QUANTUM CORRELATIONS IN SPIN CHAINS

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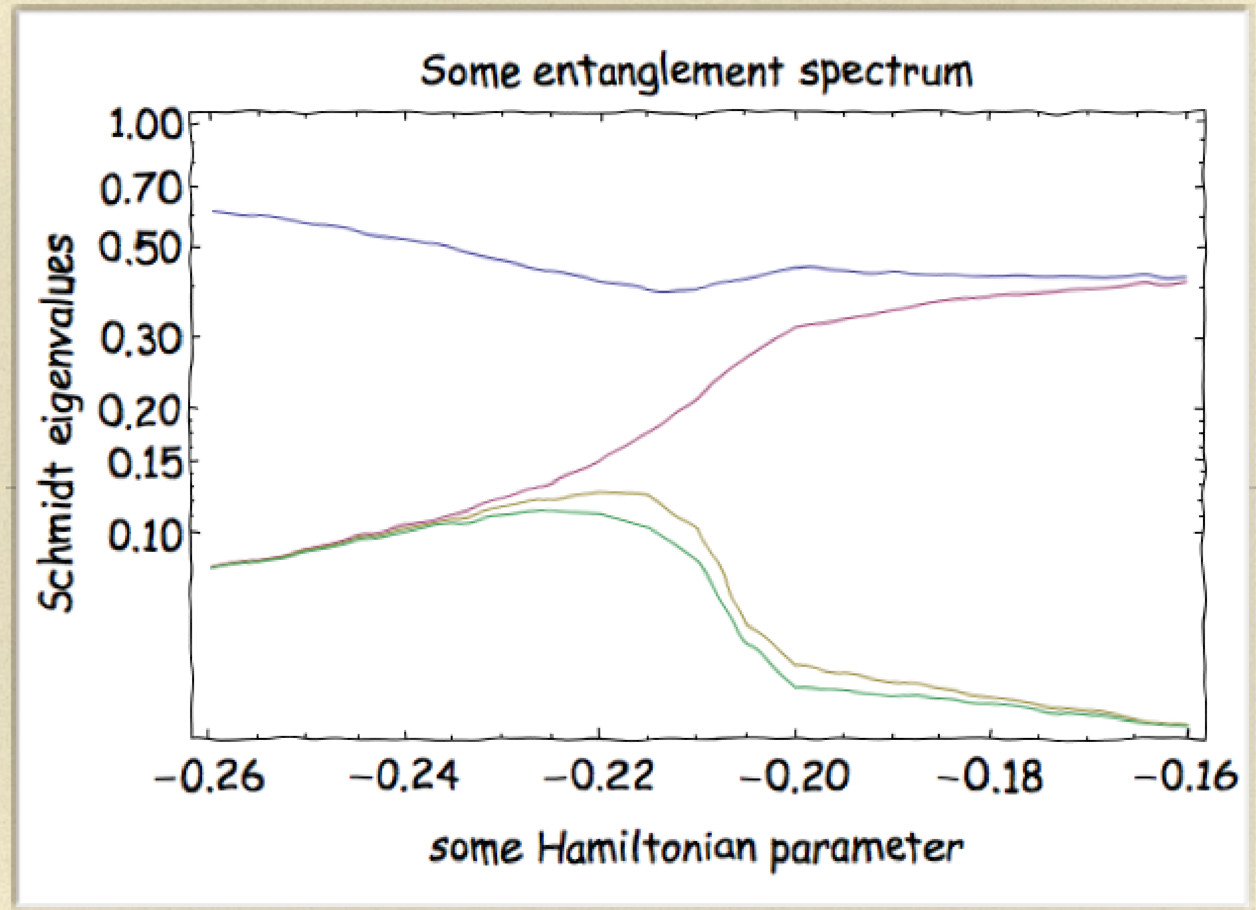
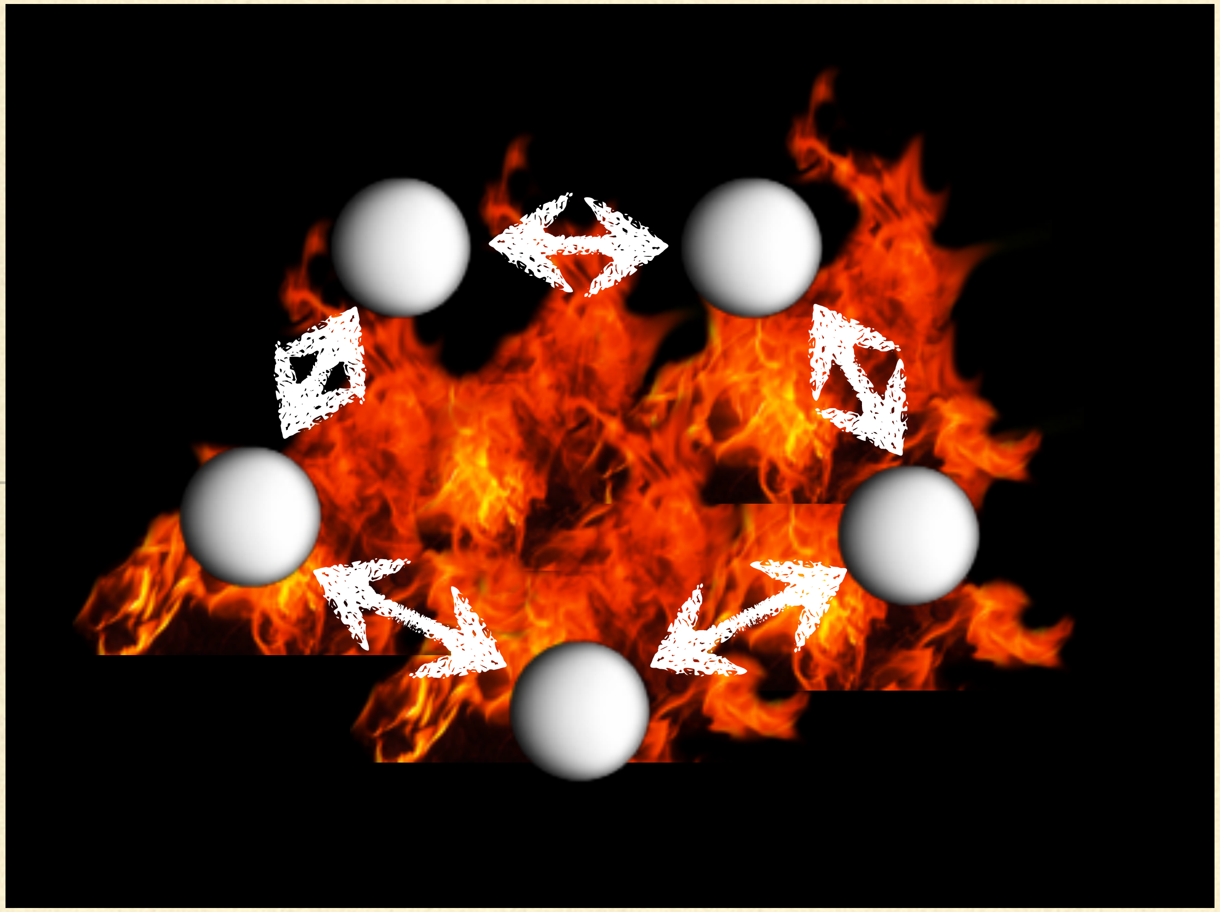
QTEQ QUANTUM TECHNOLOGY at QUEEN'S

Part I:
Entanglement Spectrum



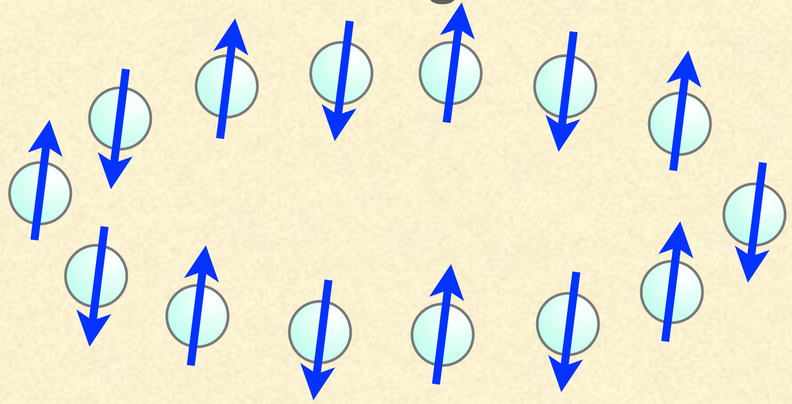
Part II:
Quantum Correlations



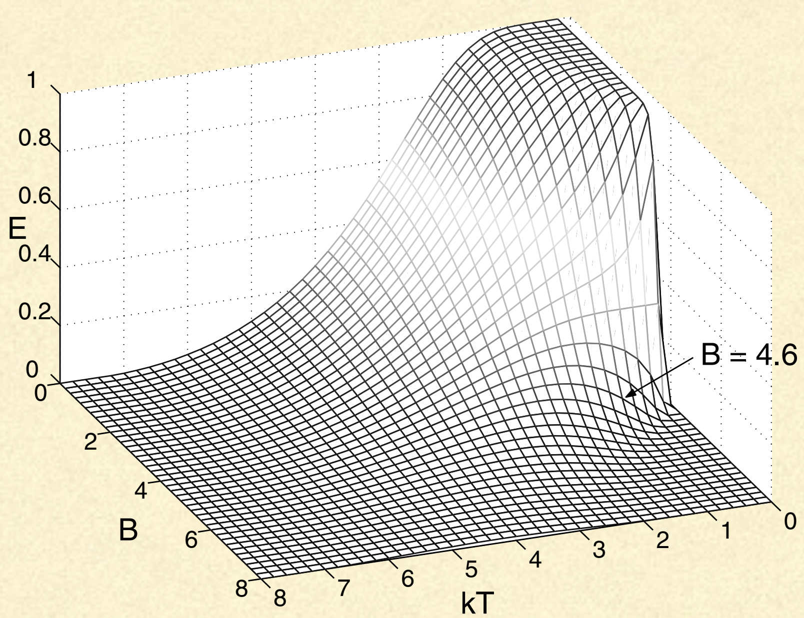
For an introduction see: Amico, Fazio, Osterloh, Vedral, RMP 2008

ENTANGLEMENT IN SPIN CHAINS

Thermal entanglement in the Heisenberg model

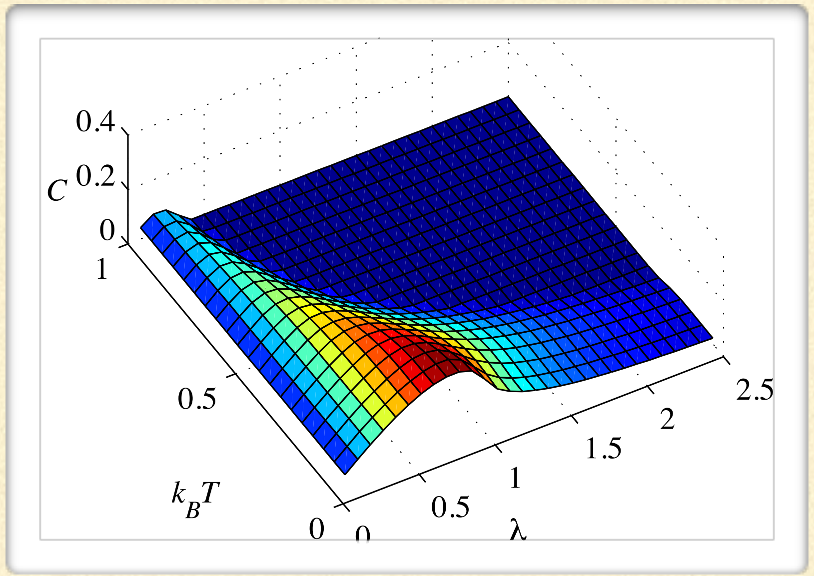


O'Connor and Wootters, PRA 2001

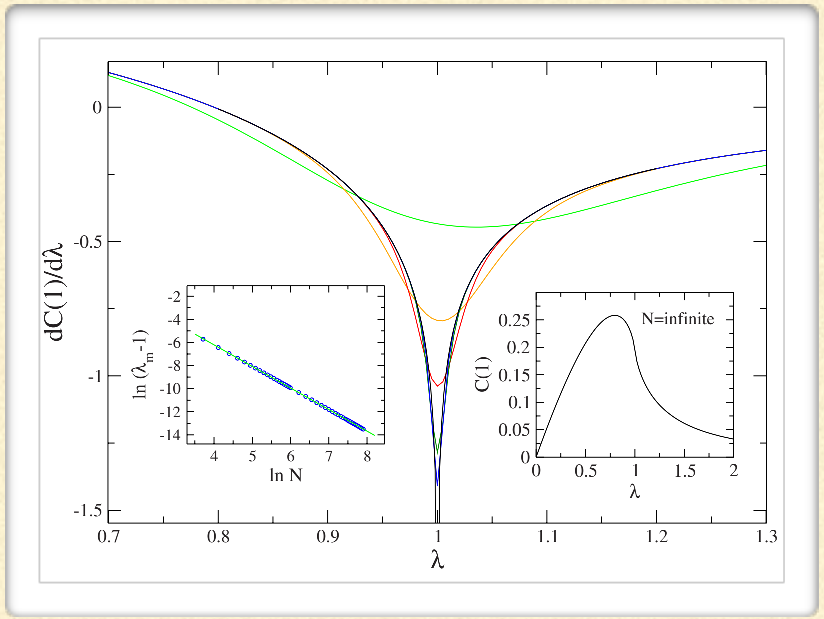


from Arnesen, Bose, Vedral, PRL 2001

Non Analytic behavior at a phase transition

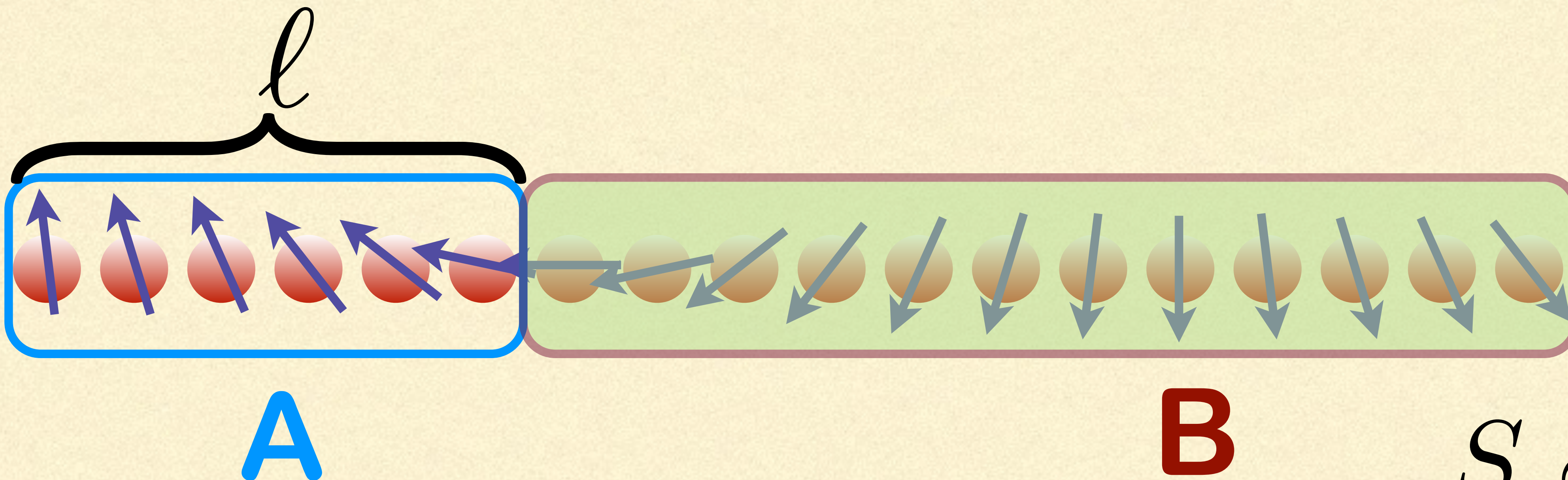


from Osborne & Nielsen, PRA 2002



from Osterloh et al., Nature 2002

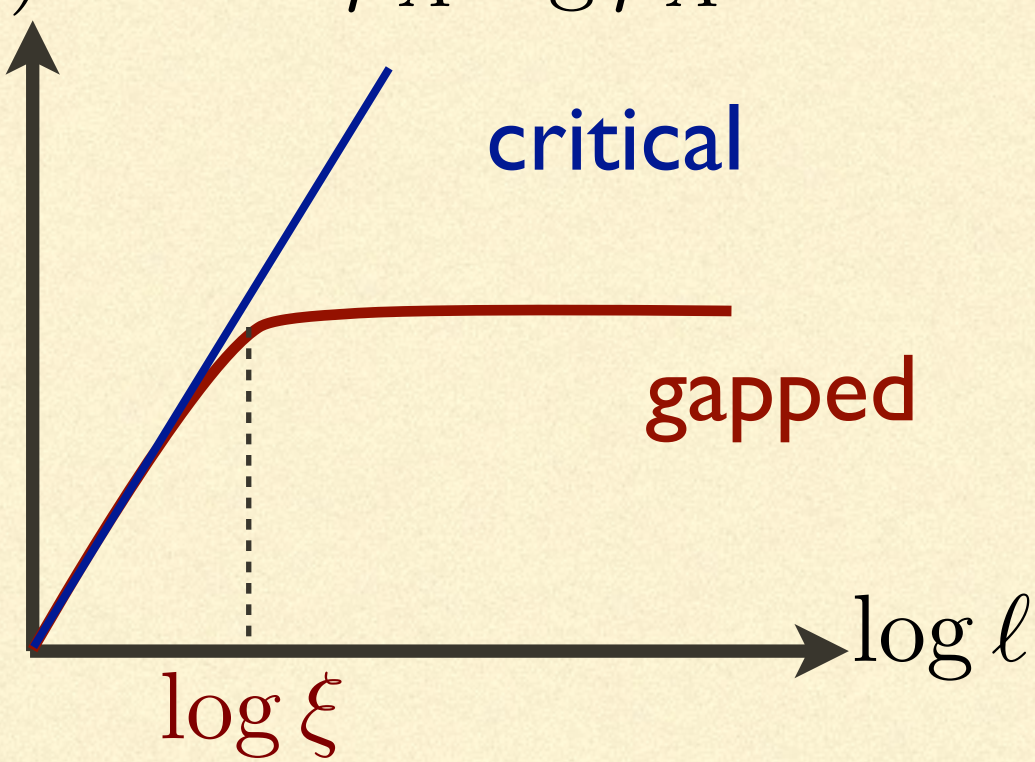
ENTANGLEMENT ENTROPY



$|\psi_G\rangle$

$$S \sim c \log \ell$$

$$S(\rho_A) = -\mathrm{Tr}\rho_A \log \rho_A$$

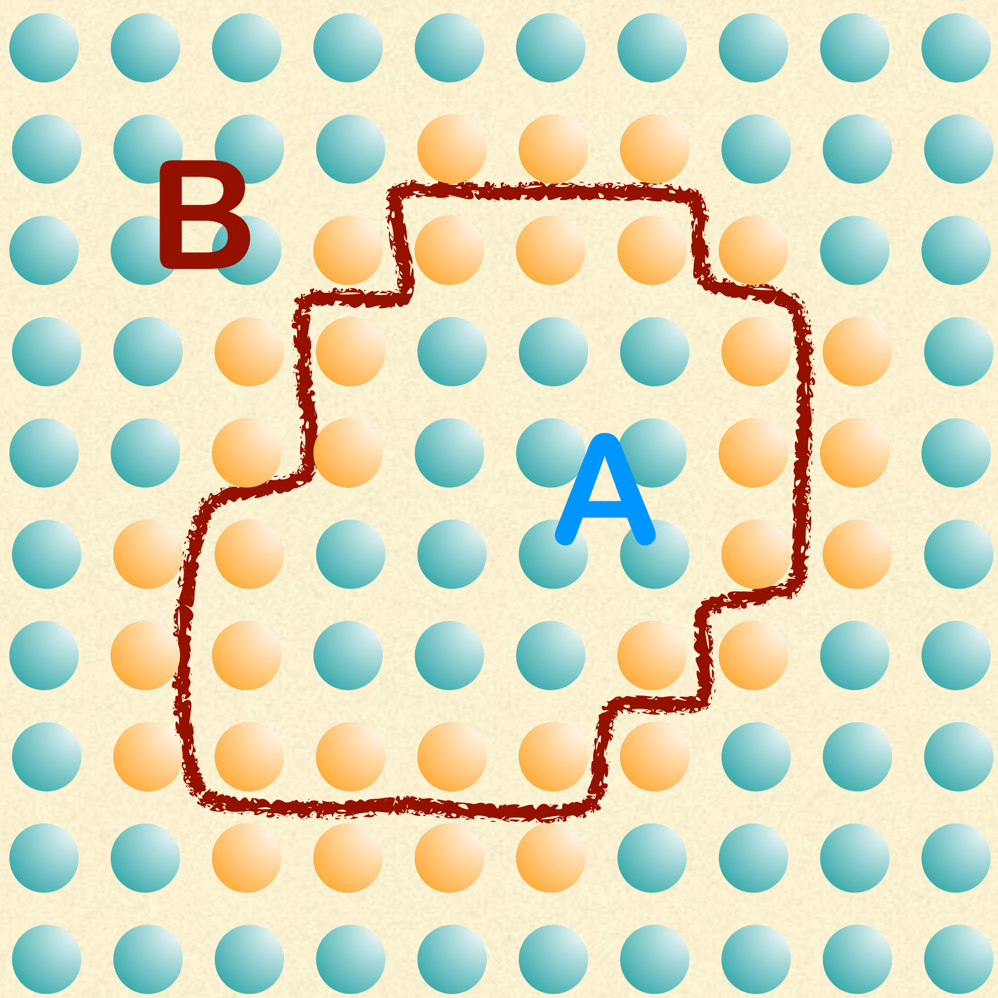


homogeneous chains:

Holzhey, Larsen, and Wilczek, Nucl Phys 1994
Vidal, Latorre, Rico, Kitaev, PRL 2003
Calabrese & Cardy, JSTAT 2004
Jin & Korepin, JSTAT 2004
Peschel 2004,2005
Keating & Mezzadri, PRL 2005
GDC, Montangero, Calabrese, Fazio, JSTAT 2006
...

Also random chains:
Refael and Moore, PRL 2004
Laflorencie, PRB 2005
Binosi, GDC, Montangero, Recati, PRB 2007
...

AREA LAW



For a cubic lattice in D-dimensions:

$$S \sim \ell^{D-1}$$

fulfilled by certain class of states and models:
Eisert, Cramer, Plenio, 2010
Verstraete & Cirac, 2004
Verstraete, Wolf, Perez-Garcia, Cirac, 2006
in general logarithmic corrections

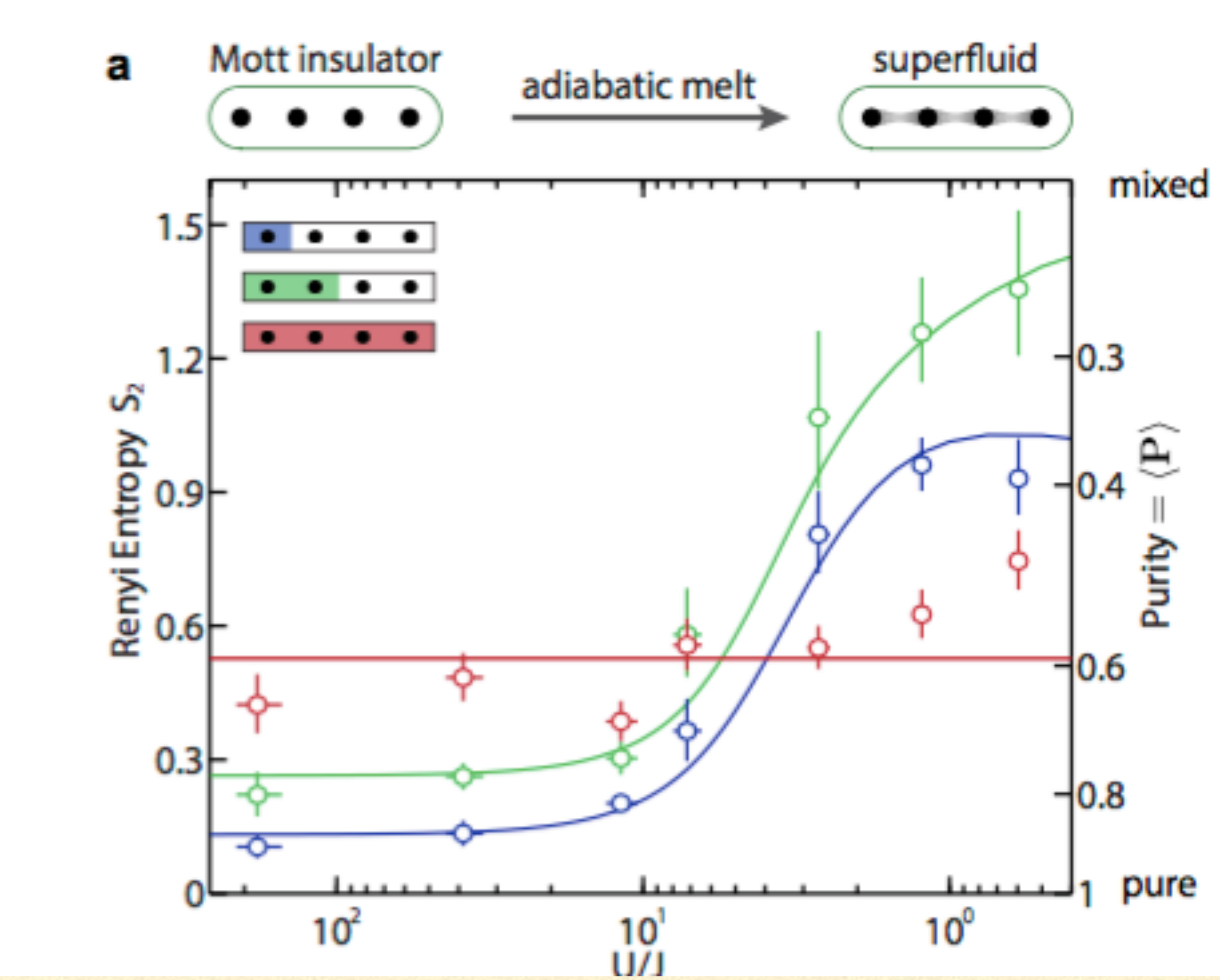
In 1D:

$$S \sim s_0 + \frac{c}{6} \log \ell$$

Hastings, 2007

Entanglement is proportional to the area of the boundary between A and B

ENTANGLEMENT ENTROPY MEASUREMENTS IN ULTRACOLD ATOMS



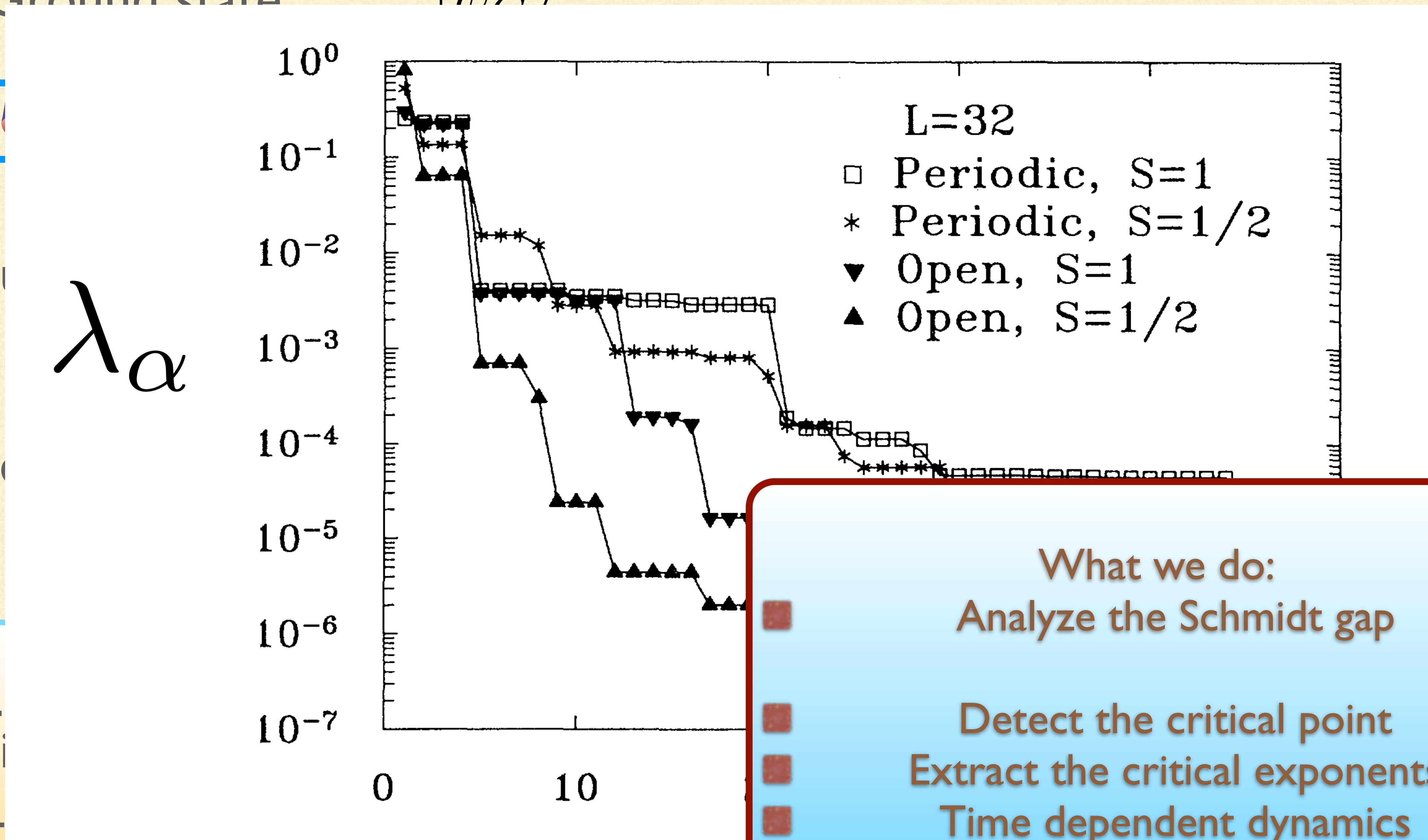
R. Islam, …, M. Greiner (Harvard), arXiv:1509.01160

ENTANGLEMENT SPECTRUM

Ground state $|\psi_G\rangle$

Redu

Eig



S.R. White, I

What we do:

- Analyze the Schmidt gap
- Detect the critical point
- Extract the critical exponents
- Time dependent dynamics

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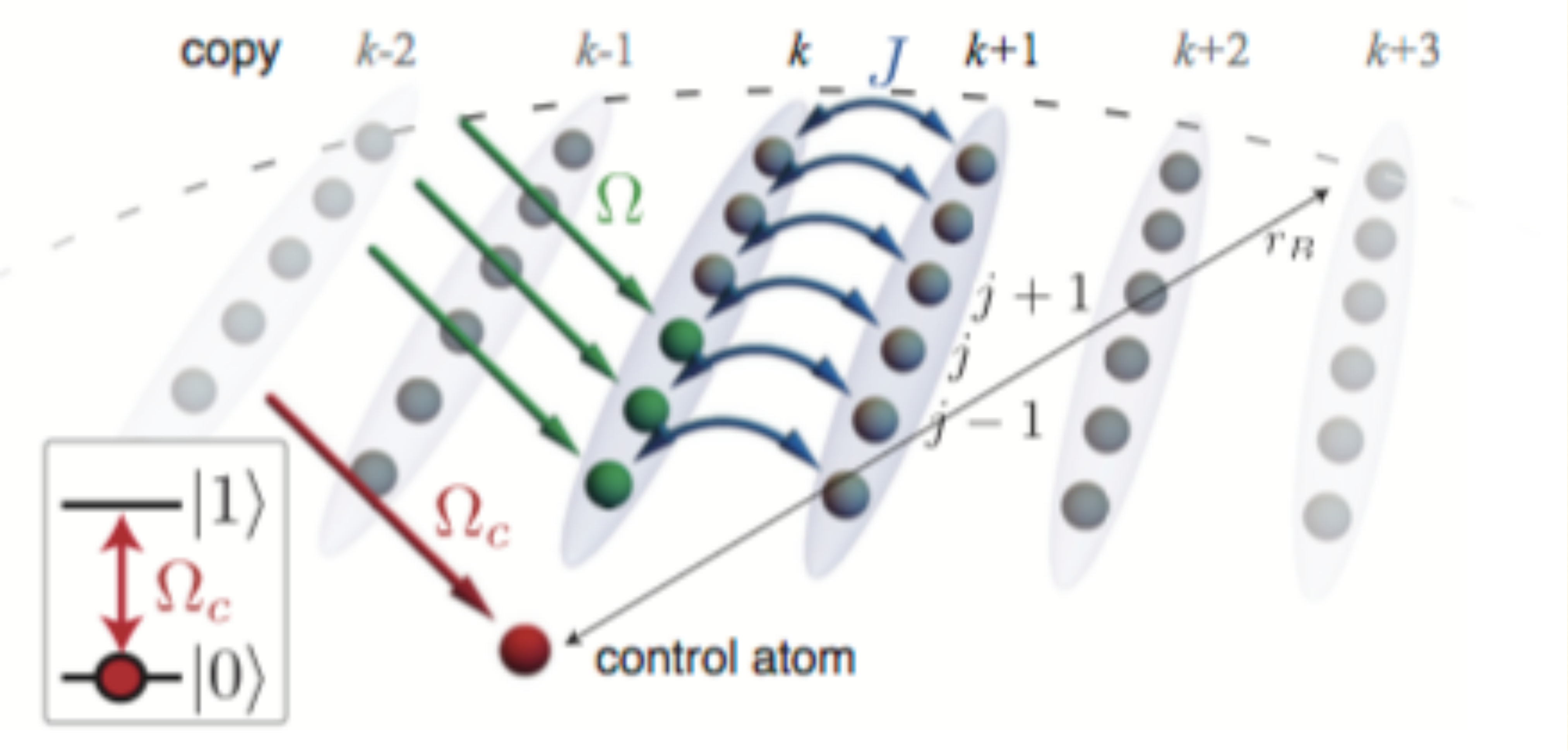
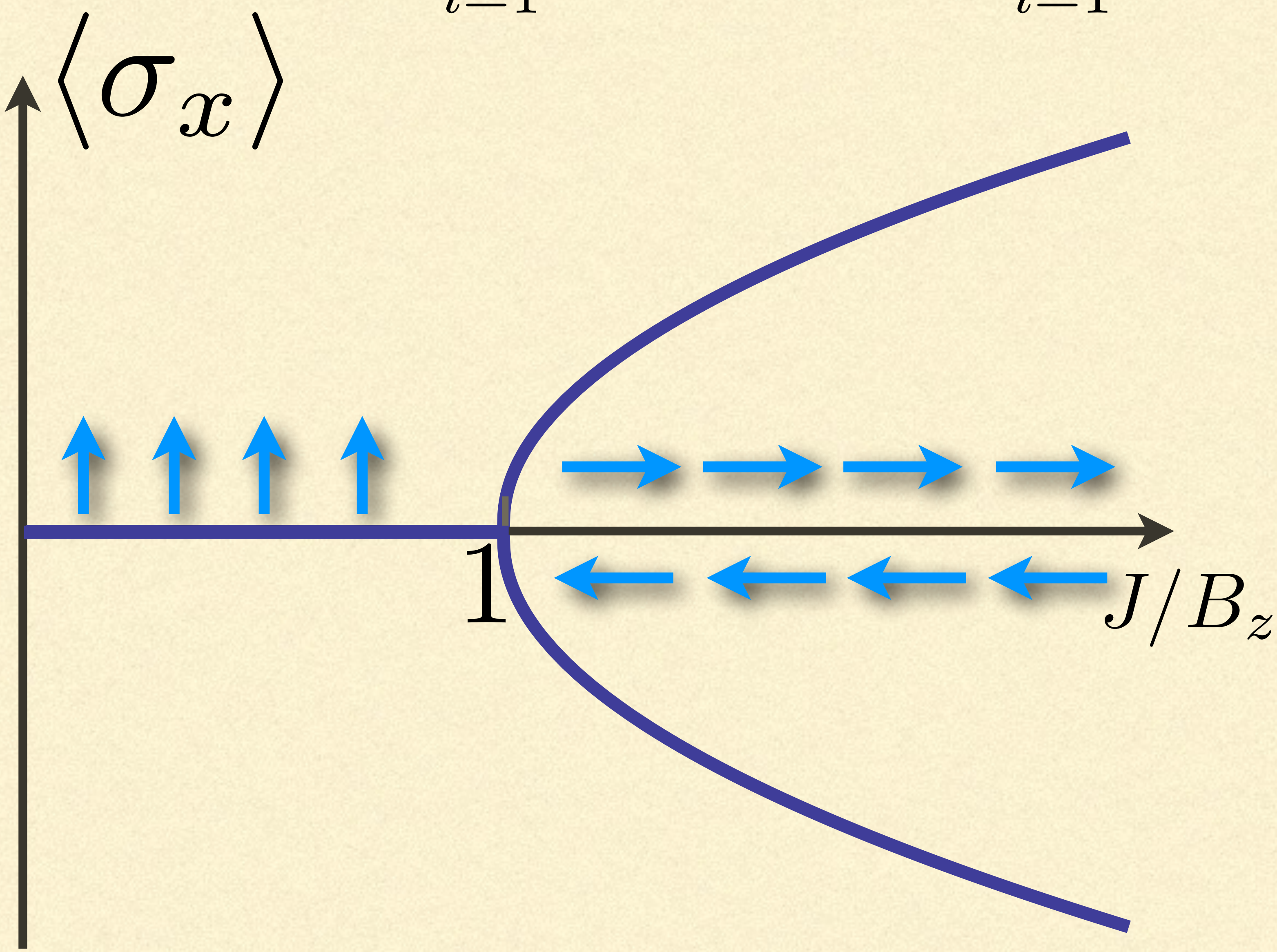
MEASURING THE ENTANGLEMENT SPECTRUM



H. Pichler, et al., arXiv:1605.08624

THE ISING MODEL

$$H = -J \sum_{i=1}^{L-1} \sigma_x^i \sigma_x^{i+1} - B_z \sum_{i=1}^{L} \sigma_z^i$$



Scaling of the order parameter

$$\langle \sigma_x \rangle \sim |B_z - J|^{\beta}$$

critical exponent

$$\beta = 1/8$$

Correlations

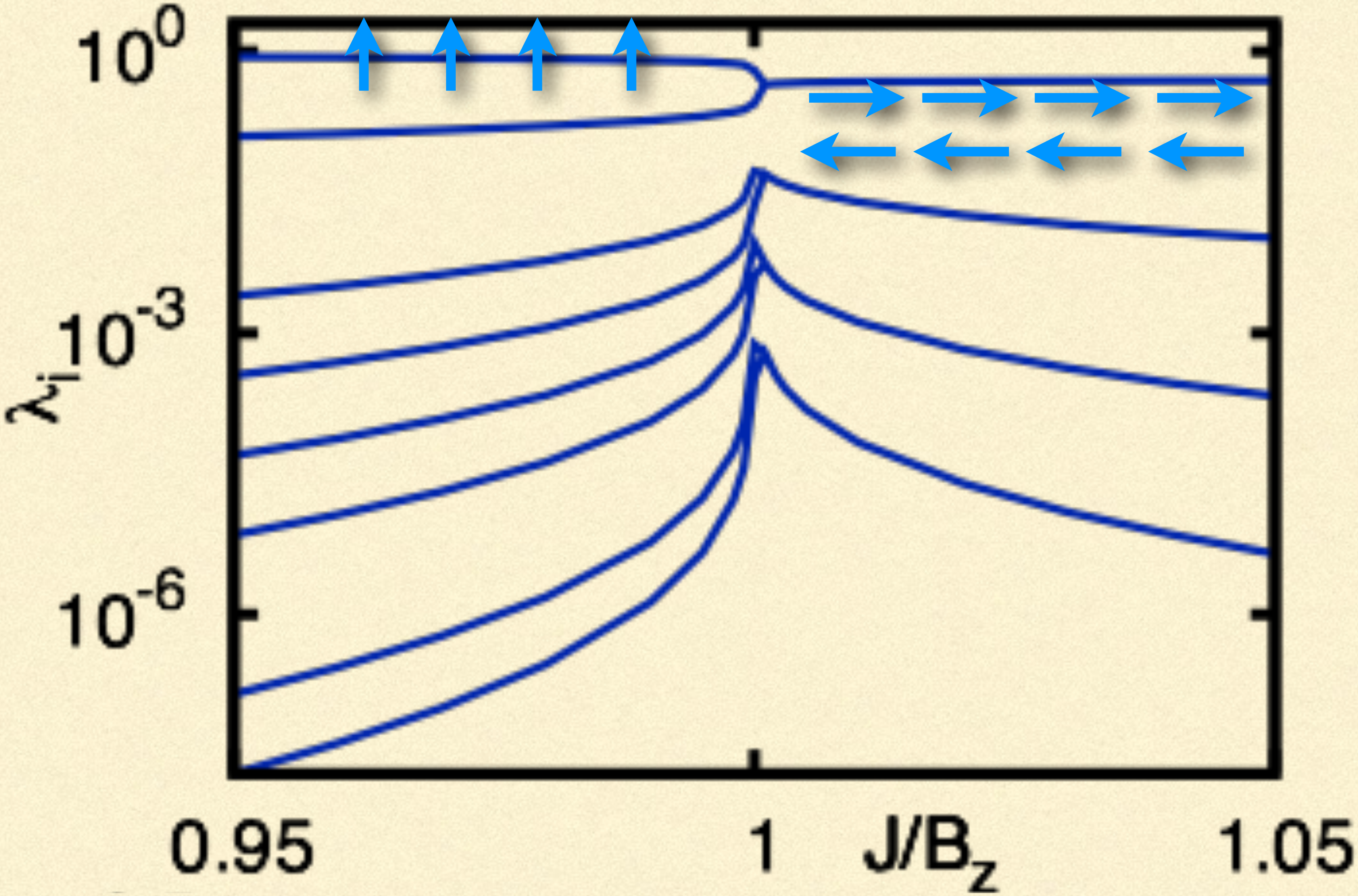
$$\langle \sigma_x^i \sigma_x^{i+r} \rangle \sim e^{-r/\xi}$$

$$\xi \sim |B_z - J|^{-\nu}$$

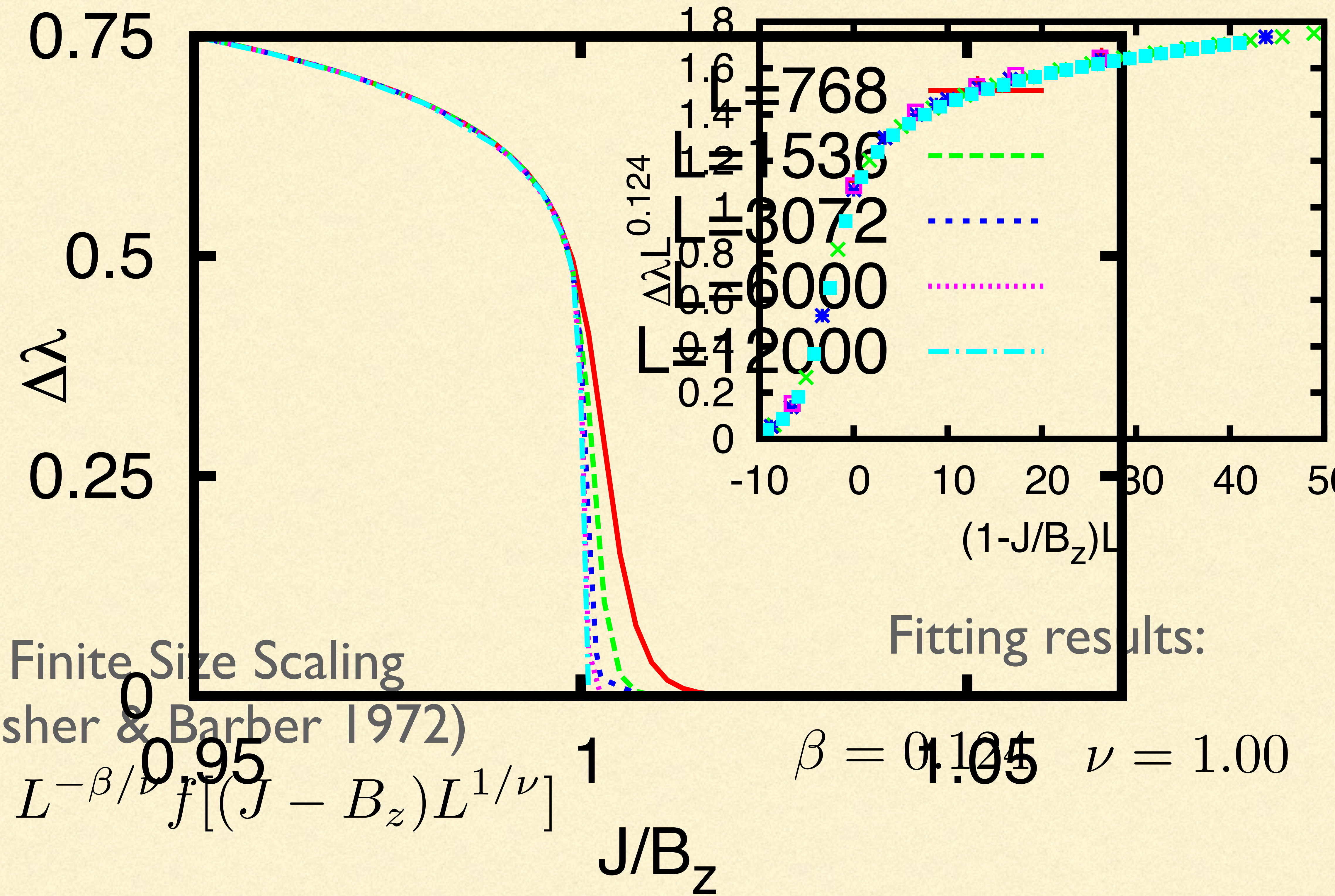
$$\nu = 1$$

ISING: ENTANGLEMENT SPECTRUM

$\ell = L/2$



ISING: SCHMIDT GAP (SCALING)



Finite Size Scaling
(Fisher & Barber 1972)

$$\Delta\lambda = L^{-\beta/\nu} f[(J - B_z)L^{1/\nu}]$$

Fitting results:

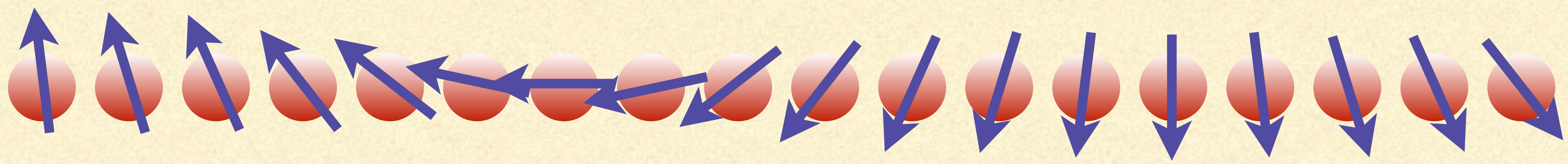
$$\beta = 0.124 \quad \nu = 1.00$$

ENTANGLEMENT SPECTRUM DYNAMICS

with Giacomo Torlai (Waterloo) & Luca Tagliacozzo (Strathclyde)

INSTANTANEOUS QUENCHES

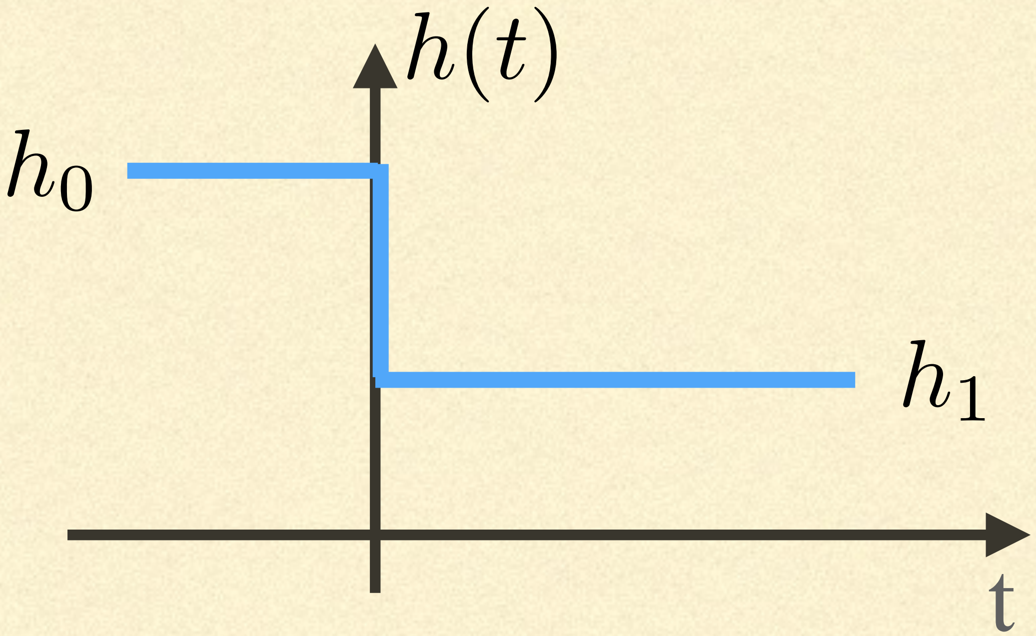
$|\psi_G\rangle$



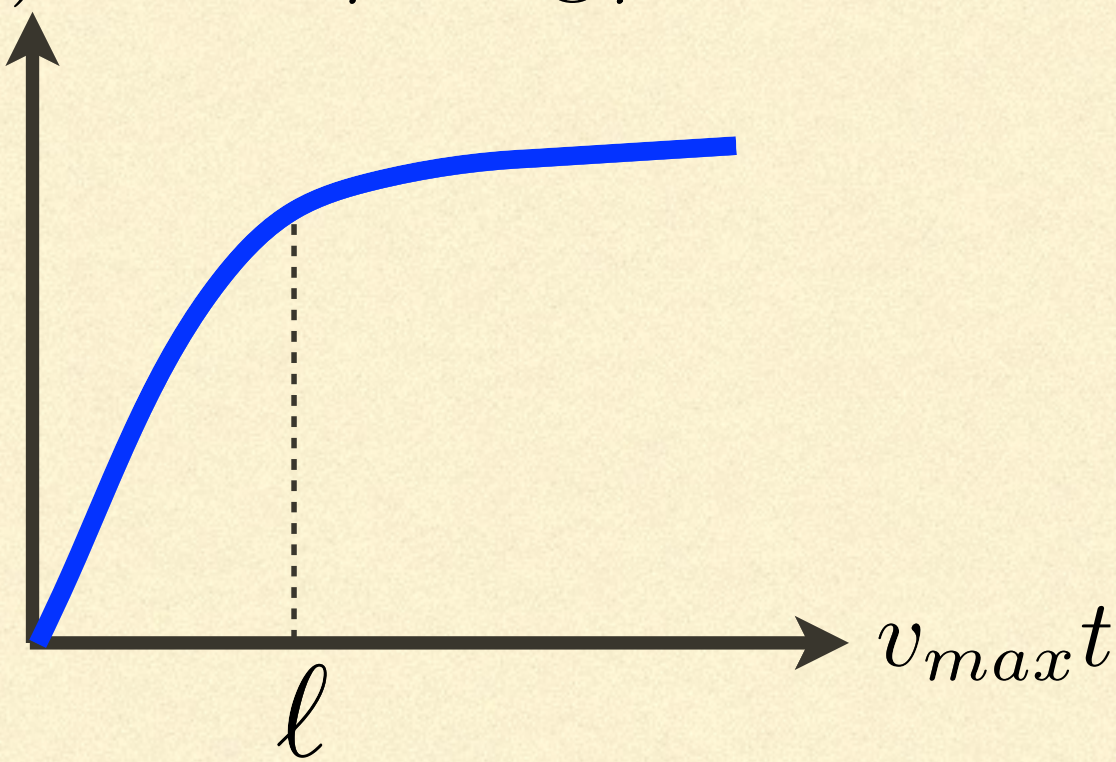
$H(h_0)$

$\Downarrow$

$H(h_1)$



$$S(\rho_A) = -\mathrm{Tr}\rho_A \log \rho_A$$



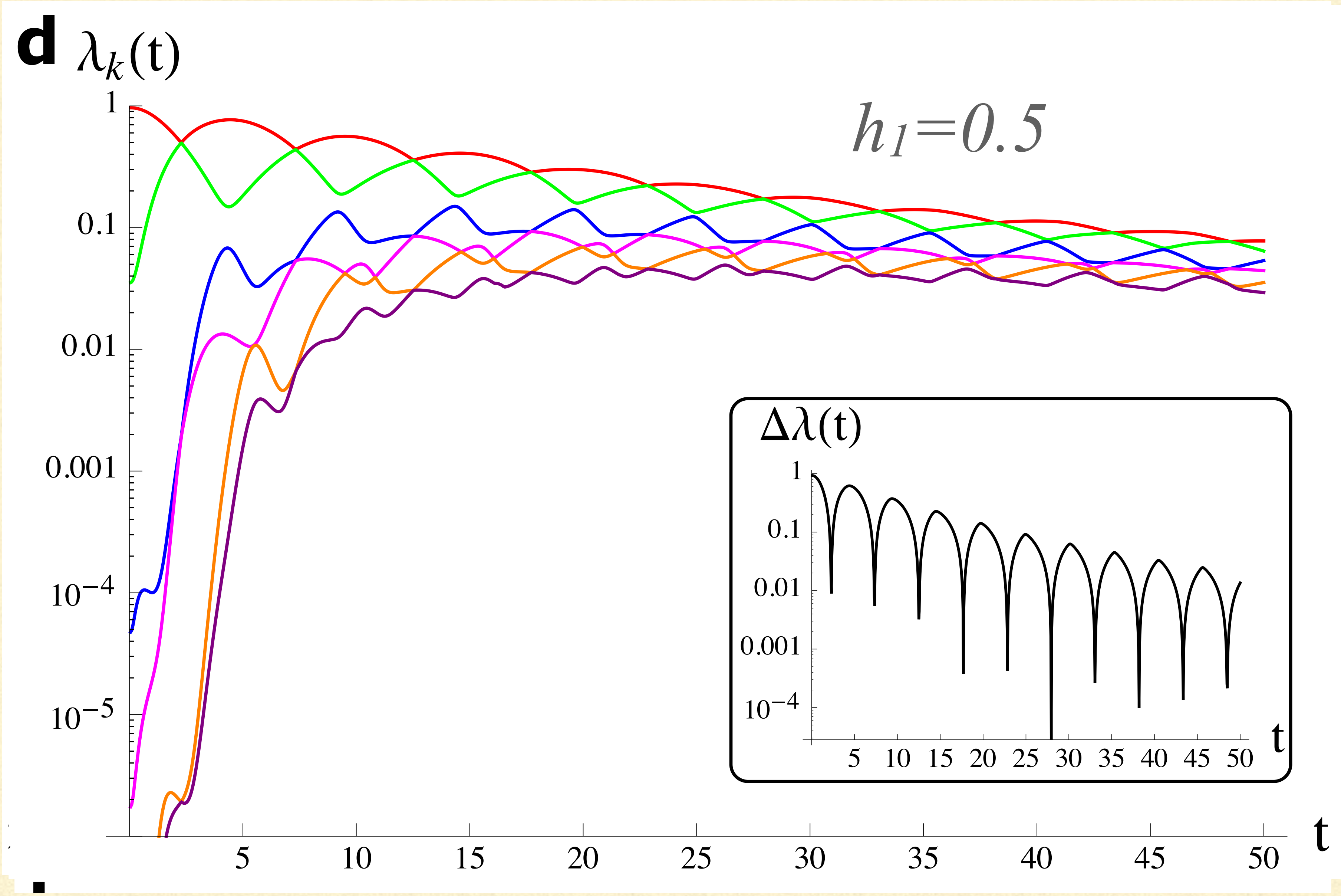
Calabrese & Cardy, JSTAT 2005
De Chiara, Montangero, Calabrese, Fazio, JSTAT 2006
Eisert & Osborne, PRL 2006
Lauchli & Kollath, JSTAT 2008
Fagotti & Calabrese PRA 2008
...

INSTANTANEOUS QUENCHES

Ising model $$\hat{H}(h) = -\frac{1}{2}\left[\sum_{i=1}^{L-1} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + h \sum_{i=1}^{L} \hat{\sigma}_i^z\right]$$

From the Paramagnetic phase

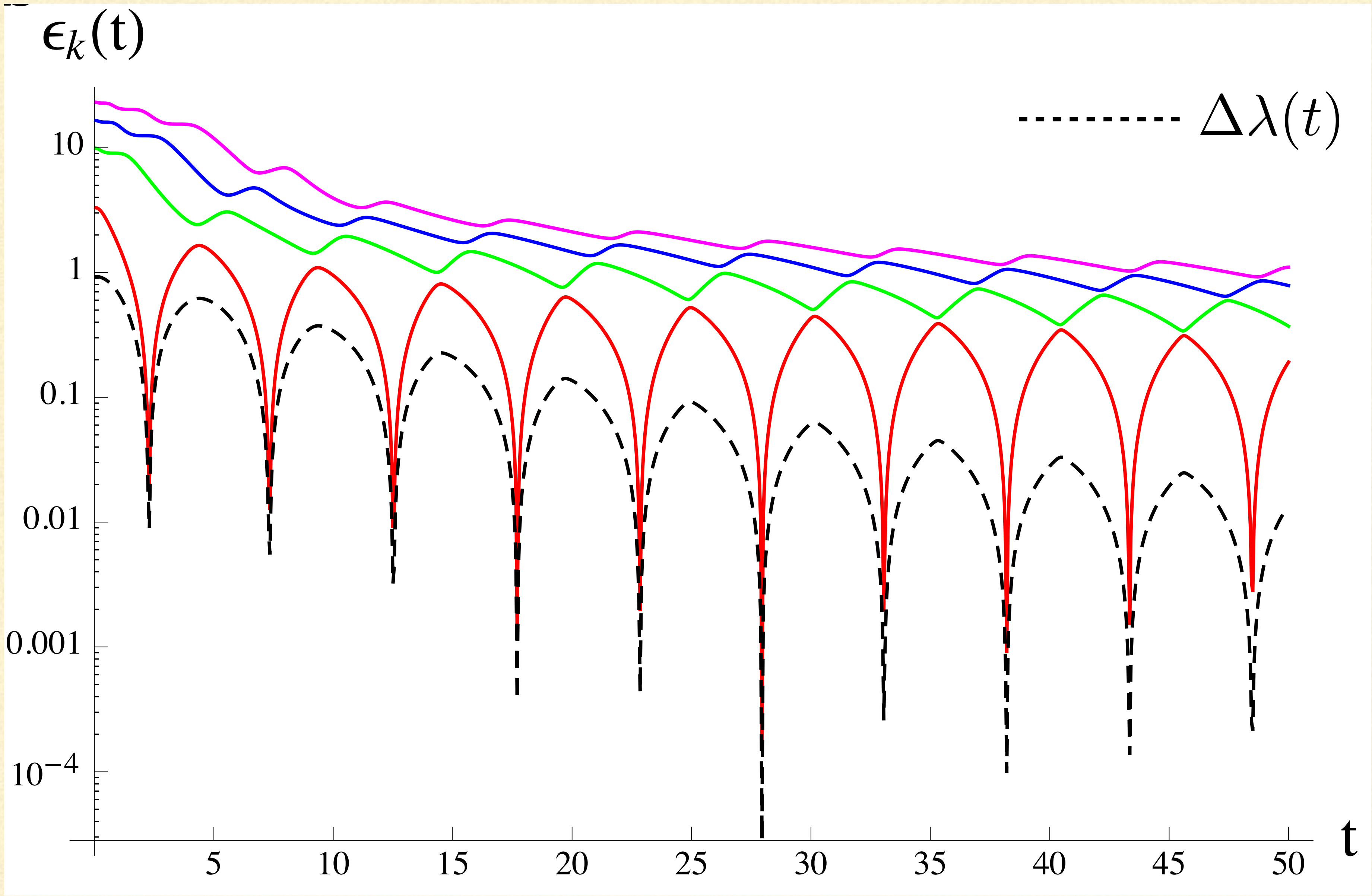
$h_0=1.5$



ORIGIN OF THE CROSSINGS

$$\rho(t) = \frac{1}{Z(t)} \exp[-\sum_k n_k(t)\epsilon_k(t)]$$

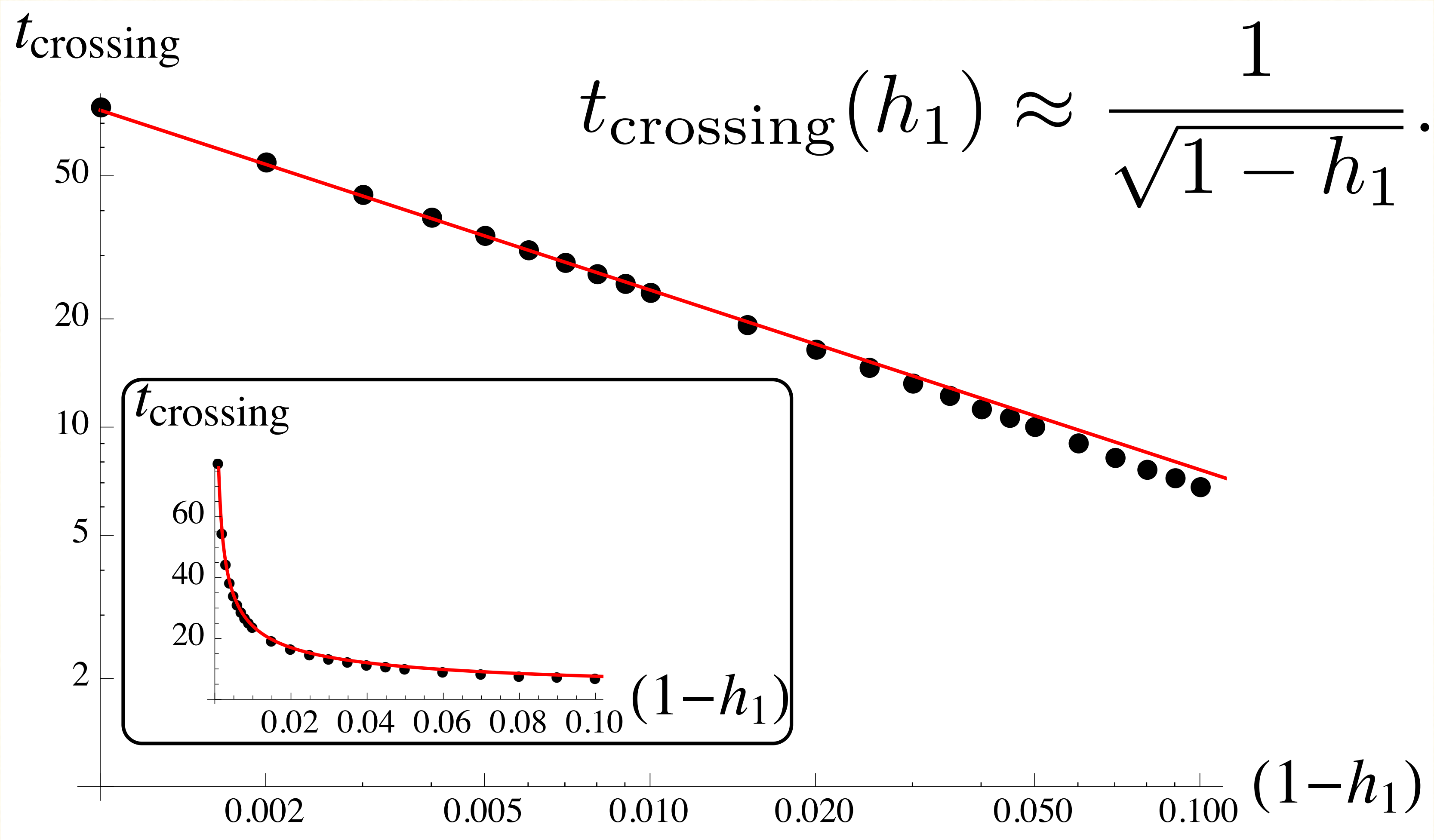
Peschel, Korepin…



The lowest single particle eigenvalue goes to zero periodically

CROSSING TIME

Close to the critical point the first crossing time diverges!

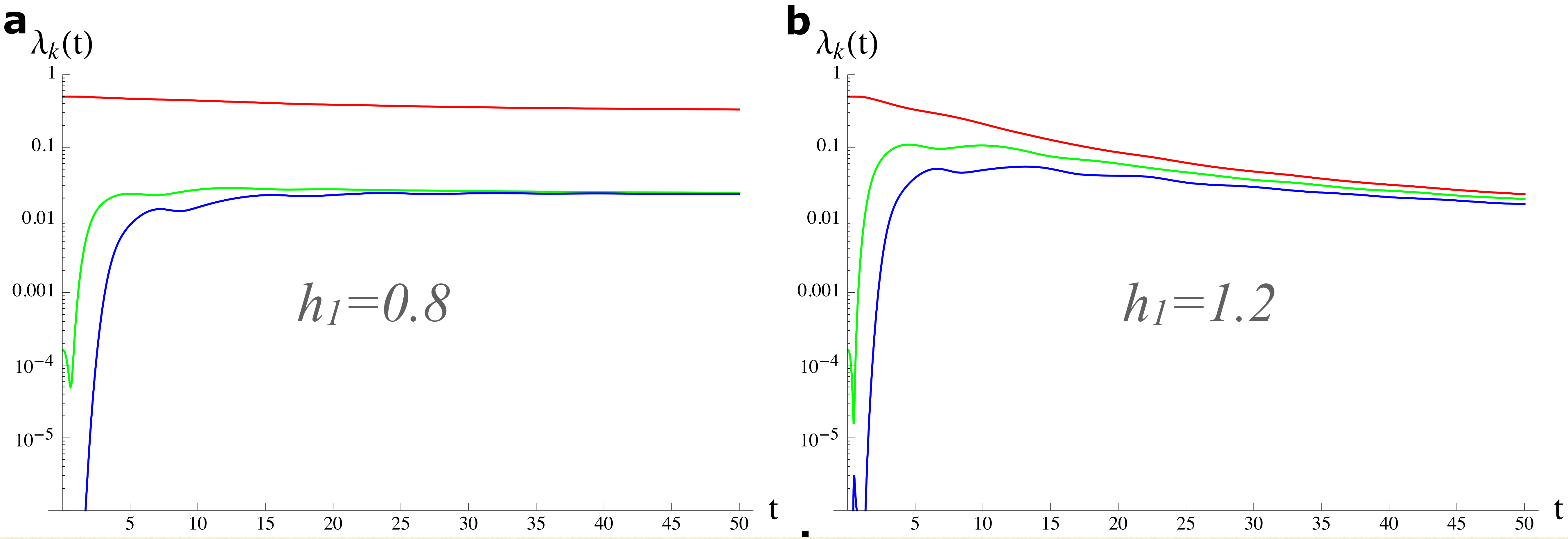


similar to the magnetisation

M. Heyl, A. Polkovnikov, S. Kehrein, Phys. Rev. Lett. 110, 135704 (2013)
P. Calabrese, F. H. L. Essler, M. Fagotti, J. Stat. Mech. (2012) P07016

QUENCHES FROM FERRO

$h_0=0.5$



No crossing!

- Not due to Z_2 symmetry (tDMRG calculations)
- Has to do with criticality
- gap related to final H

CONCLUSIONS (I)

- First finite size scaling analysis of the **entanglement spectrum**
- Extraction of the **critical exponents**
- **Entanglement Spectrum Dynamics (also XXZ)**
- **Slow quenches: Kibble-Zurek mechanism**
- Open Questions: dynamics in **non-integrable** models? random systems?

GDC, Lewenstein & Sanpera, PRB 2011
GDC, Lepori, Lewenstein & Sanpera, PRL 2012
Lepori, GDC & Sanpera, PRB 2013
Torlai, Tagliacozzo, GDC, JSTAT 2014

QUANTUM CORRELATIONS

with
Matthew Power (Belfast),
Steve Campbell (Belfast),
Mariona Moreno-Pardoner (Belfast→Barcelona)

QUANTUM CORRELATIONS AND DISCORD

- **Entanglement does not capture all quantum correlations**, especially for **thermal states**: a non-entangled state can still be quantum correlated

- Many forms of quantum correlations have been proposed. Here we consider **quantum discord** (Ollivier-Zurek 2001; Henderson-Vedral 2001)

- QD (and not entanglement) is a **resource** for certain quantum computational tasks (DQC1)

- Interestingly QD **doesn't fulfil monogamy** relations

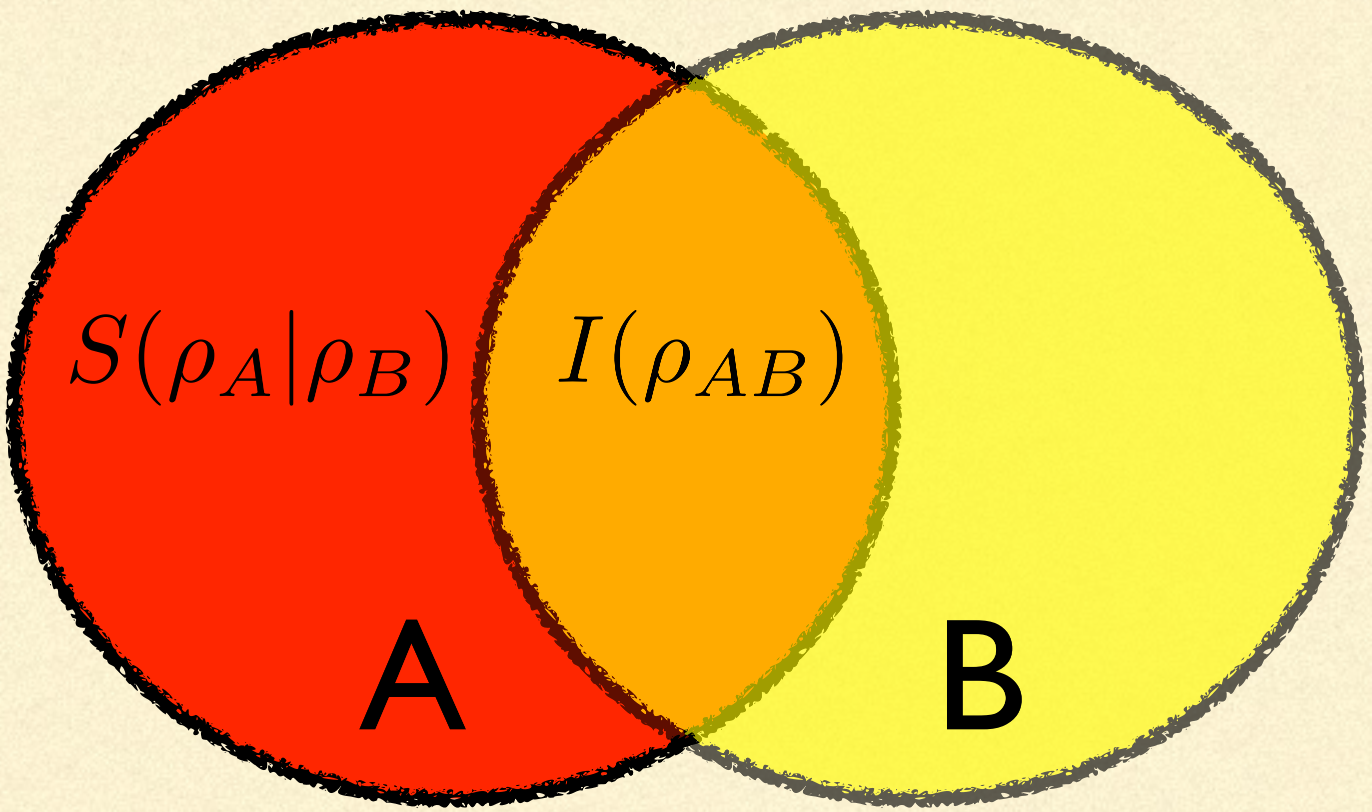
BIPARTITE QUANTUM DISCORD

Quantum discord is the difference of two classically equivalent ways to measure correlations:

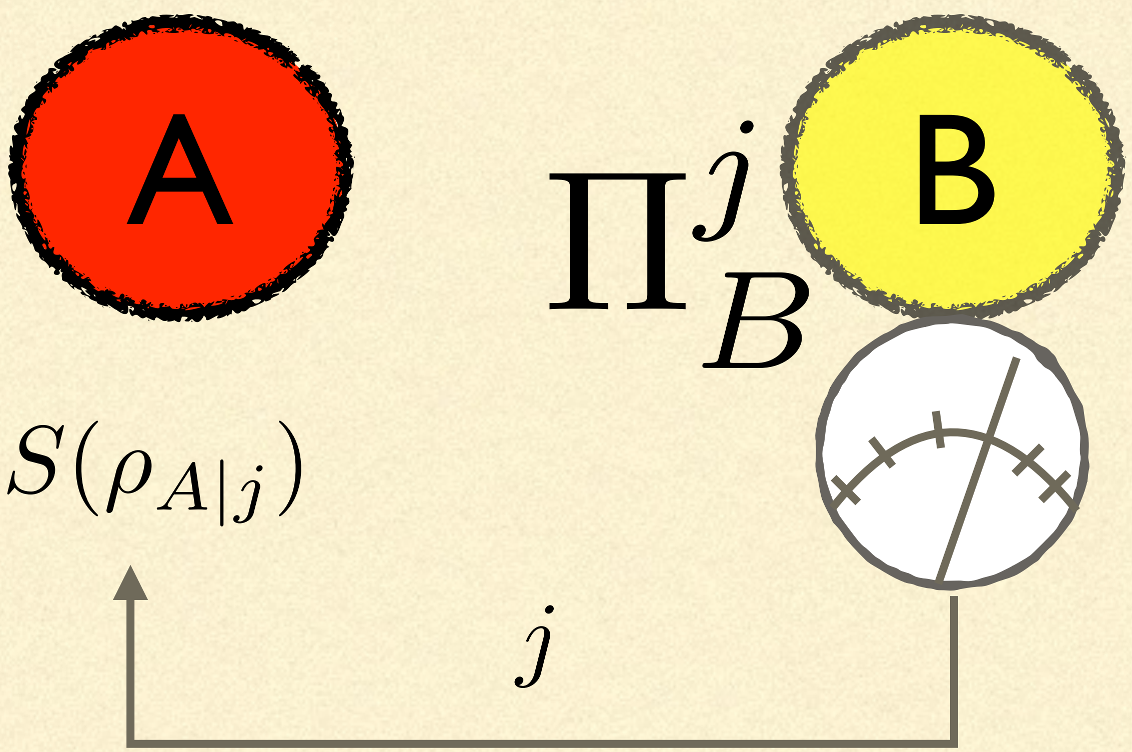
mutual information $$I(\rho_{AB}) = S(\rho_A) + \underbrace{S(\rho_B) - S(\rho_{AB})}$$

conditional entropy

$$S(\rho_A|\rho_B) = S(\rho_{AB}) - S(\rho_B)$$



BIPARTITE QUANTUM DISCORD



$$S(\rho_{AB}|\Pi_B^j) = \sum_j p_j S(\rho_{A|j})$$

one-way classical information

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{AB}|\Pi_B^j)$$

quantum discord

$$\mathcal{D}^{B\to A}(\rho_{AB}) = \inf_{\{\hat{\Pi}_B^j\}} [I(\rho_{AB}) - J(\rho_{AB})]$$

In the classical world $I(\rho_{AB})$ and $J(\rho_{AB})$ coincide

DISCORD FOR SPIN-1 CHAINS

Spin-1 Heisenberg chain with uniaxial field:

$$\mathcal{H} = \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + S_i^z S_{i+1}^z + U \sum_i \left(S_i^z\right)^2$$

NÉEL **HALDANE** **LARGE D**

$|-1,+1,-1,+1,\ldots\rangle$ U_{NH}

$|+1,-1,+1,-1,\ldots\rangle$

$|+1,0,0,\ldots,0,-1\rangle$

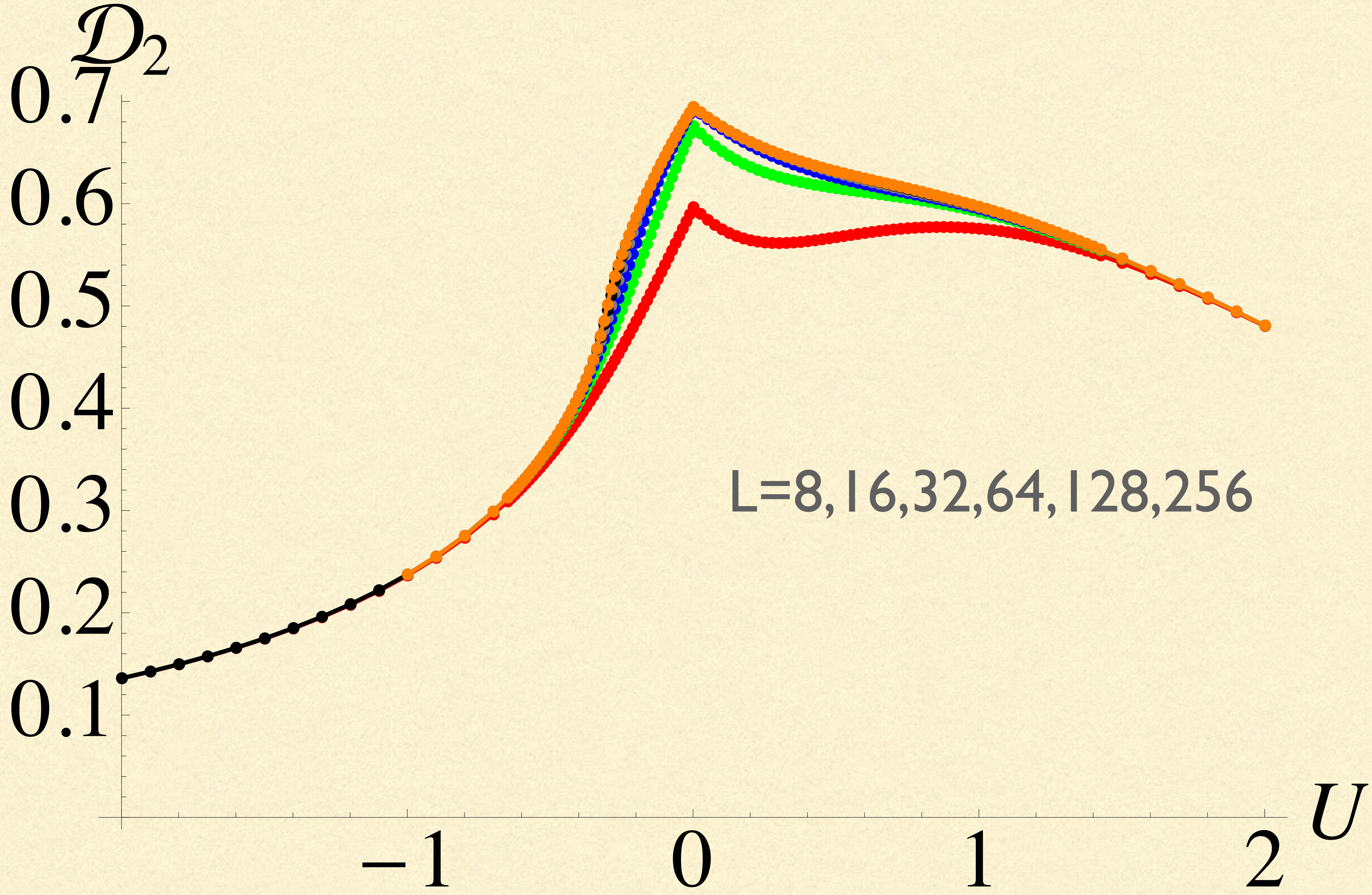
U_{HL}

$|000\ldots0\rangle$

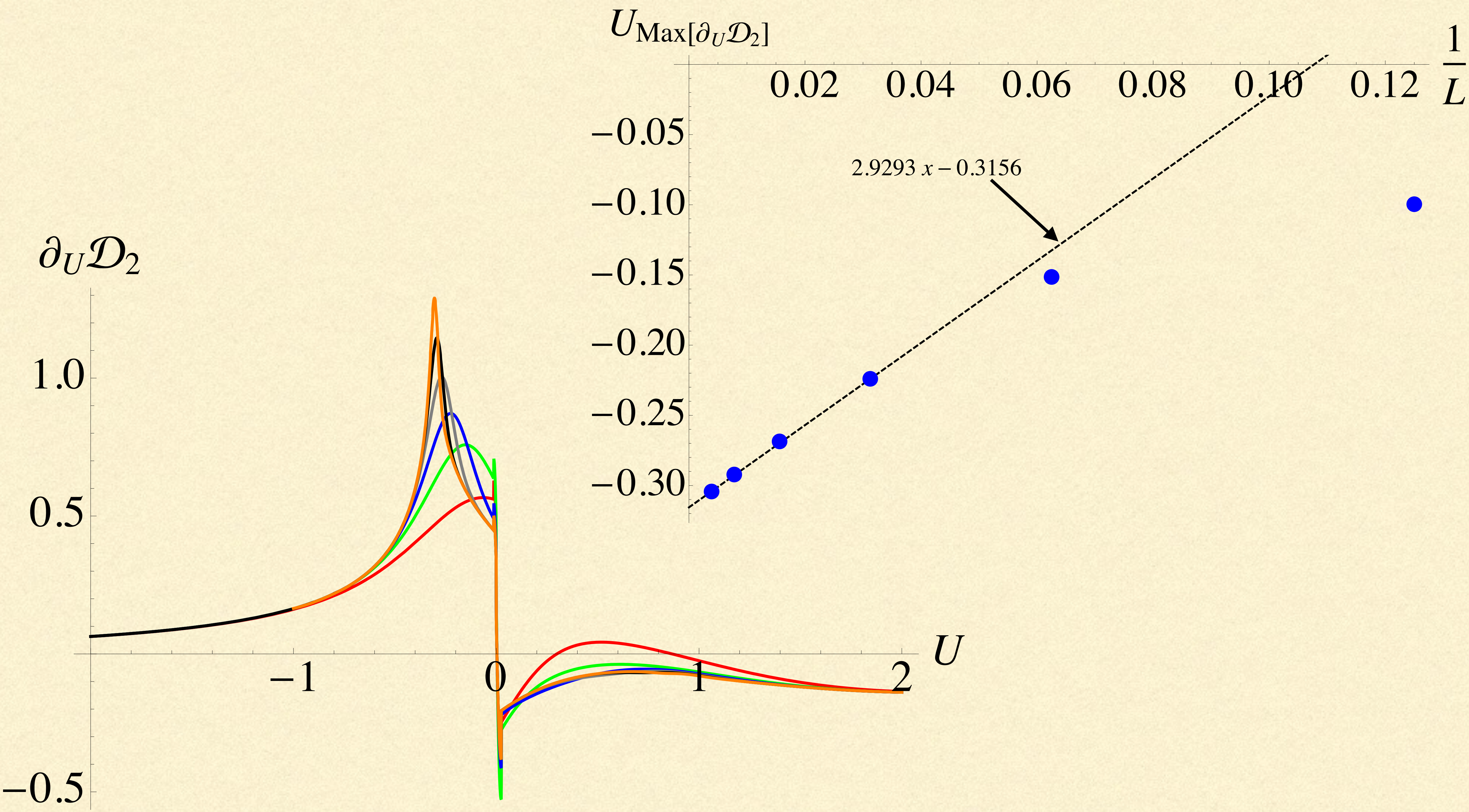
Ising transition

Gaussian, 3rd order

RESULTS: BIPARTITE DISCORD



FINITE SIZE-SCALING: NEEL-HALDANE



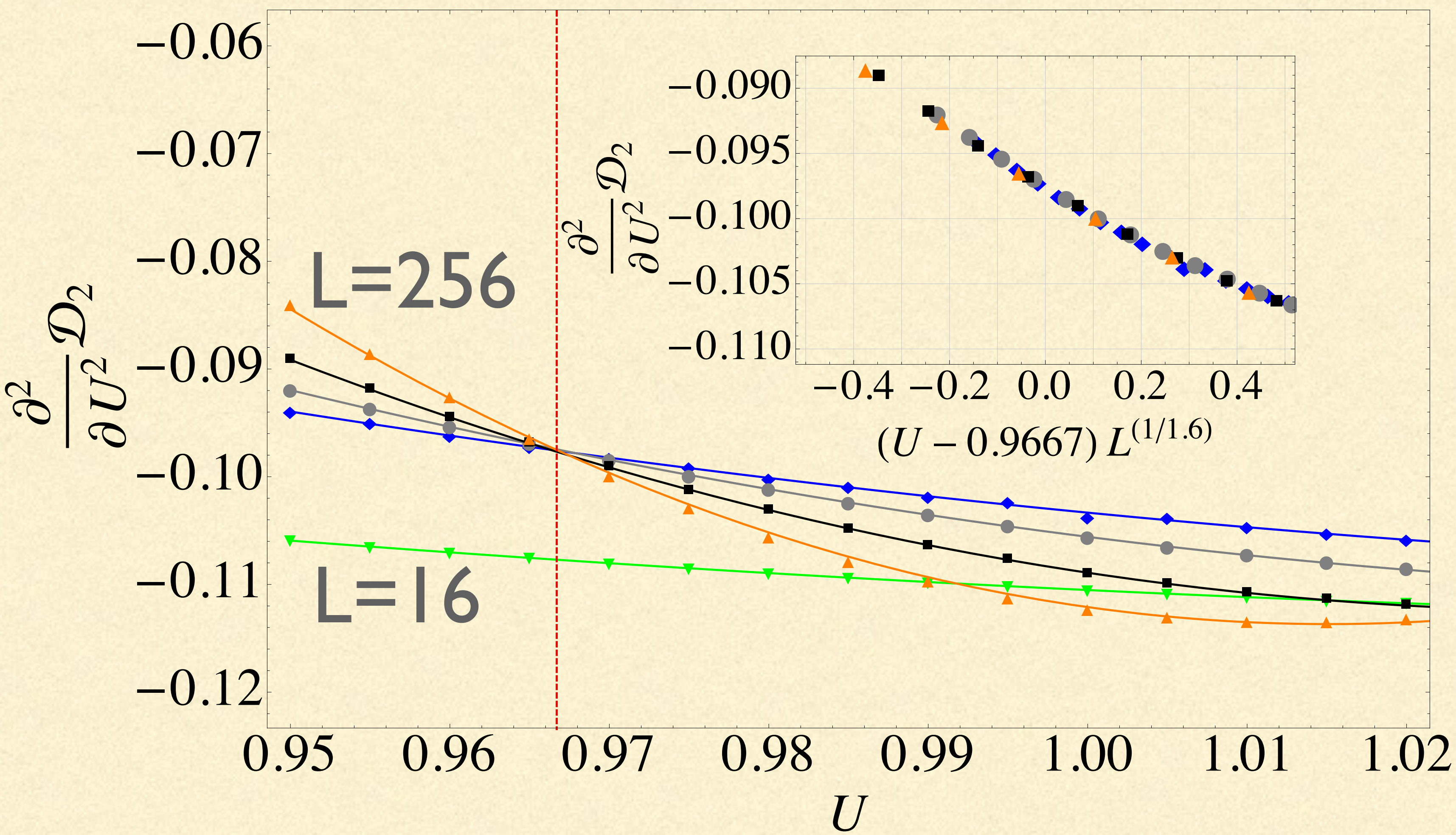
[1] Hu, Normand, Wang, Yu, PRB (2011)

FINITE SIZE-SCALING: HALDANE-LARGE D

3rd order-Gaussian transition at U = 0.96845 [1]

at criticality: **a point of inflection** in the second order derivative of discord

$$\frac{\partial^2 \mathcal{D}_2}{\partial U^2} = f[(U - 0.9667)L^{1/\nu}]$$

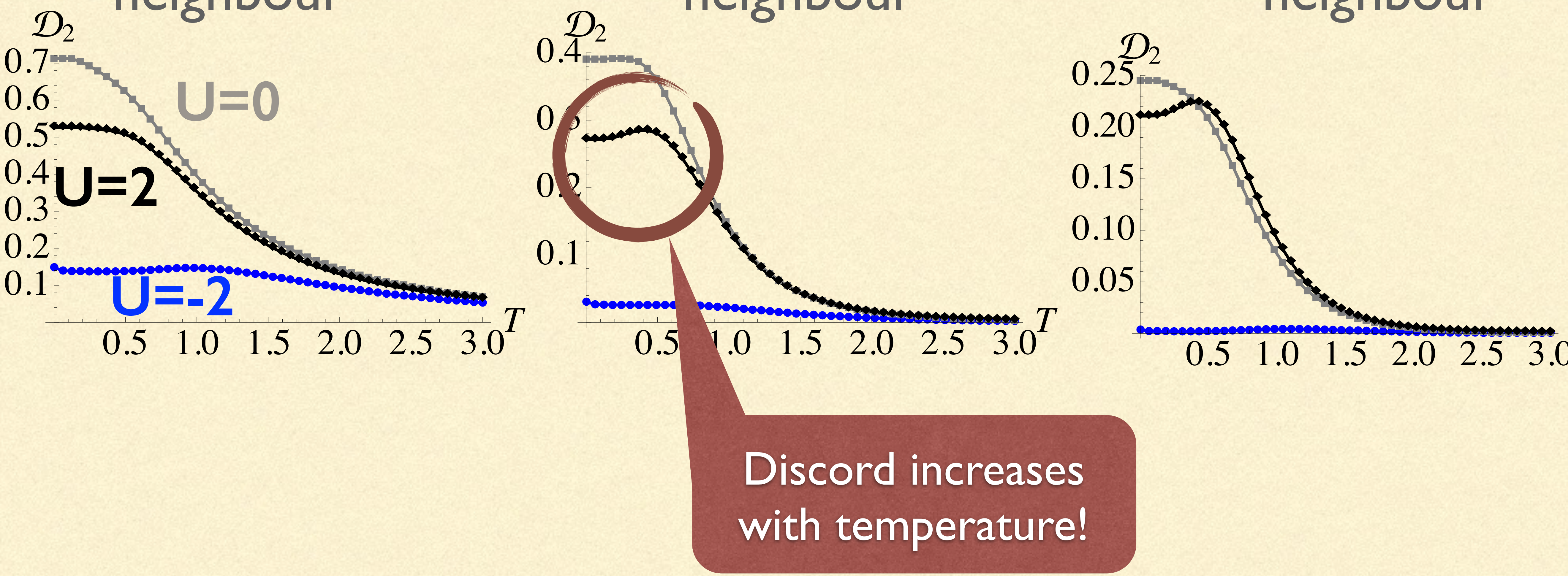


TEMPERATURE EFFECTS

nearest neighbour

next-nearest neighbour

next-next-nearest neighbour



L=6

CONCLUSIONS (II)

- Entanglement is not the end of the story!
- **Quantum correlations** could be more useful in the case of thermal states.
- In the many-body case and like entanglement there is no unique definition and calculations are demanding.

M. Power, S. Campbell, M. Moreno-Cardoner, GDC,
PRB 91, 214411 (2015)

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