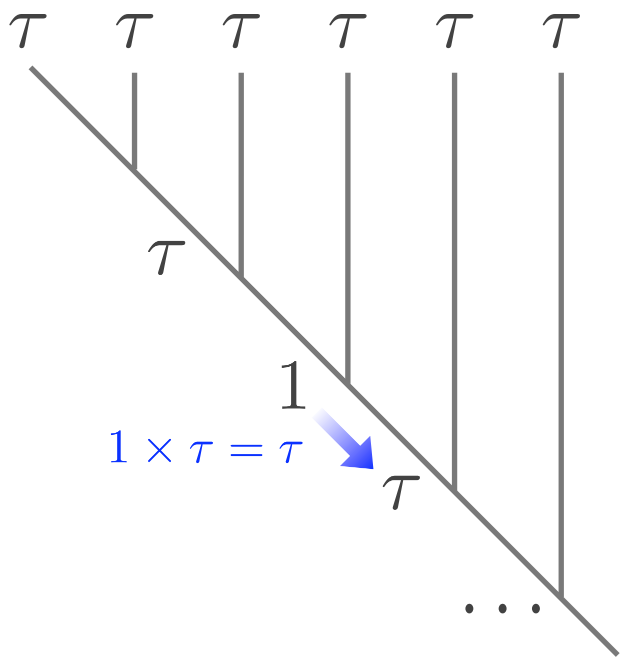

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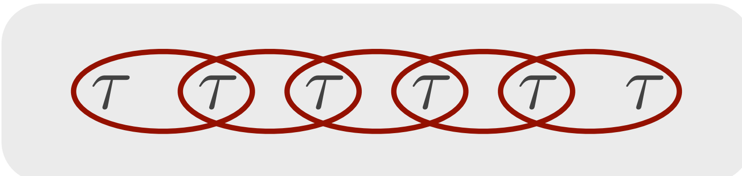

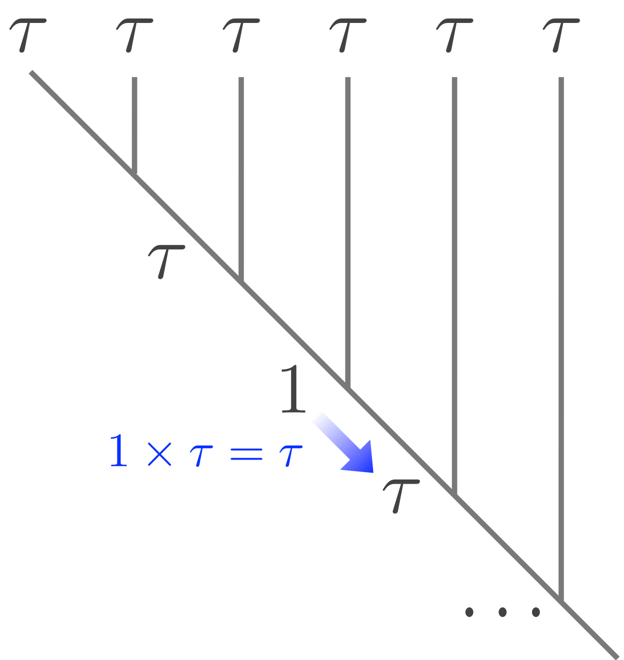

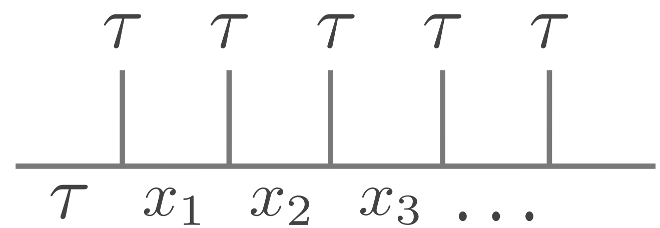

Hilbert space: $|x_1, x_2, x_3, \ldots\rangle$

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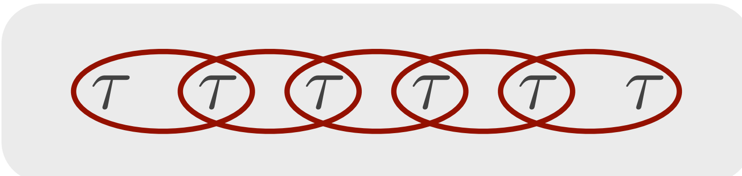

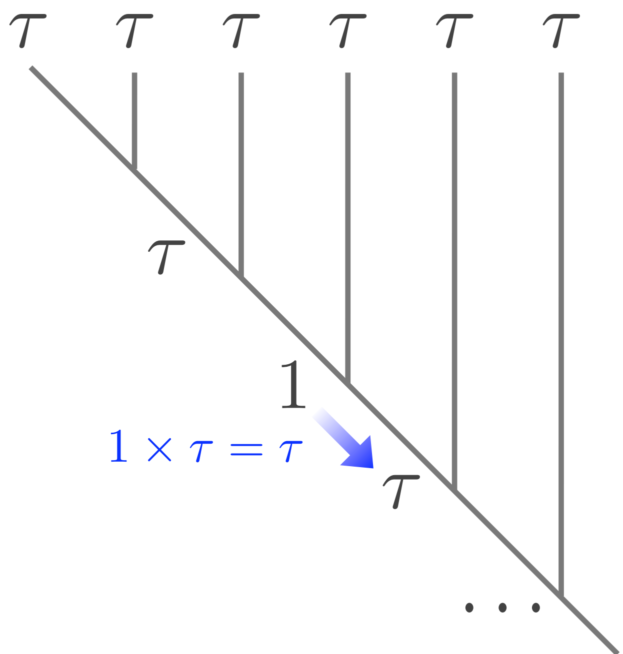

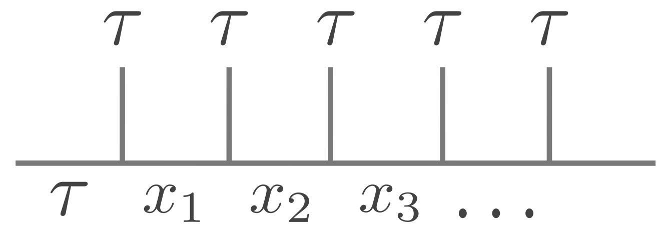

Hilbert space: $|x_1, x_2, x_3, \ldots\rangle$

$$\dim_L = F_{L+1} \propto \phi^L$$

$$\phi = \frac{1+\sqrt{5}}{2} = 1.618\ldots$$

Hilbert space has **no natural decomposition** as tensor product of single-site states.

The golden chain

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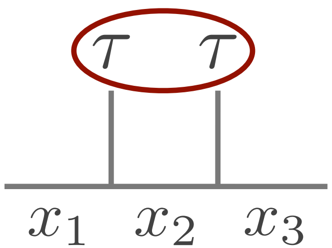

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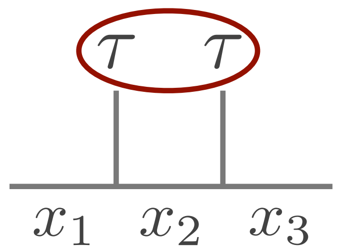

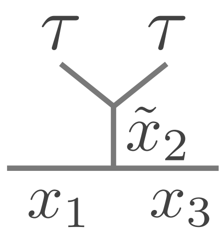

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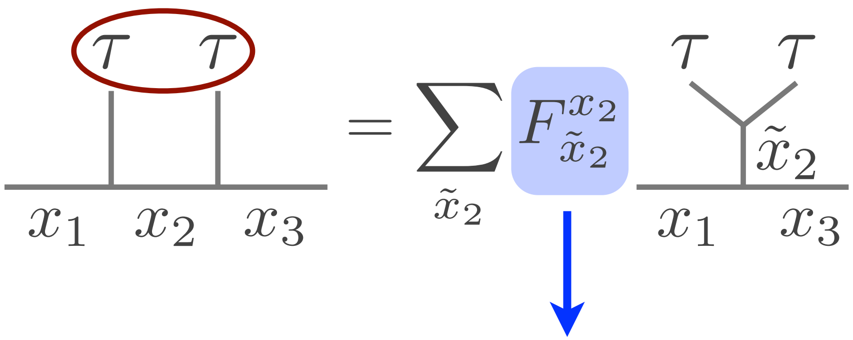

F-matrix

$$F = \begin{pmatrix} \phi^{-1} & \phi^{-1/2} \\ \phi^{-1/2} & -\phi^{-1} \end{pmatrix}$$

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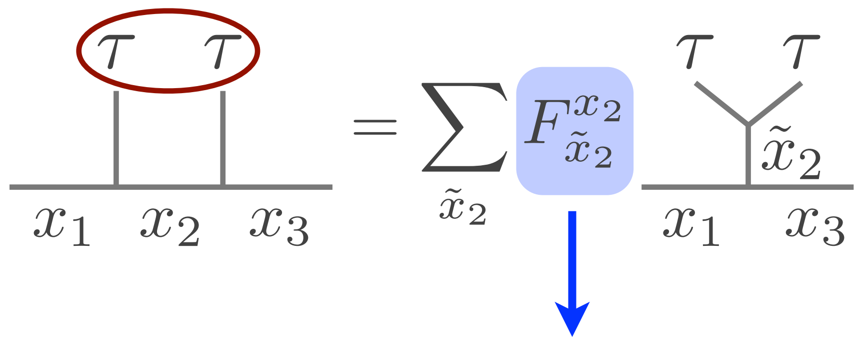

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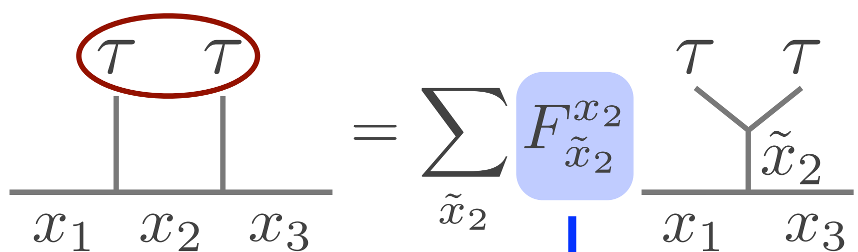

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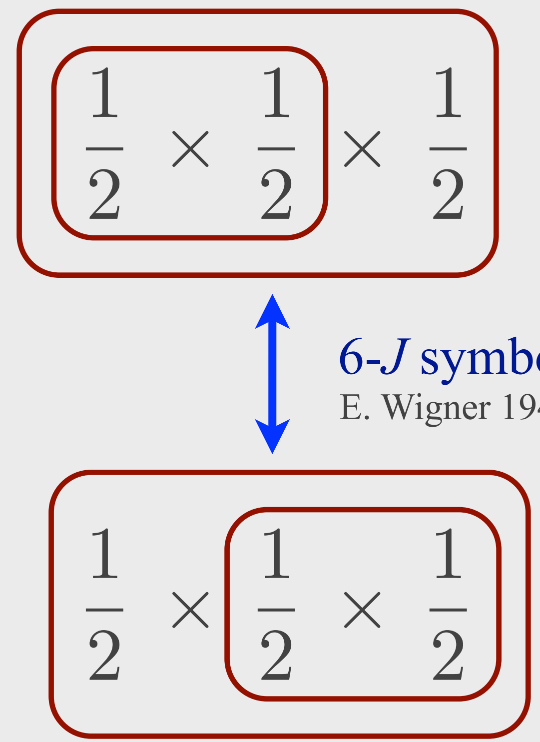
SU(2) spins



The golden chain

Local Hamiltonian: $H_i = F_i \, \Pi_i^1 \, F_i$

$$H_i = \begin{pmatrix} \phi^{-2} & \phi^{-3/2} \\ \phi^{-3/2} & \phi^{-1} \end{pmatrix}$$

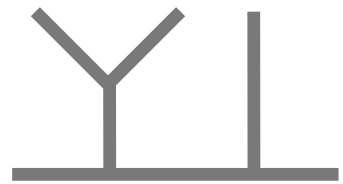
Explicit form:

$$\begin{aligned} H_{2-\mathrm{body}} &= \mathcal{P}_{1\tau\tau} + \mathcal{P}_{\tau\tau 1} + \phi^{-2}\mathcal{P}_{\tau 1 \tau} + \phi^{-1}\mathcal{P}_{\tau\tau\tau} \\ &\quad + \phi^{-3/2}\left(|\tau 1 \tau\rangle \langle \tau\tau\tau| + \mathrm{h.c.}\right) \end{aligned}$$

Feiguin et.al., PRL (2007)

The 3-body interaction

We need to transform twice to find the fusion channel of three neighbouring anyons:


$= \sum F$

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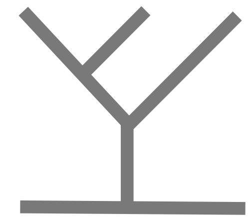

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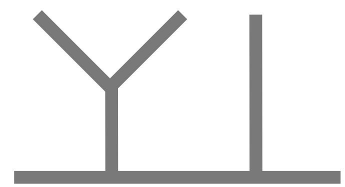
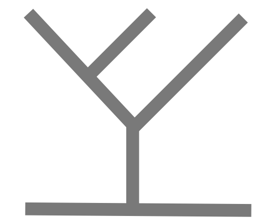
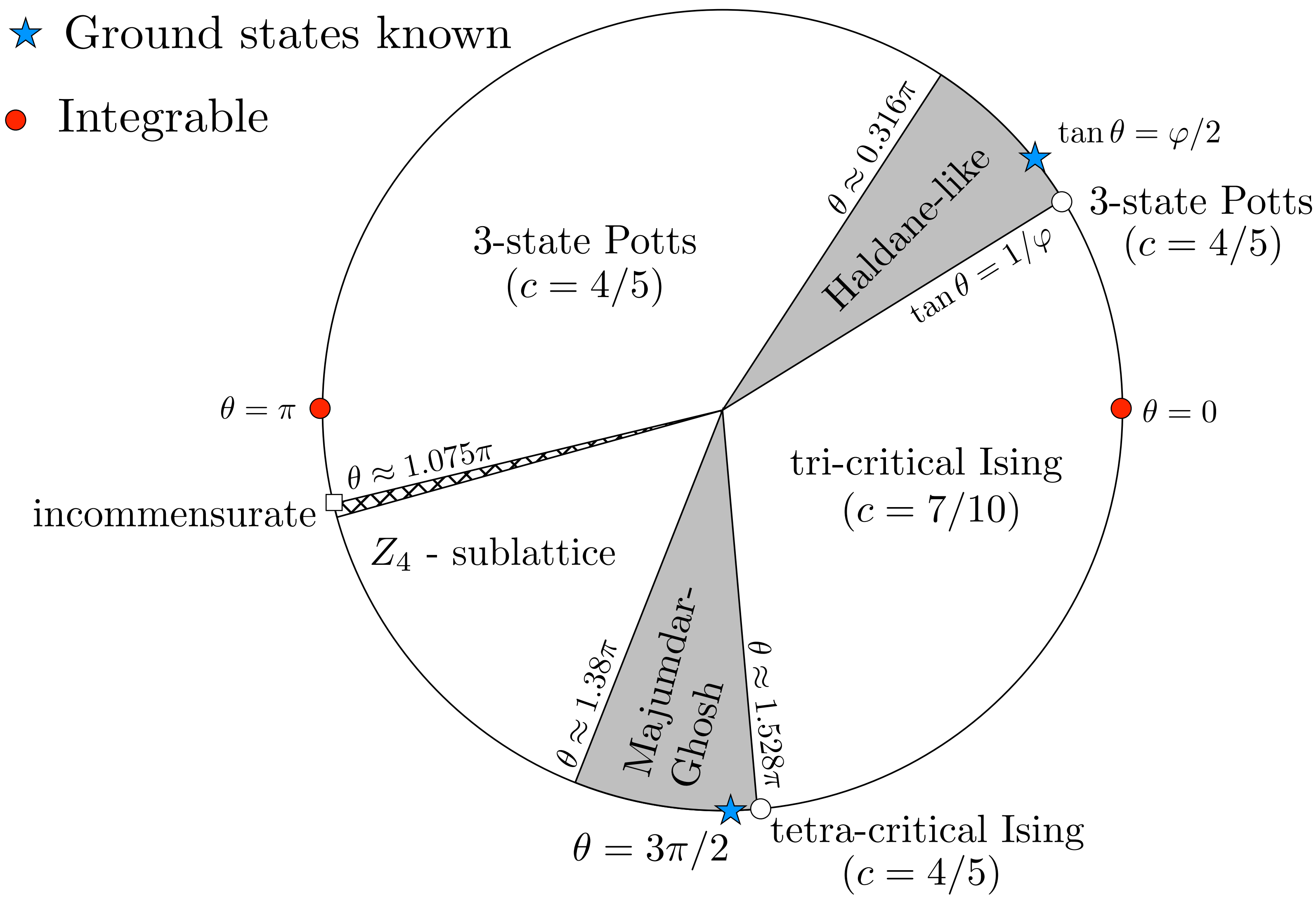
 $= \sum F$  $= \sum FF$ 

Explicitly, we find (please forget!):

$$
\begin{aligned}
H_{3-\text{body}} \quad = \quad & \mathcal{P}_{\tau 1 \tau 1} + \mathcal{P}_{1 \tau 1 \tau} + \mathcal{P}_{\tau \tau \tau 1} + \mathcal{P}_{1 \tau \tau \tau} + 2\phi^{-2} \mathcal{P}_{\tau \tau \tau \tau} + \\
& \phi^{-1} \left(\mathcal{P}_{\tau 1 \tau \tau} + \mathcal{P}_{\tau \tau 1 \tau} \right) - \phi^{-2} \big(\left| \tau \tau 1 \tau \right\rangle \left\langle \tau 1 \tau \tau \right| + \text{h.c.} \big) - \\
& \phi^{-5/2} \left(\left| \tau 1 \tau \tau \right\rangle \left\langle \tau \tau \tau \tau \right| + \left| \tau \tau 1 \tau \right\rangle \left\langle \tau \tau \tau \tau \right| + \text{h.c.} \right)
\end{aligned}
$$

Phase diagram of the model

$$H_{J_2-J_3} = \sum_i \cos\theta H_{2-\mathrm{body},i} + \sin\theta H_{3-\mathrm{body},i}$$



Trebst et.al., PRL (2008)

Integrability of the Golden chain

The operators $e_i = -\phi H_i$ form a representation of the Temperly-Lieb algebra:

$$e_i^2 = de_i \qquad e_i e_{i\pm1} e_i = e_i$$

Pasquier (1987)

$$[e_i, e_j] = 0 \quad \text{for } |i-j| \geq 2$$

$d = \phi$ is the isotopy parameter

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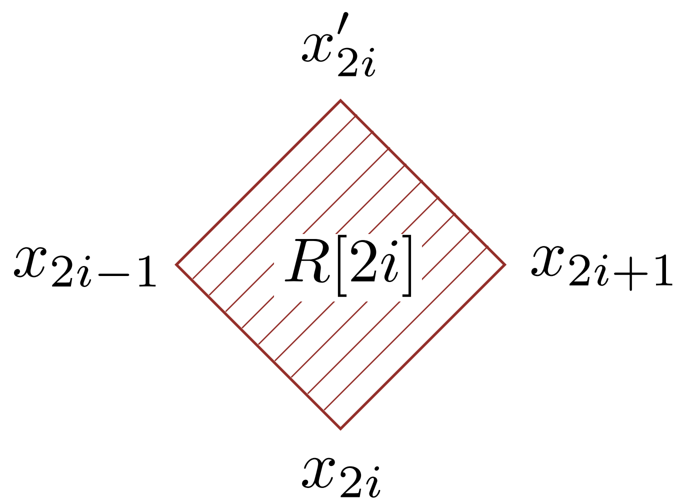
$d = \phi$ is the isotopy parameter

With the e's, we can construct an R-matrix (plaquette weights), which satisfies the Yang-Baxter equation!

The R-matrix

$$R_i(u)^{\vec{x}'}_{\vec{x}} = \Big(\frac{\sin(\frac{\pi}{k+2} - u)}{\sin(\frac{\pi}{k+2})}\mathbf{1}^{\vec{x}'}_{\vec{x}} + \frac{\sin(u)}{\sin(\frac{\pi}{k+2})}e(i)^{\vec{x}'}_{\vec{x}}\Big)$$

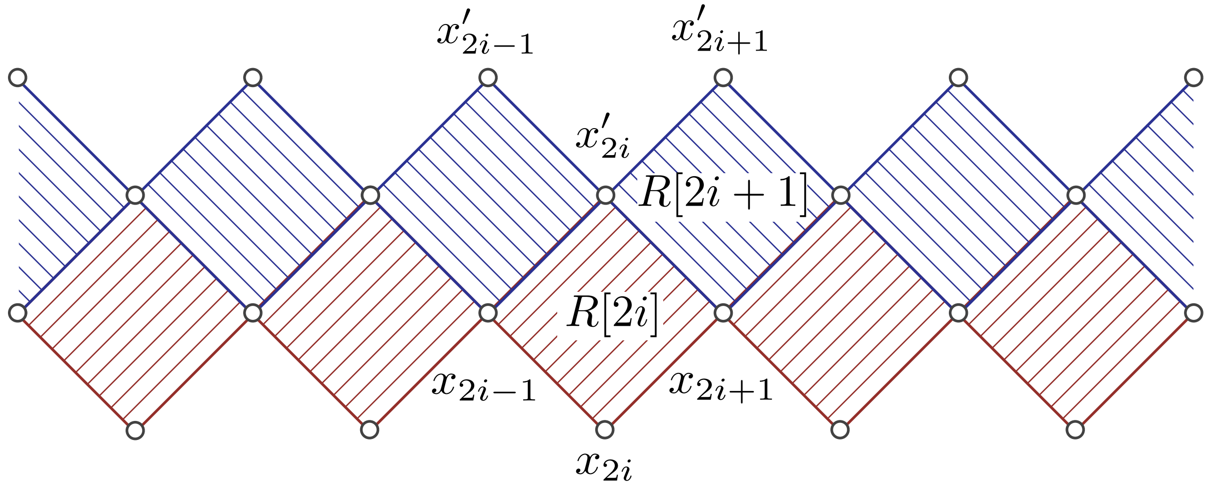
$$e(i)^{\vec{x}'}_{\vec{x}} = \Big(\prod_{j\neq i}\delta_{x'_j,x_j}\Big)e(i)^{x_{i+1}}_{x_{i-1}} \qquad \mathbf{1}^{\vec{x}'}_{\vec{x}} = \prod_j \delta_{x'_j,x_j}$$



R-matrix satisfies the Yang-Baxter equation:

$$R_j(u)R_{j+1}(u+v)R_j(v) = R_{j+1}(v)R_j(u+v)R_{j+1}(u)$$

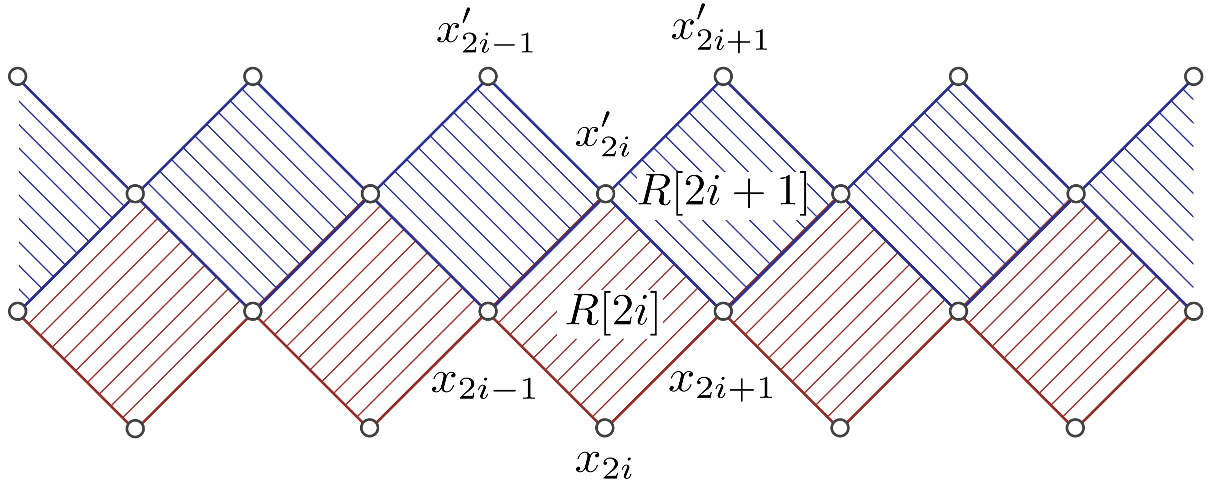
The transfer matrix



$$T(u) = \prod_i R[2i+1](u) R[2i](u)$$

The different transfer matrices commute, because R satisfies the Yang-Baxter equation.

The transfer matrix



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Connection with the hamiltonian:

$$\frac{d \ln T(u)}{du}\Big|_{u=0} = c_1 H_{2-\text{body}} + c_2$$

Composite R-matrix

The 3-body term requires a composite R-matrix:

$$\tilde{R}_j(u,\phi) = R_{2j+1}(u-\phi)R_{2j}(u)R_{2j+2}(u)R_{2j+1}(u+\phi)$$

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The Hamiltonian indeed contains 2 and 3-body terms:

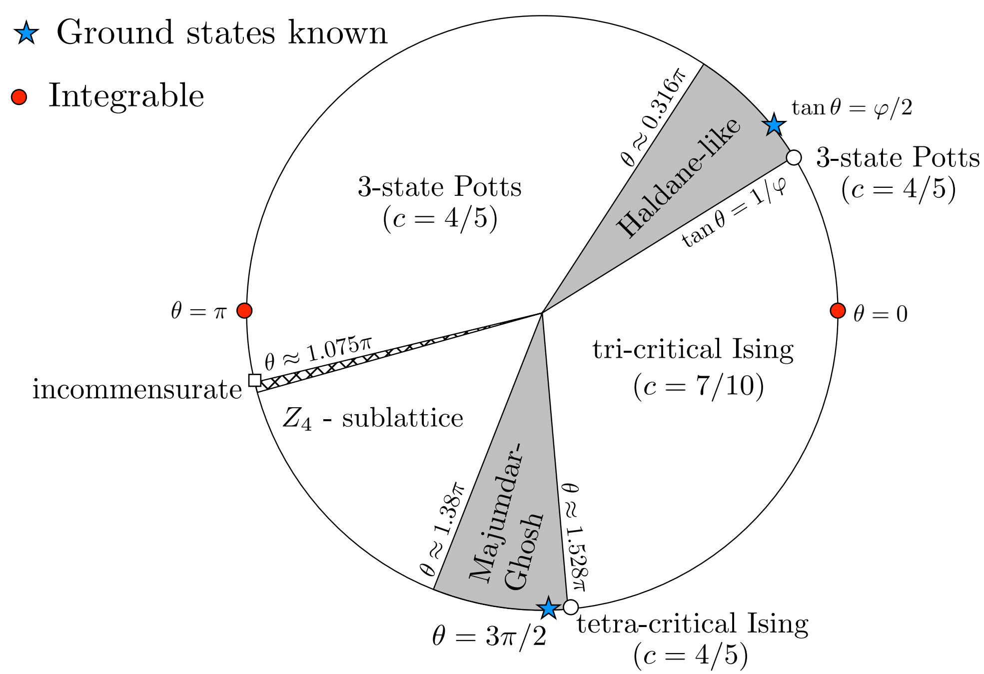
$$H = c_3 + \sum_i c_1(e_i + e_{i+1})/2 + c_2(e_i e_{i+1} + e_{i+1} e_i)$$

See also Yklhef et.al., JPA (2009)

Composite R-matrix

ϕ ranges over $0 \le \phi \le \pi/2$ and is related to θ :

$$\tan\theta = \frac{(d^2-1)\sin(\phi)^2}{(4-d^2)-(4-2d^2)\sin(\phi)^2}$$



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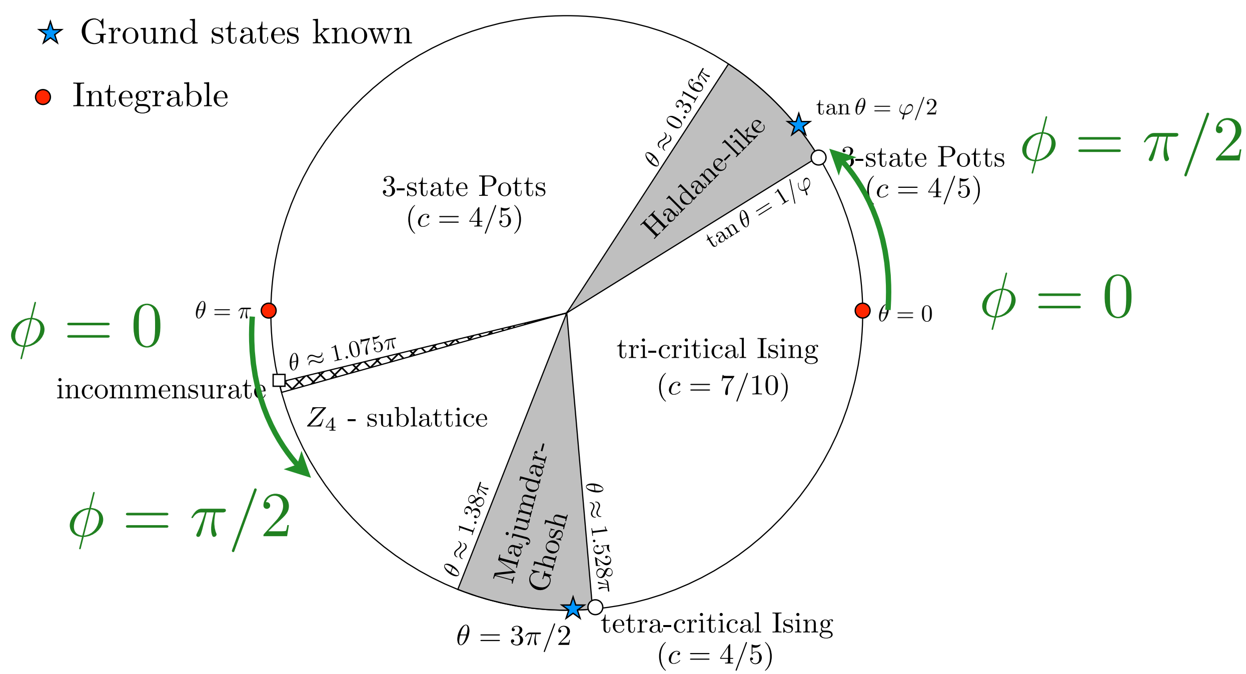

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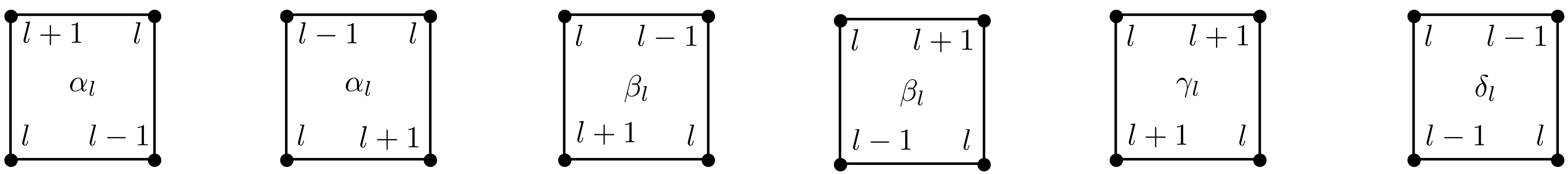
Andrews-Baxter-Forrester model

Solvable height model on square lattice.

Heights take the values $l = 1, 2, \ldots, r-1$
Neighbouring heights differ by one!



ABF, Nucl.Phys B (1984)

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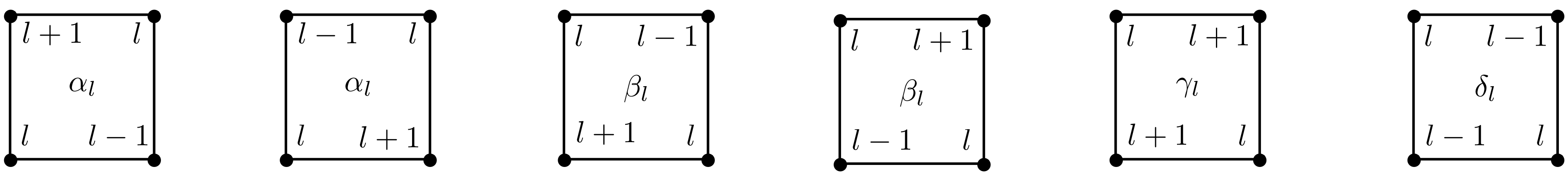
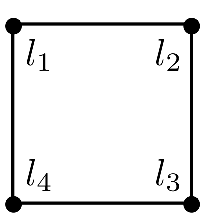


$$Z = \sum_{\text{configurations}} \prod_{\text{plaquettes}} W(l_{j_1}, l_{j_2}, l_{j_3}, l_{j_4})$$



$$\leftrightarrow W(l_1, l_2, l_3, l_4)$$

ABF, Nucl.Phys B (1984)

Parameters in the model

Two parameters in the model:

$-1 \leq p \leq 1$ drives a phase transition at $p = 0$

u is related to the anisotropy of the lattice

The critical behaviour of this model describes the golden chain, for both signs of the interaction

Form of the weights

The weights are given in terms of elliptic functions:

$$
\begin{aligned}
h(u) &= \theta_1(\frac{u\pi}{2K}, p)\theta_4(\frac{u\pi}{2K}, p) \\
&= 2p^{1/4}\sin(\frac{\pi u}{2K})\prod_{n=1}^{\infty}(1-2p^n\cos(\frac{\pi u}{K})+p^{2n})(1-p^{2n})^2
\end{aligned}
$$

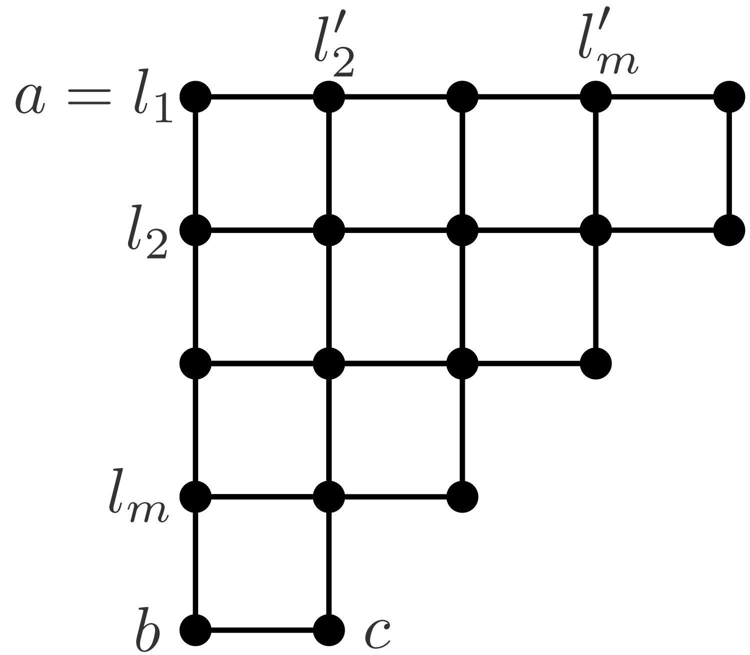
$$
\begin{aligned}
\alpha_l &= \frac{h(2\eta-u)}{h(2\eta)}, \\
\beta_l &= \frac{h(u)}{h(2\eta)}\frac{h(w_{l-1})^{1/2}h(w_{l+1})^{1/2}}{h(w_l)}, \\
\gamma_l &= \frac{h(w_l+u)}{h(w_l)}, \\
\delta_l &= \frac{h(w_l-u)}{h(w_l)}.
\end{aligned}
$$

$$\eta = \frac{K}{r}$$

$$w_l = 2\pi\eta l$$

Corner transfer matrices

$A_{l,l'} =$



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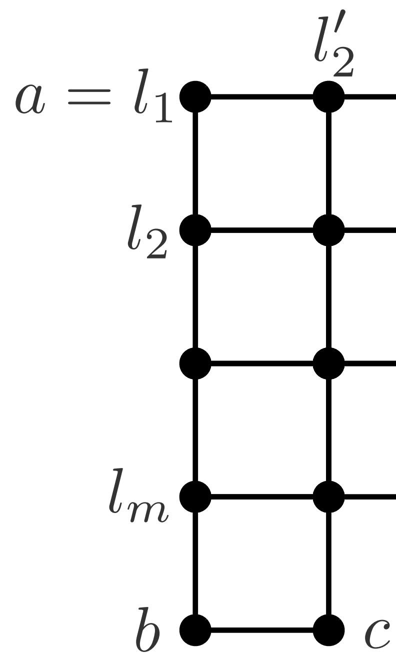

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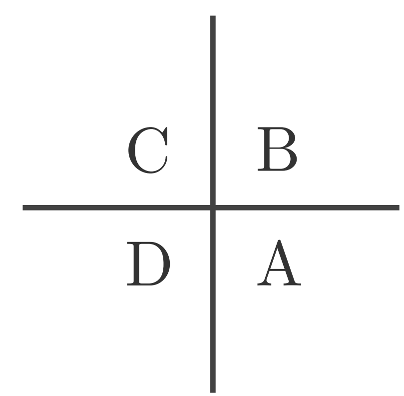
B, C and D are defined analogously, by rotating successively over 90 degrees



Corner transfer matrix method

One can ‘solve’ the model by calculating the probability for the height of the center vertex.

In terms of corner transfermatrices, Z reads $Z = Tr(ABCD)$

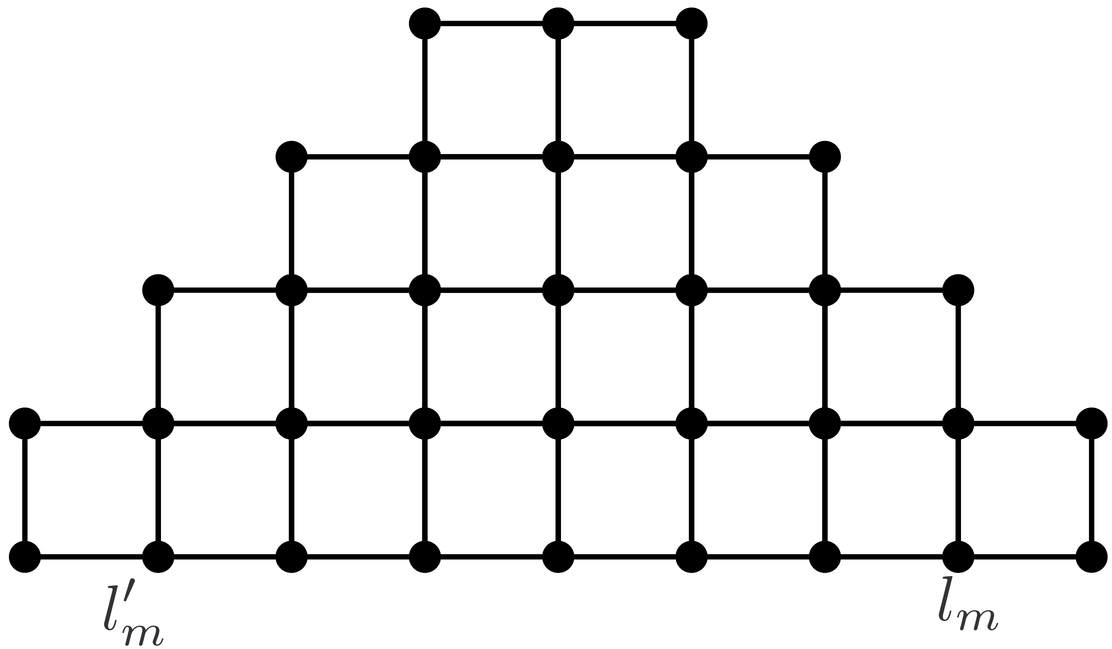
The height probabilities are $$P_a = \frac{Tr(S_a ABCD)}{Tr(ABCD)}$$

S_a is a diagonal matrix, with 1’s on the diagonal for the block with $l_1 = a$

How the method works

Equate, in the large lattice limit, the following:

$BC =$



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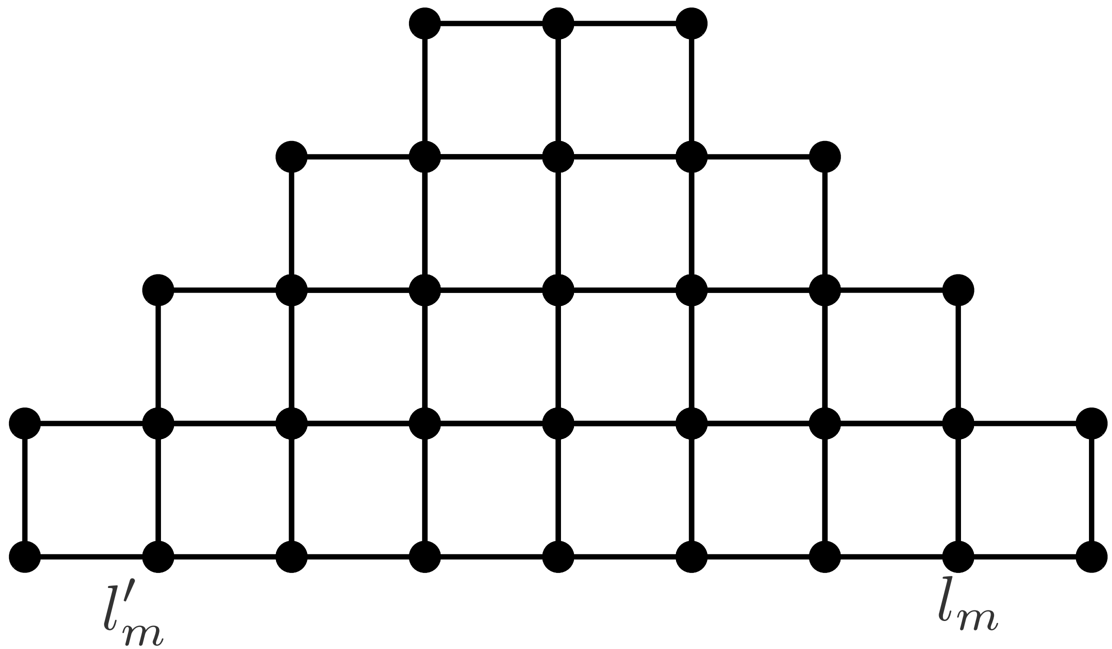
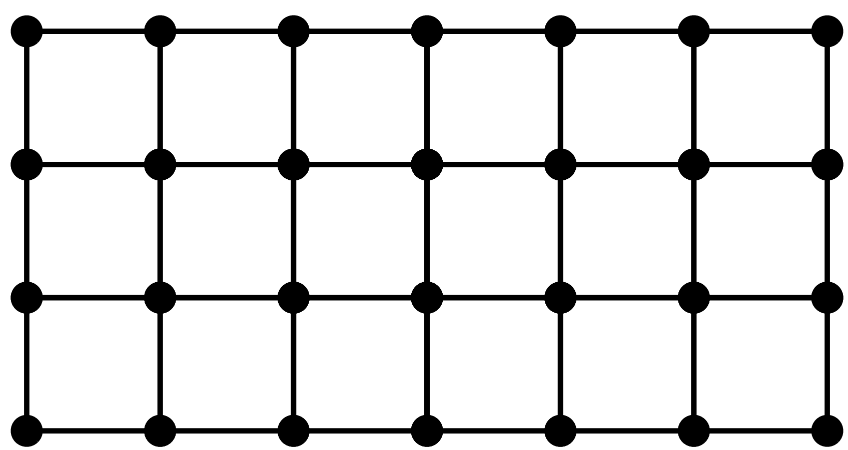


$T^n =$



Diagonal form of the CTM's

It follows that one can write A,B,C,D in diagonal form

$$\begin{aligned} A(u) &= Q_1 M_1 e^{-u\mathcal{H}} Q_2^{-1} \ , \\ B(u) &= Q_2 M_2 e^{u\mathcal{H}} Q_3^{-1} \ , \\ C(u) &= Q_3 M_3 e^{-u\mathcal{H}} Q_4^{-1} \ , \\ D(u) &= Q_4 M_4 e^{u\mathcal{H}} Q_1^{-1} \ , \end{aligned}$$

Baxter's book (1984)

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Baxter's book (1984)

$\mathcal{H}$ and the M_i are diagonal, dependence on anisotropy u is explicit

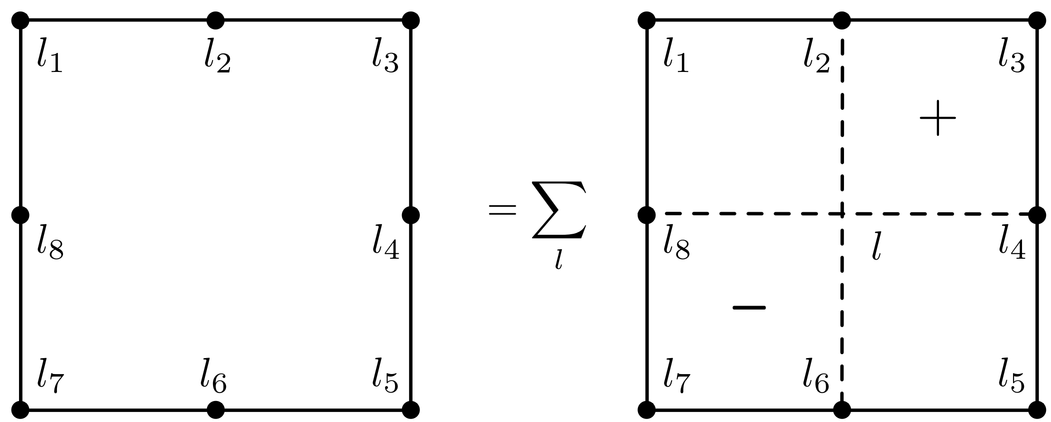
The height probabilities take the following form:

$$P_a = Tr(S_a M_1 M_2 M_3 M_4)/Tr(M_1 M_2 M_3 M_4)$$

The final result follows from considering various limits, such as $u = 0$, $u = (2 \pm r)\eta$ and $p = 1$

New lattice model

We introduce the following new lattice model with the following plaquettes:



$$\tilde{W}(l_1, \ldots, l_8) =$$
$$\sum_l W(l_1, l_2, l, l_8; u) W(l_2, l_3, l_4, l; u + K) W(l, l_4, l_5, l_6; u) W(l_8, l, l_6, l_7; u - K)$$

Not 6, but 66 different types of plaquettes!

Height probability

When the dust settles, the height probability is given by:

$$P_a = \frac{1}{\mathcal{N}} v_a X_m(a; b, c, d, e; x^t) \qquad x = e^{-4\pi\eta/K'}$$

$$X_m(a; b, c, d, e; q) = \sum_{l_2, \ldots, l_m} q^{\phi(\mathbf{l})} \qquad \phi(\mathbf{l})\text{: next slide}$$

$\mathcal{N}$ normalization

v_a depends only on the central height

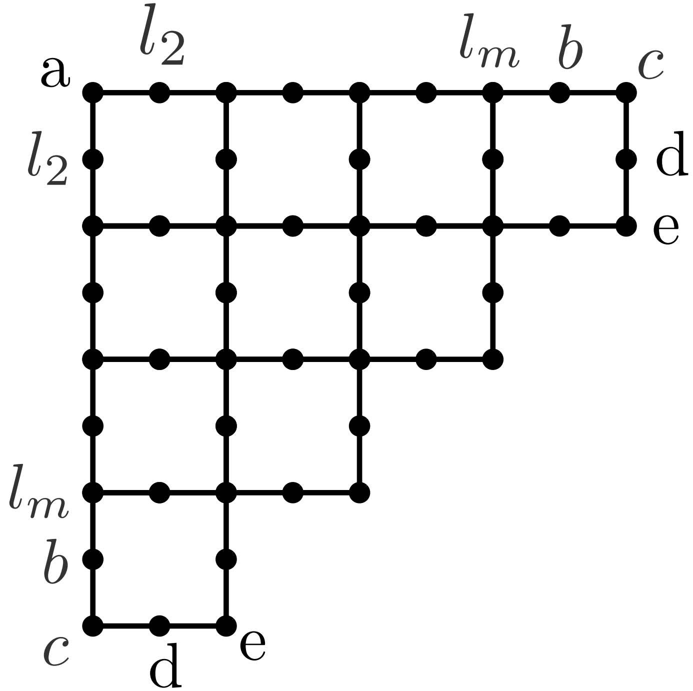
t depends on the region:

$$t = \begin{cases} 2 - r & \text{for } u > 0 \\ 2 + r & \text{for } u < 0 \end{cases}$$

Height probability

$$\phi(\mathbf{l}) = \sum_{j=1}^{(m+1)/2} j\Big(\frac{|l_{2j+3}-l_{2j-1}|}{4} + \delta_{l_{2j-1},l_{2j+1}}\delta_{l_{2j+1},l_{2j+3}}\delta_{l_{2j},l_{2j+2}}\Big)$$

Calculated from the limit $p=1$, in which only 'diagonal' plaquettes contribute



Ordered phases

The ordered phases at $p = 1$ are obtained by:
maximizing $\phi(\mathbf{l})$ for $u > 0$

1	2	1	2	1	2	1	2	1
2		2		2		2		2
1	2	1	2	1	2	1	2	1
2		2		2		2		2
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\begin{array}{ccccccccc}
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2 &   & 2 &   & 2 &   & 2 &   & 2 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
2 &   & 2 &   & 2 &   & 2 &   & 2 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1
\end{array}
$$

minimizing $\phi(\mathbf{l})$ for $u < 0$

$$
\begin{array}{ccccccccc}
1 & 2 & 3 & 2 & 1 & 2 & 3 & 2 & 1 \\
2 &   & 2 &   & 2 &   & 2 &   & 2 \\
3 & 2 & 1 & 2 & 3 & 2 & 1 & 2 & 3 \\
2 &   & 2 &   & 2 &   & 2 &   & 2 \\
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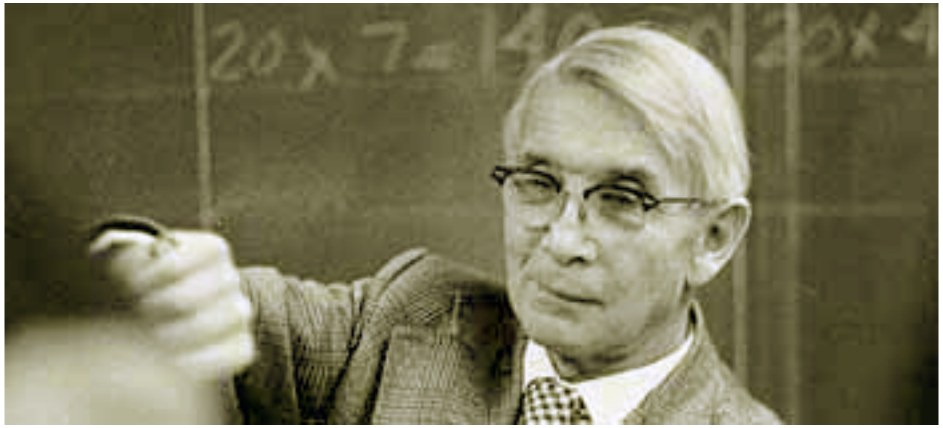
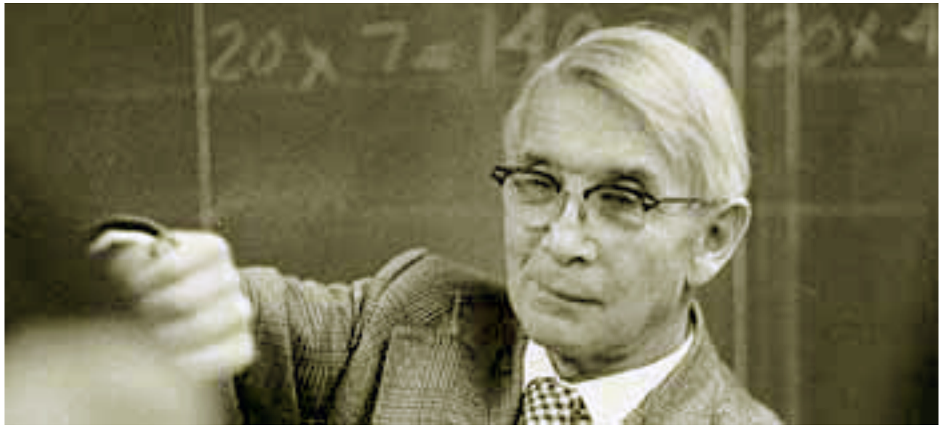
\end{array}
$$

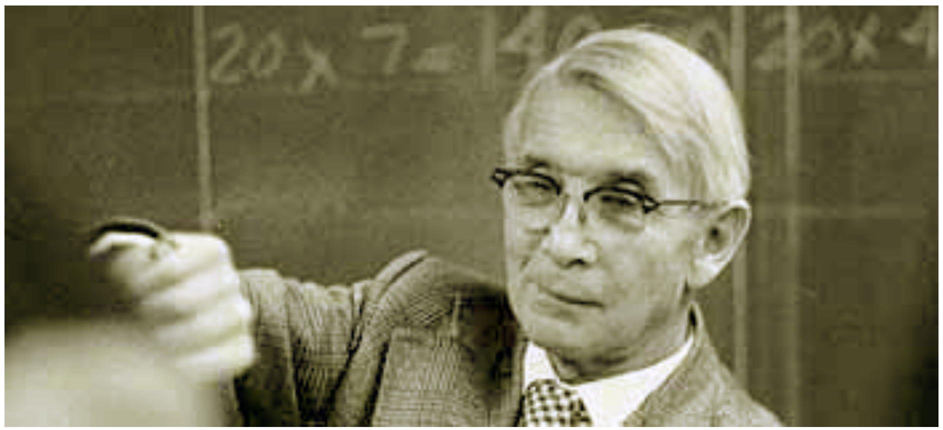
Intermezzo:



Intermezzo:

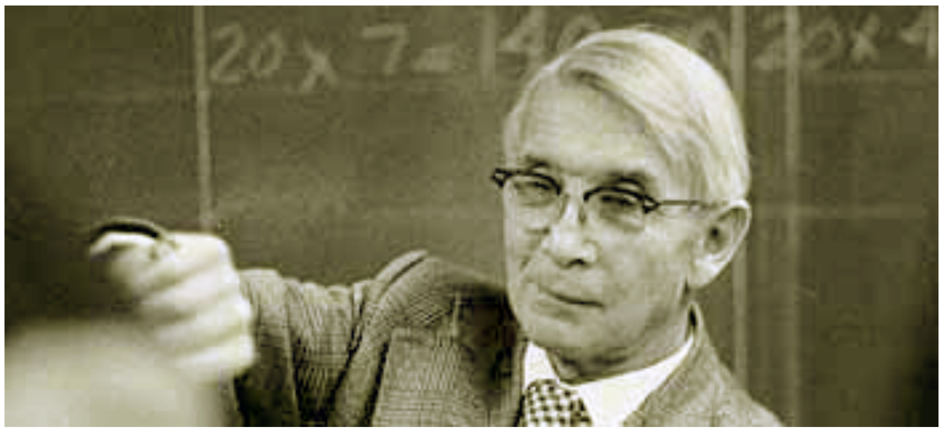


Intermezzo:



meets

Intermezzo:



meets

Ising meets Fibonacci:
Relation between characters of theories with Ising and Fibonacci particles

Grosfeld & Schoutens, PRL (2008)

Intermezzo:
Fibonacci meets Leonardo Pisano

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meets

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meets

Critical behaviour

For the critical behaviour at $p = 0$, we need information about the whole function X_m

$$X_m(a; b, c, d, e; q) = \sum_{l_2, \ldots, l_m} q^{\phi(\{l\})}$$

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Relevant for $u < 0$

$$X_{43}(1;2,1,2,3;q) =$$
$$1 + 3q^2 + 4q^3 + 9q^4 + 12q^5 + 22q^6 + 30q^7$$
$$+ \ldots + 5875310q^{121} + \ldots +$$
$$+8q^{235} + 7q^{236} + 4q^{237} + 3q^{238} + 2q^{239} + q^{240} + q^{242}$$

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So, Fibonacci meets Fibonacci!

Connection with CFT

For $r = 5$ $(k = 3)$, we have explicit formulas for the functions X_m ‘in the groundstates’

These reproduce all the characters of the Z_3 and $su(3)_2$ parafermions.

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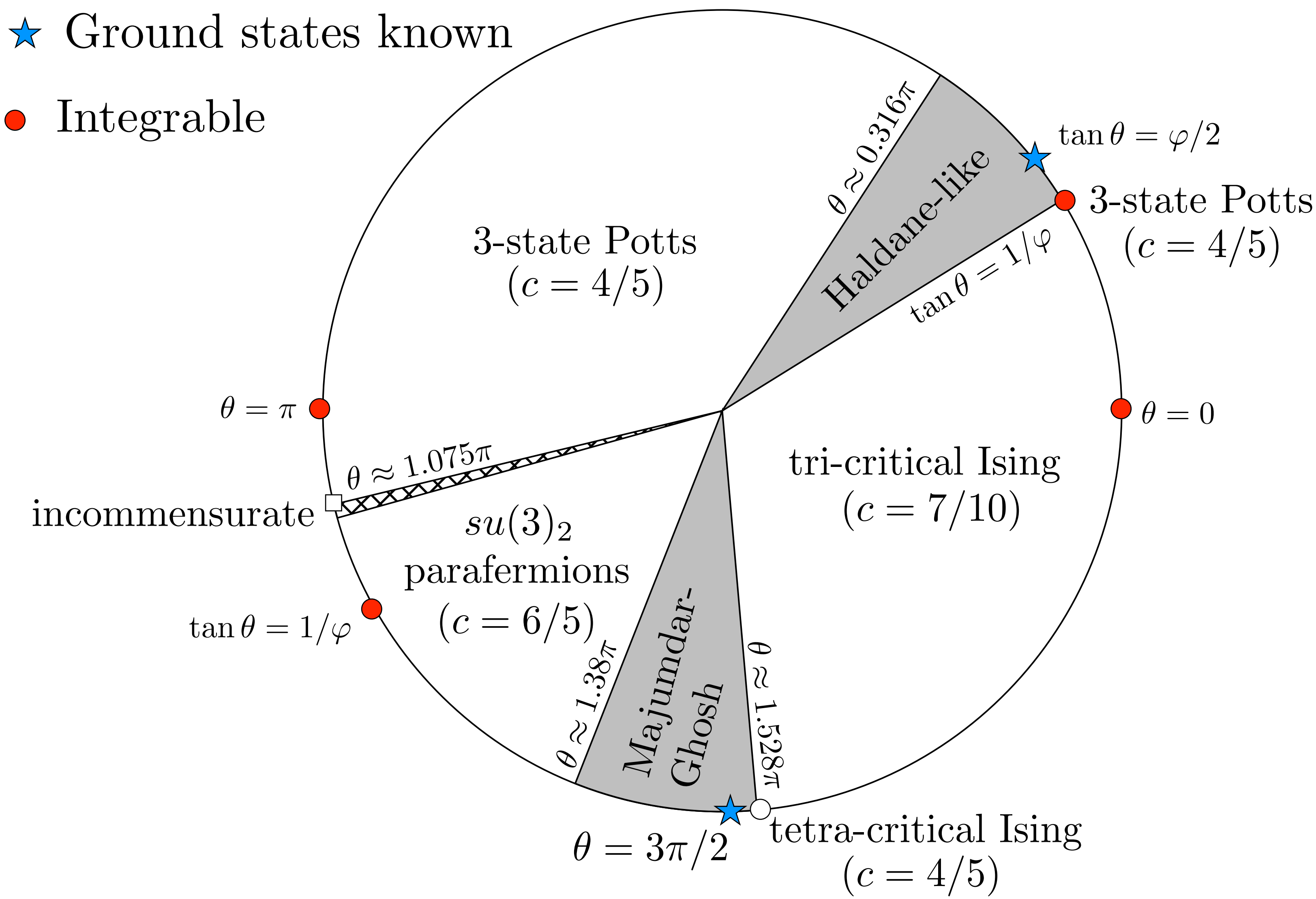
The critical behaviour for arbitrary k is given by:

Z_k parafermions for $u > 0$

$$\frac{su(2)_1 \times su(2)_1 \times su(2)_{k-2}}{su(2)_k}$$ for $u < 0$

Updated phase diagram



Conclusions

- Studied an exactly solvable point in an anyonic chain with competing interactions.
- Introduced a new 2-d, solvable height model
- Obtained the critical behaviour, explaining an extended critical region in the chain.
- Connection with CFT was made

Outlook

- Connection with SU(2) Heisenberg chains
- Understanding of (topological) phase transitions
- Connection with related loop models?
- Other types of anyonic chains
- Relation with Rogers-Ramanujan identities?
- Finitization of characters might have other (qHe) applications

Nordita program

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www.nordita.org/~ardonne/workshops.html