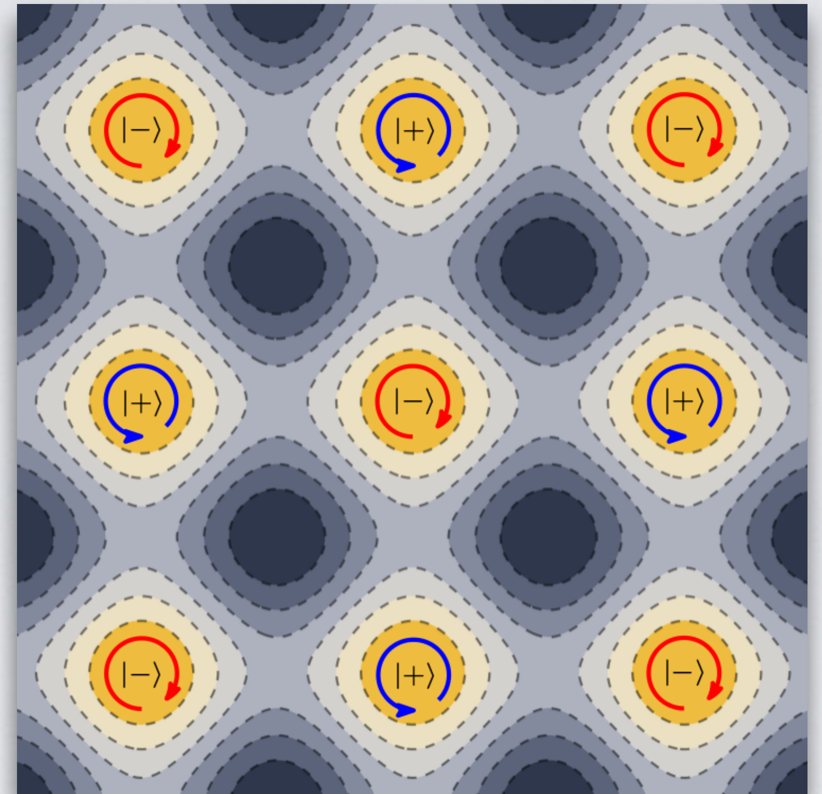


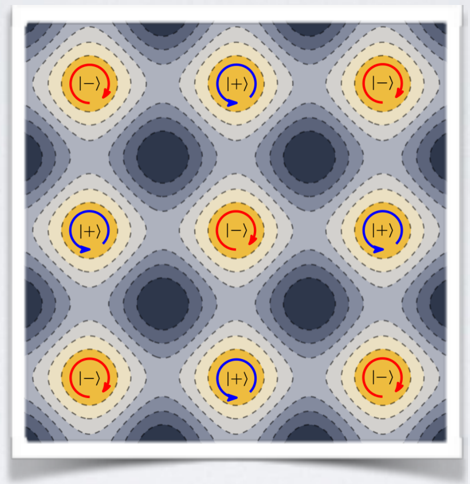

# SHAKEN, NOT STIRRED: CREATING EXOTIC ANGULAR MOMENTUM STATES BY SHAKING AN OPTICAL LATTICE

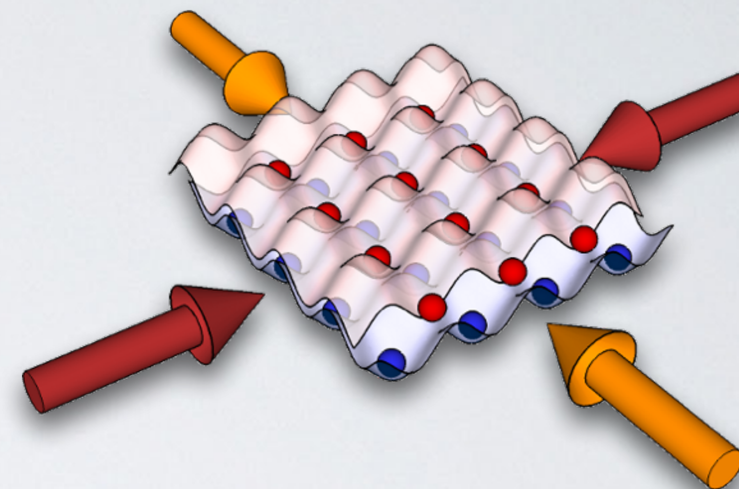
Anthony Kiely
**University College Cork, Ireland**

A Kiely, A Bensey, T Busch and A Ruschhaupt arXiv:1603.05927

# OUTLINE

- The Goal
- The Model
  - Optical lattice potential
  - Four-level approximation
- Shortcuts to Adiabaticity
  - Lewis-Riesenfeld Invariants
- Shaking Schemes
- Numerical Simulations
- Summary





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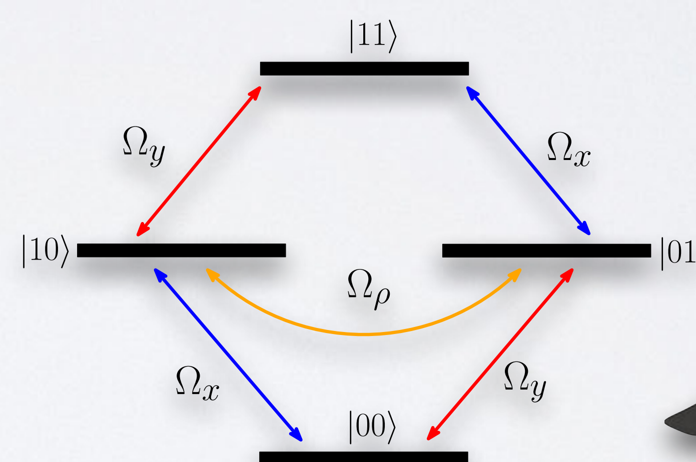
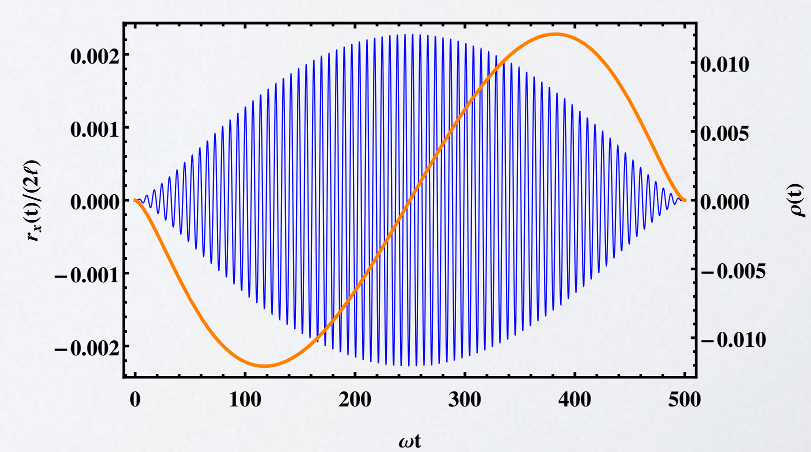
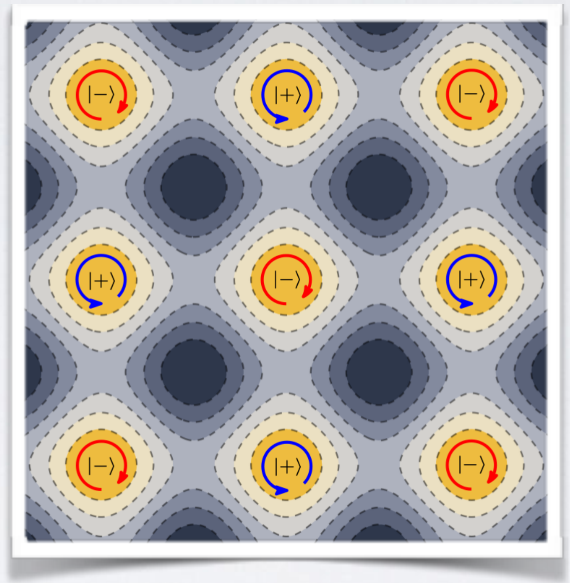
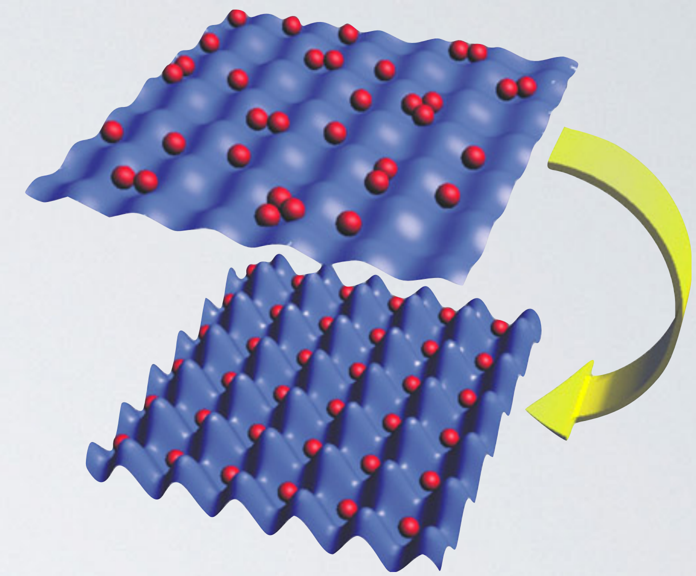


# THE GOAL

- Start with a 2D optical lattice with atoms in the Mott regime
- We want to create higher orbital states in an optical lattice
- Example: anti–ferromagnetic ordered angular momentum state*
- We do this by time dependent control of the position of the trap minima and relative phase between of the lasers



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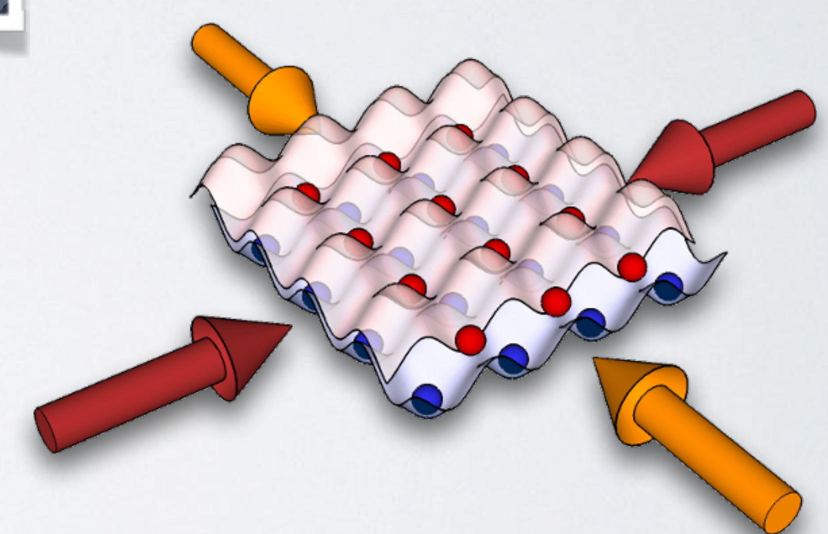

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*arXiv:1508.06285

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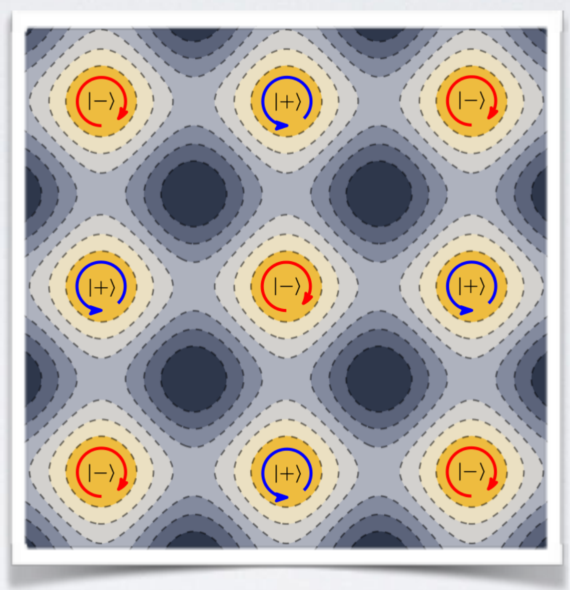

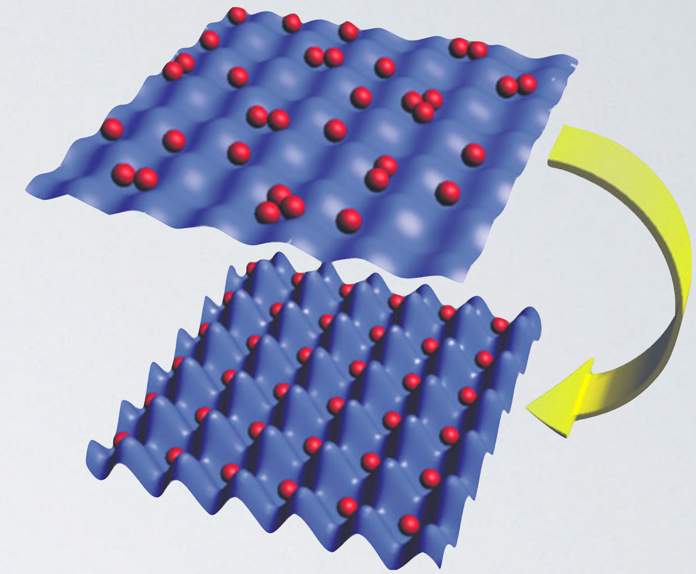

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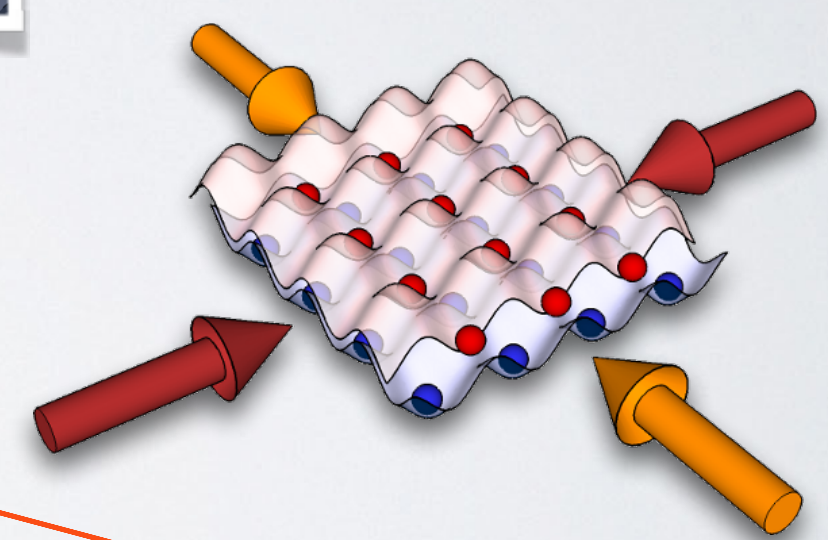

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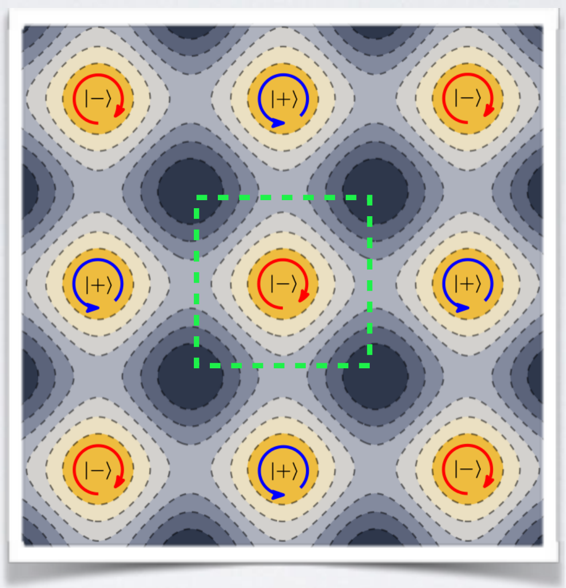

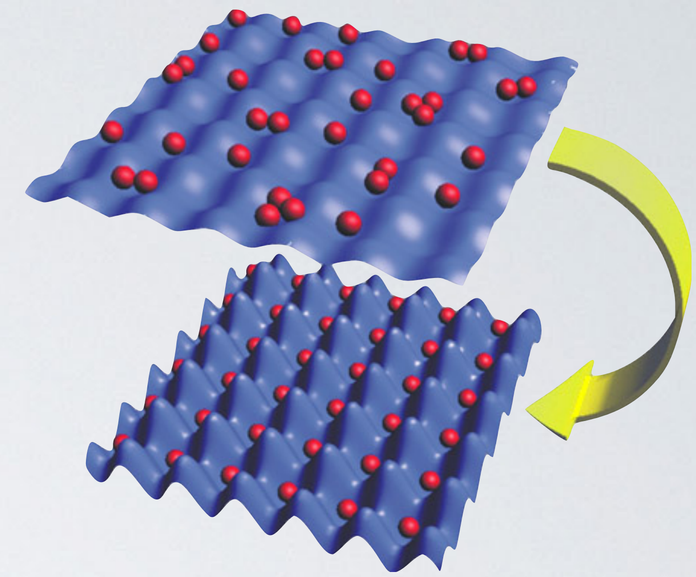

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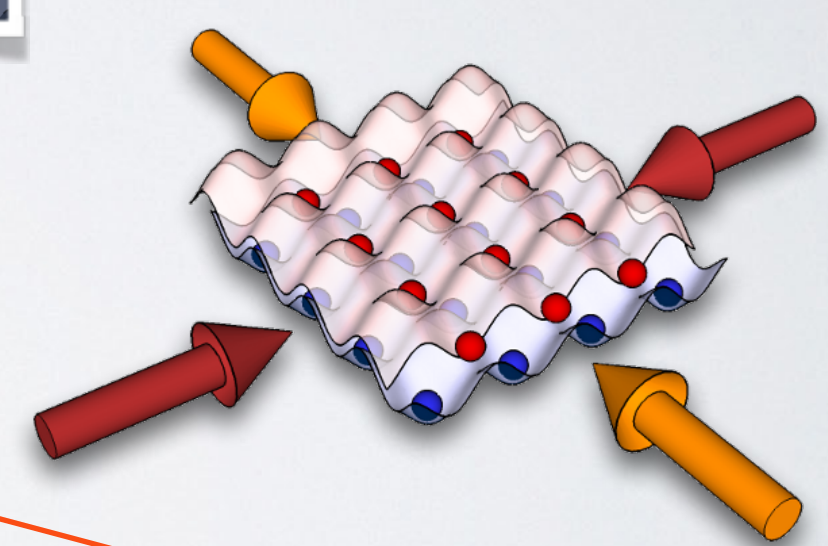

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# THE MODEL

The potential for a single atom

$$V(x,y) = V_0 \sin^2\{k[x - r_x(t)]\} + V_0 \sin^2\{k[y - r_y(t)]\} + 2V_0 \sin[\rho(t)] \sin\{k[x - r_x(t)]\} \sin\{k[y - r_y(t)]\}$$

The shaking of the lattice is of the form:

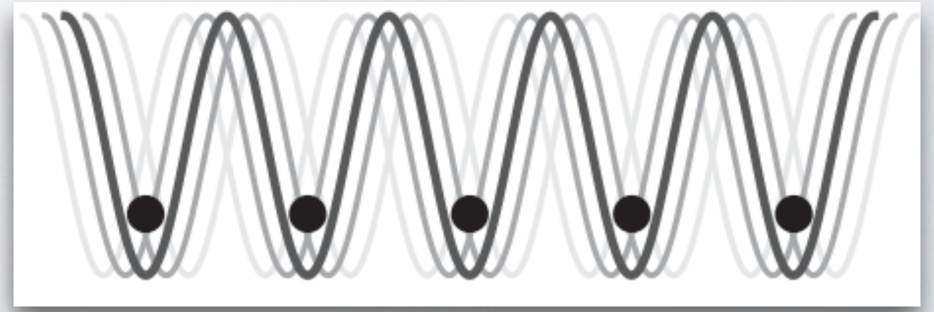
$$r_x(t) = -g_x(t)\cos(\omega_x t)$$
$$r_y(t) = g_y(t)\sin(\omega_y t)$$

The Hamiltonian in the lattice rest frame:

$$H_{\text{lattice}}(t) = H_0 + H_1(t)$$

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V_0 \sin^2(kx) + V_0 \sin^2(ky)$$

$$H_1(t) = m\ddot{r}_x(t)x + m\ddot{r}_y(t)y + 2V_0 \sin[\rho(t)] \sin(kx)\sin(ky).$$



# FOUR-LEVEL MODEL

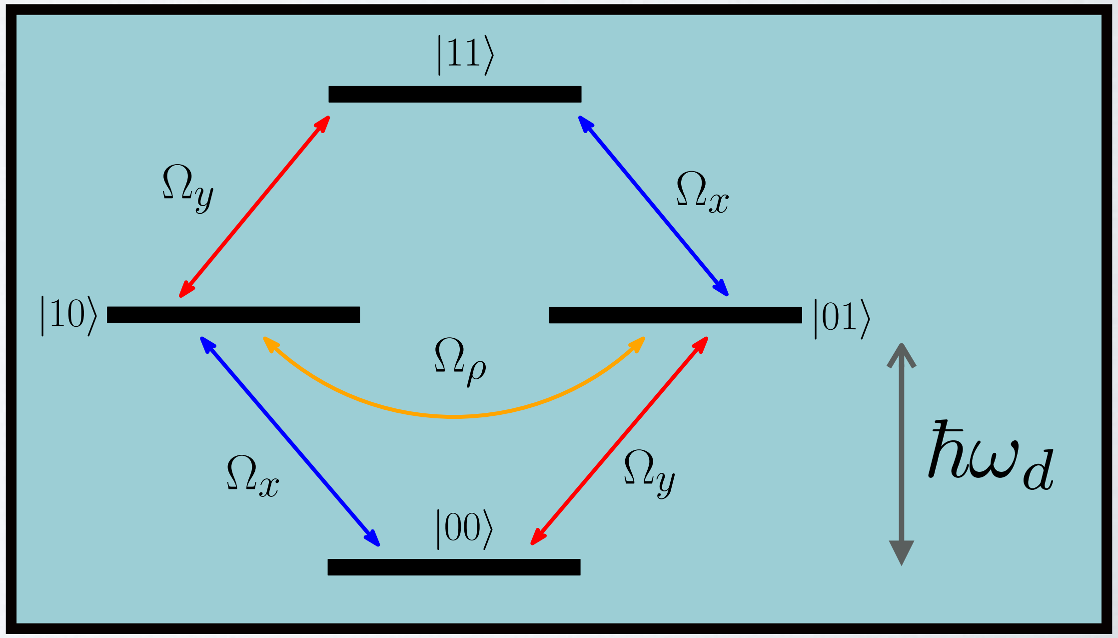
**Need the following assumptions**

- The envelope of the shaking varies slowly
- Shaken on resonance
- Only four relevant basis states $\langle \vec{r} | ij \rangle = \Gamma_i(x)\Gamma_j(y)$
- Neglect fast oscillating terms

$e^{\pm 2i\omega_{x,y}t} \approx 0$ for long times

$$r_x(t) = -g_x(t)\cos(\omega_x t)$$
$$r_y(t) = g_y(t)\sin(\omega_y t)$$

$$\omega_{x,y} = -\omega_d$$



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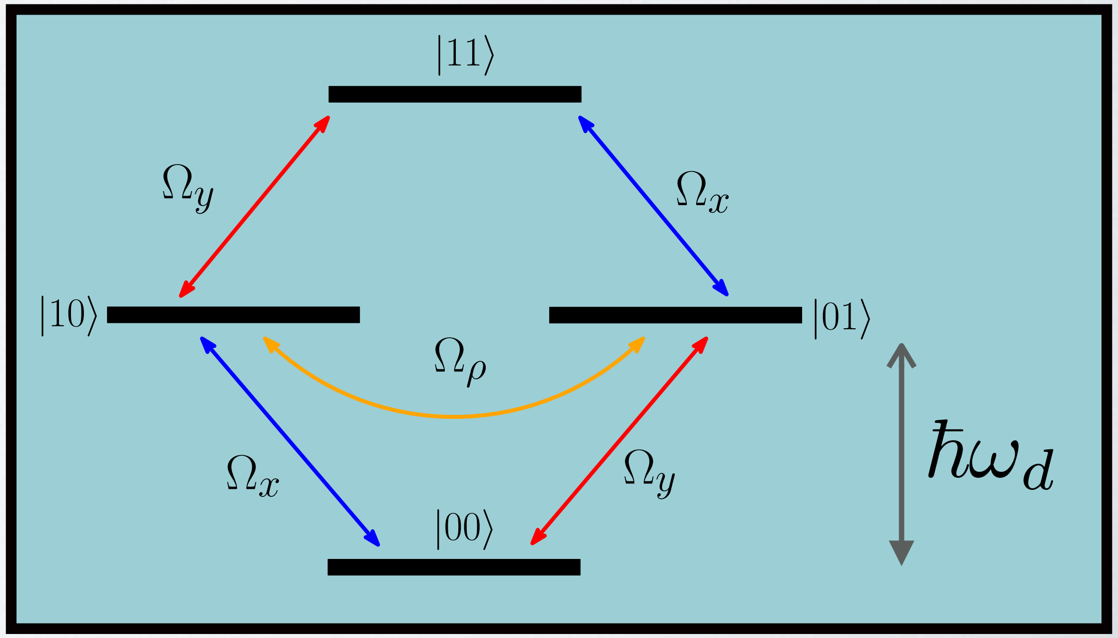
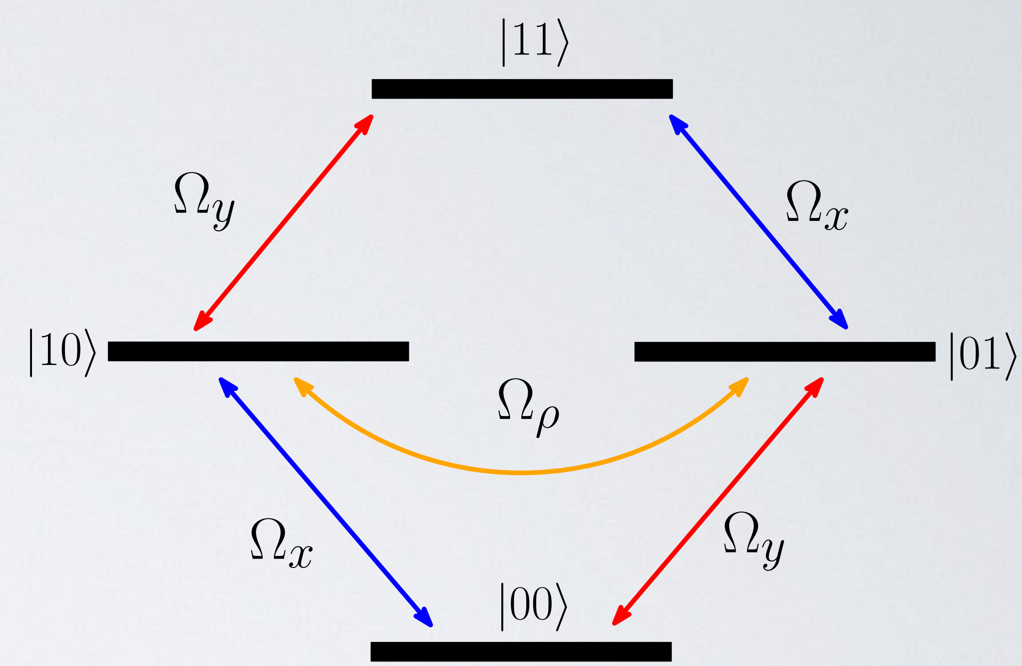


# FOUR-LEVEL MODEL

Final Hamiltonian:

$$H_{4L}(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_x & \Omega_\rho & -i\Omega_y \\ \Omega_x & 0 & -i\Omega_y & 0 \\ \Omega_\rho & i\Omega_y & 0 & \Omega_x \\ i\Omega_y & 0 & \Omega_x & 0 \end{pmatrix}$$




Couplings are directly related to the control parameters

$$\Omega_{x,y}(t) = m\omega_d^2\gamma_1 g_{x,y}(t)/\hbar$$
$$\Omega_\rho(t) = 4V_0 \sin\left[\rho(t)\right]\gamma_2/\hbar$$

$$|10\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \ |00\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \ |01\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \ |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Constants

$$\gamma_1 = \int_{-\ell}^{\ell} \Gamma_0(x)x\Gamma_1(x)dx$$
$$\gamma_2 = \left[\int_{-\ell}^{\ell} \Gamma_0(x)\sin(kx)\Gamma_1(x)dx\right]^2$$

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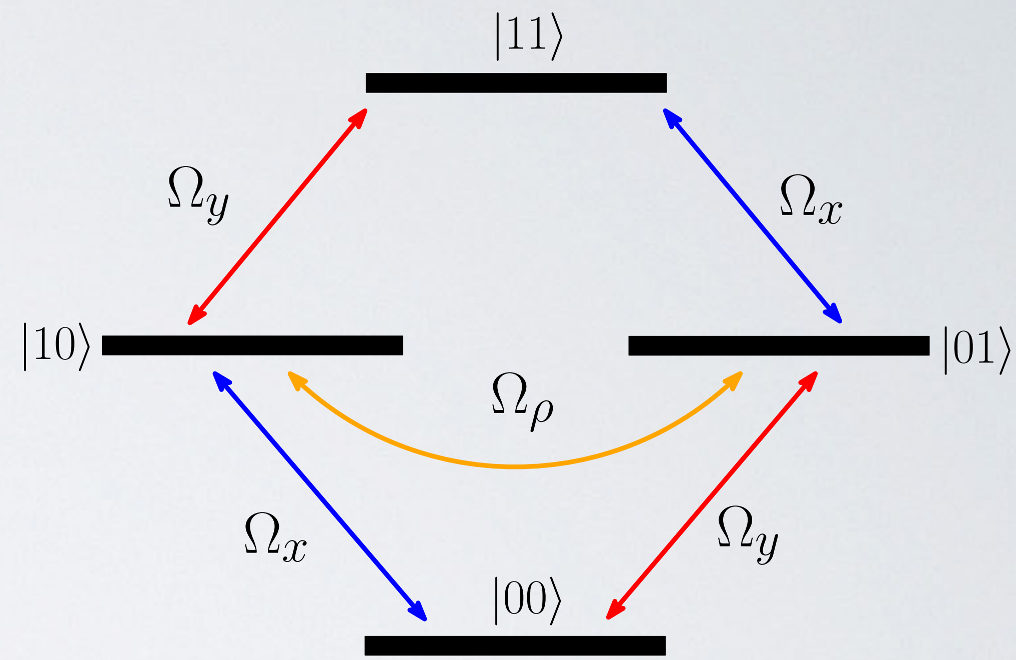


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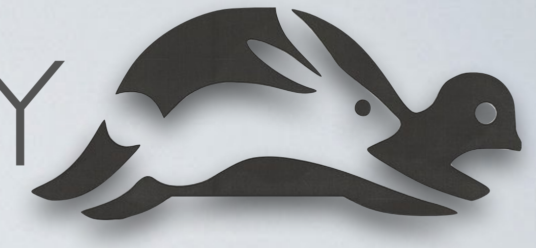
# SHORTCUTS TO ADIABATICITY



- These are processes which achieve the same final state as an adiabatic process in a much shorter time.
- One method uses Hermitian operators called Lewis-Riesenfeld Invariants.
- Its eigenstates are solutions of the Schrödinger equation.
- We follow the instantaneous eigenstate of the invariant (up to a phase). This has no requirement on the total time!
- The invariant and the Hamiltonian must commute at the start and end of the process.

**Adiabatic Theorem**

A system remains in its instantaneous eigenstate if the Hamiltonian changes slowly and there is a gap between the eigenvalues.

$$\frac{\partial I}{\partial t} + \frac{i}{\hbar}[H, I] = 0$$

$$|\psi_n(t)\rangle = e^{i\beta_n(t)} |\phi_n(t)\rangle$$

$$\beta_n(t) = \frac{1}{\hbar}\int_0^t \langle \phi_n(s) |[i\hbar\partial_s - H(s)]| \phi_n(s)\rangle \, ds$$

$$[I(0), H(0)] = [I(T), H(T)] = 0$$

Phys. Rev. Lett. 104 063002 (2010),
Adv. At. Mol. Opt. Phys 62, 117 (2013)

# LEWIS-RIESENFELD INVARIANTS

Case for $\Omega_y = 0$

**General form**

$$H\left(t\right) = \frac{\hbar}{2}\Omega_x\left(t\right)G_1 + \frac{\hbar}{2}\Omega_\rho\left(t\right)G_2$$

$$I\left(t\right) = \sum_{i=1}^{4}\alpha_i\left(t\right)G_i$$

**Closed set**

$$G_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, G_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$G_3 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, G_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

**Solve for couplings**

$$\Omega_x\left(t\right) = -\frac{\dot{\alpha}_2(t)}{\alpha_3(t)}$$

$$\Omega_\rho\left(t\right) = \frac{2\dot{\alpha}_1(t)}{\alpha_3(t)}$$

$$\frac{\partial I}{\partial t} + \frac{i}{\hbar}\left[H, I\right] = 0$$

**Solution and boundary conditions**

$$|\psi\left(t\right)\rangle = \frac{1}{\sqrt{2}}\left[-|\phi_1\left(t\right)\rangle\, e^{i\beta_1(t)} + |\phi_4\left(t\right)\rangle\, e^{i\beta_4(t)}\right]$$

$$|\psi(0)\rangle = |00\rangle$$

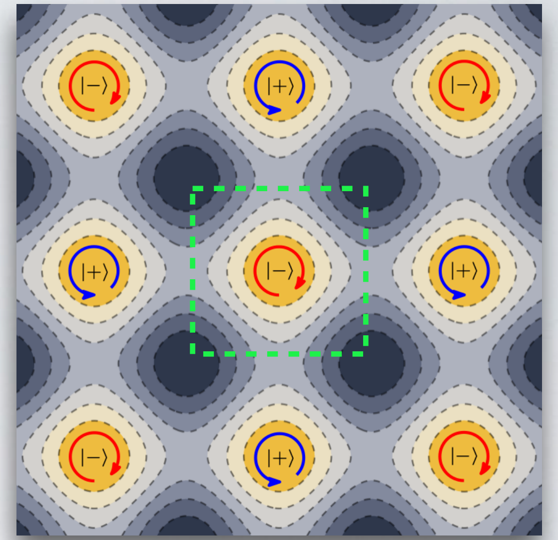
$$|\psi(T)\rangle = |-\rangle$$

$$[I(0), H(0)] = [I(T), H(T)] = 0$$

# STATE TRANSFER

The desired state transfer is:

$$|00\rangle \rightarrow |\pm\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle \pm i|01\rangle\right)$$



$\Omega_\rho$ alternates sign at each site

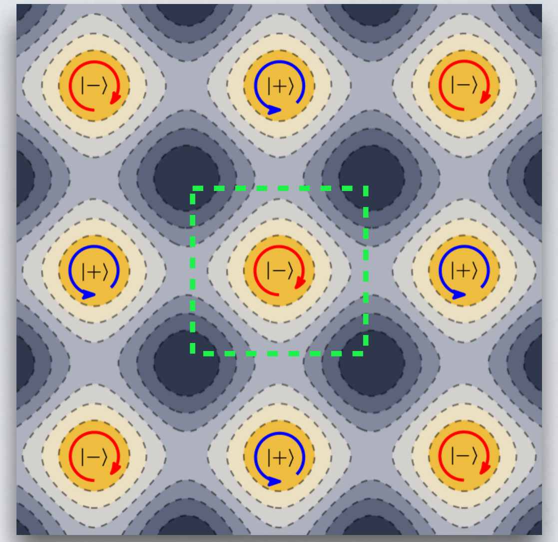
$$H_1(t) = m\ddot{r}_x(t)x + m\ddot{r}_y(t)y + 2V_0 \sin\left[\rho(t)\right]\sin(kx)\sin(ky).$$

Change of basis

$$|01\rangle \rightarrow -|01\rangle$$
$$|11\rangle \rightarrow -|11\rangle$$

Alternates $|+\rangle$ or $|-\rangle$ at over the lattice sites

# STATE TRANSFER

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$$|11\rangle \to -|11\rangle$$

Alternates $|+\rangle$ or $|-\rangle$ at over the lattice sites

# STATE TRANSFER

**Polynomial scheme (dashed lines)**

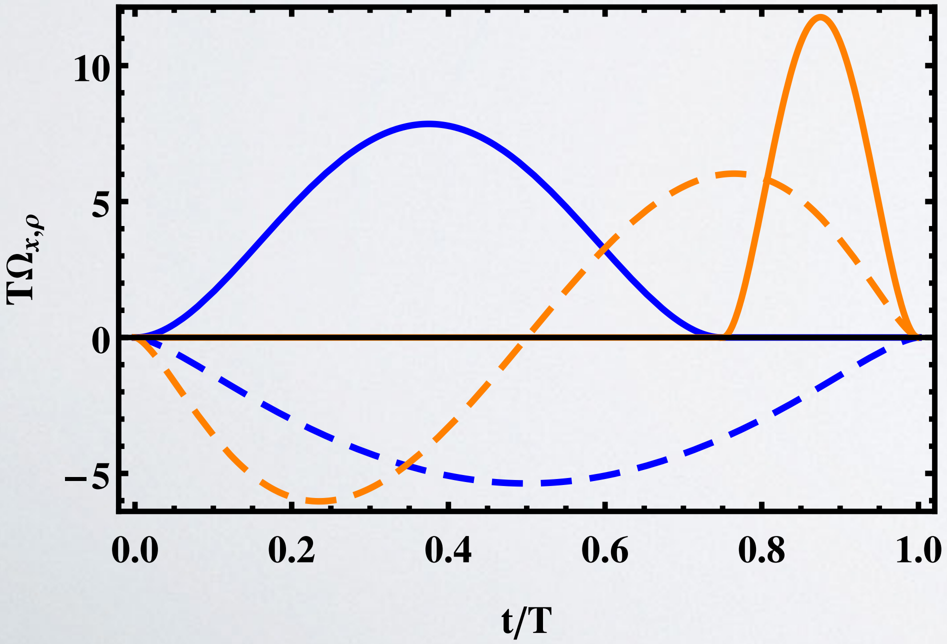
$$\alpha_1(sT) = 1024W(-s^{10} + 5s^9 - 10s^8 + 10s^7 - 5s^6 + s^5),$$
$$\alpha_2(sT) = \frac{1}{2}(C_1 - Q) + 70Qs^9 - 315Qs^8 + 540Qs^7 - 420Qs^6 + 126Qs^5$$

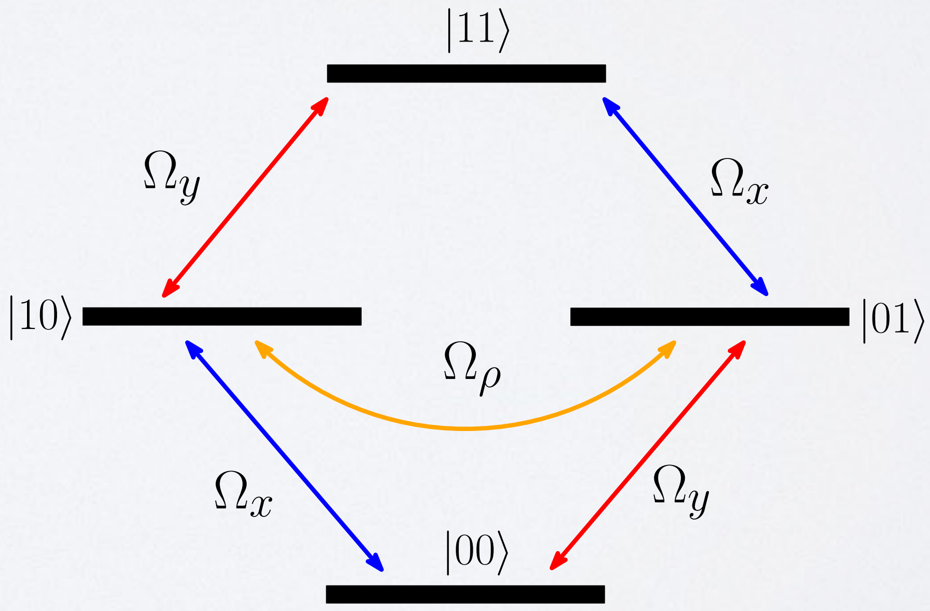
**Piecewise scheme (solid lines)**

$$\alpha_1(t) = \begin{cases} \epsilon & 0 \le t \le t_S \\ \epsilon \cos\left[\frac{1}{2}\int_{t_S}^{t} \Omega_\rho(t')\,dt'\right] & t_S < t \le T \end{cases}$$

$$\alpha_2(t) = \begin{cases} \frac{1}{2}\left\{C_1 - \sqrt{C_1^2 + 8C_2 - 4\epsilon^2}\cos\left[\int_0^t \Omega_x(t')\,dt'\right]\right\} & 0 \le t < t_S \\ \frac{1}{2}\left(C_1 + \sqrt{C_1^2 + 8C_2 - 4\epsilon^2}\right) & t_S \le t \le T \end{cases}$$

Schemes



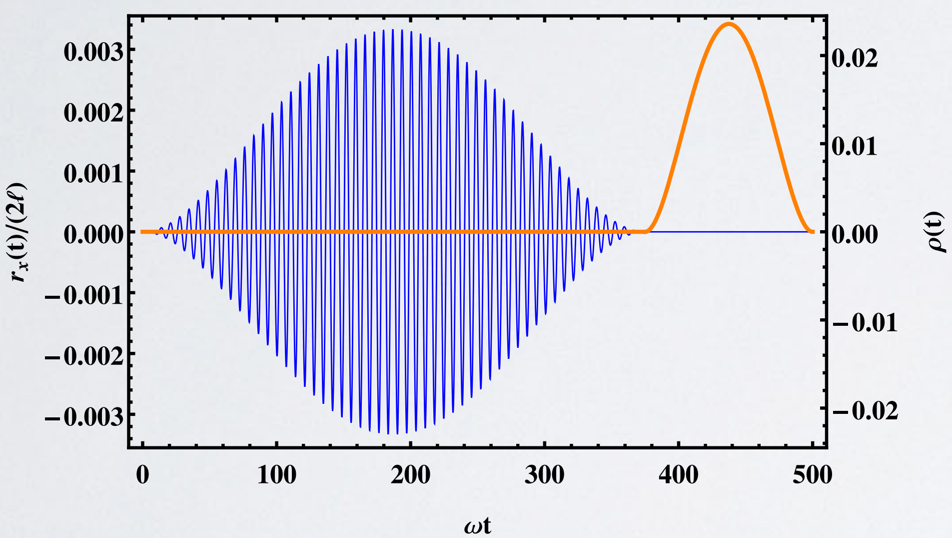



# NUMERICAL SIMULATION

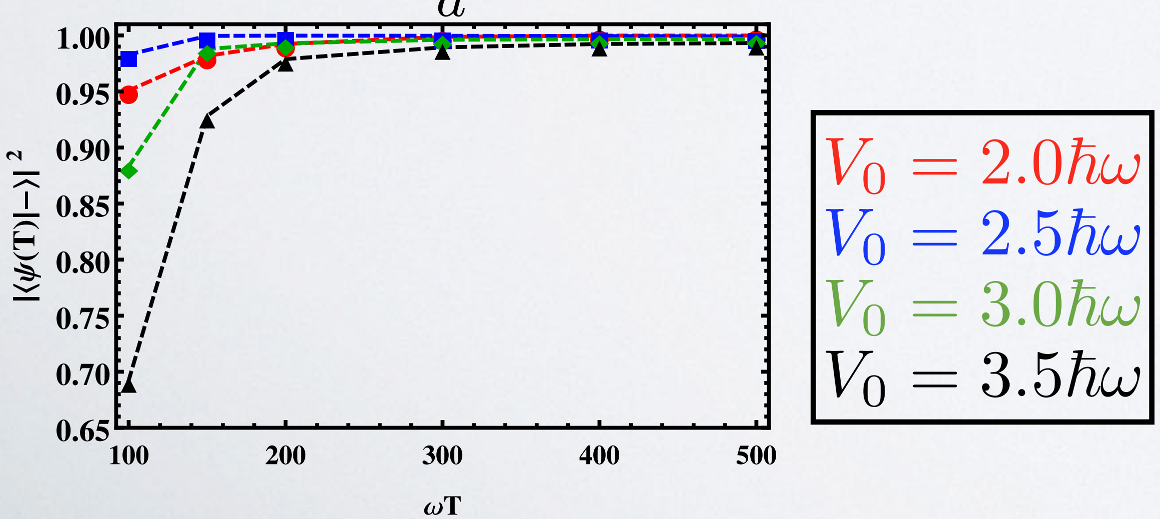
## Piecewise physical implementation

$$r_x(t) = -\frac{\hbar}{m\omega_d^2\gamma_1}\Omega_x(t)\cos(\omega_x t)\,,$$

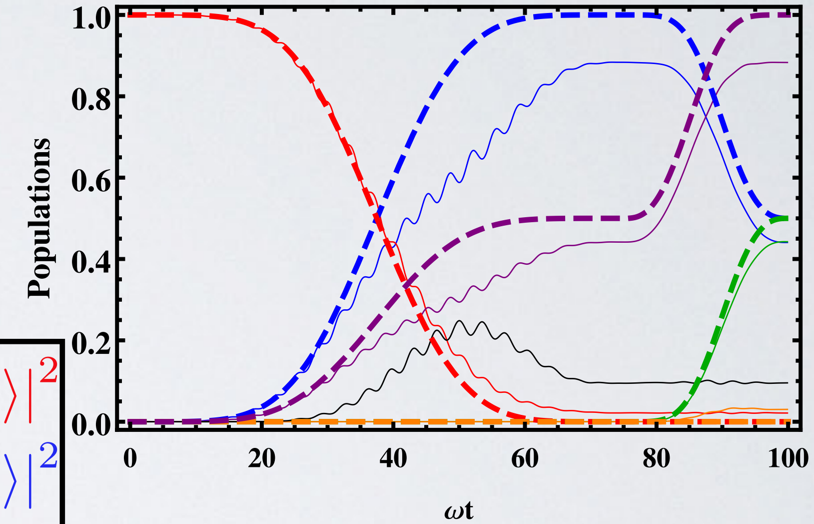
$$\rho(t) = \arcsin\left(\frac{\hbar}{4V_0\gamma_2}\Omega_\rho(t)\right).$$

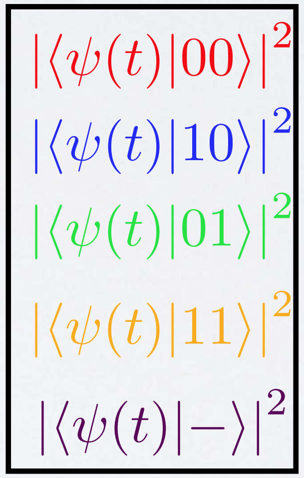



$$T \gg \omega_d^{-1} \approx \omega^{-1}$$

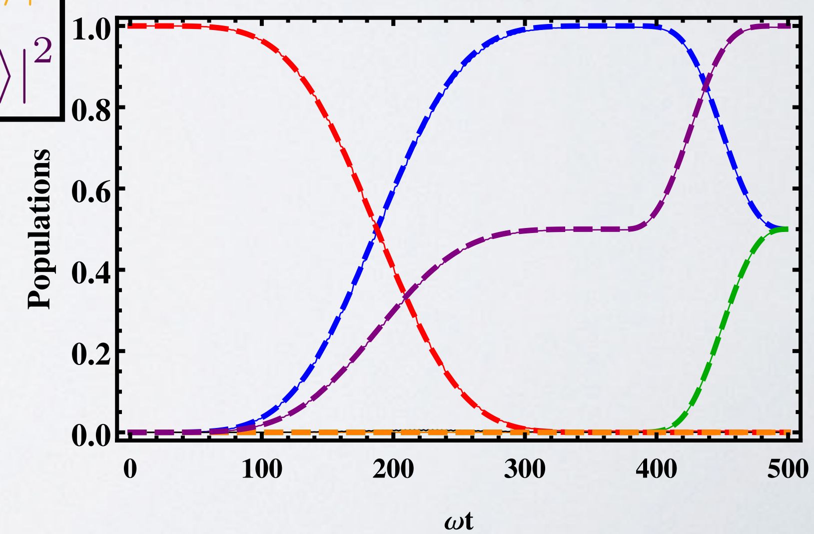



## Four-level model approximates the dynamics well




$$T = 100\omega^{-1}$$




$$T = 500\omega^{-1}$$

# NUMERICAL SIMULATION

## Polynomial physical implementation

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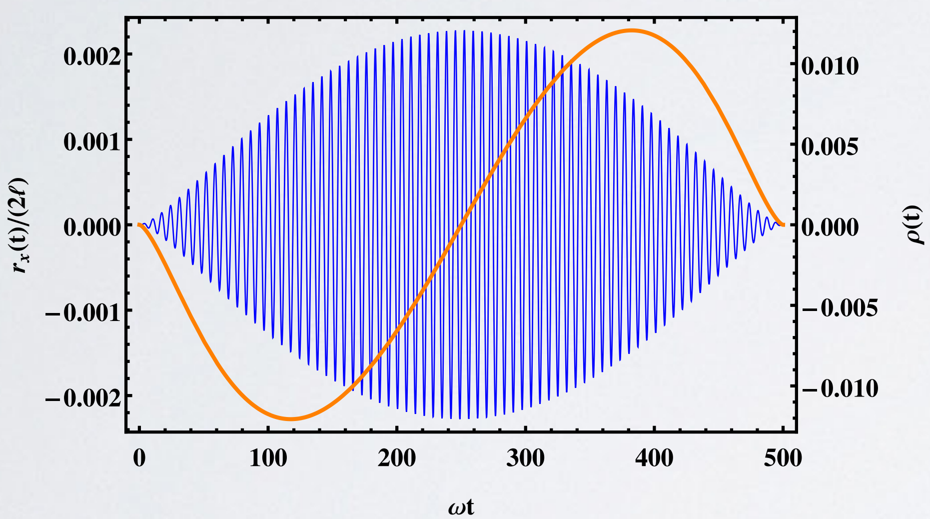


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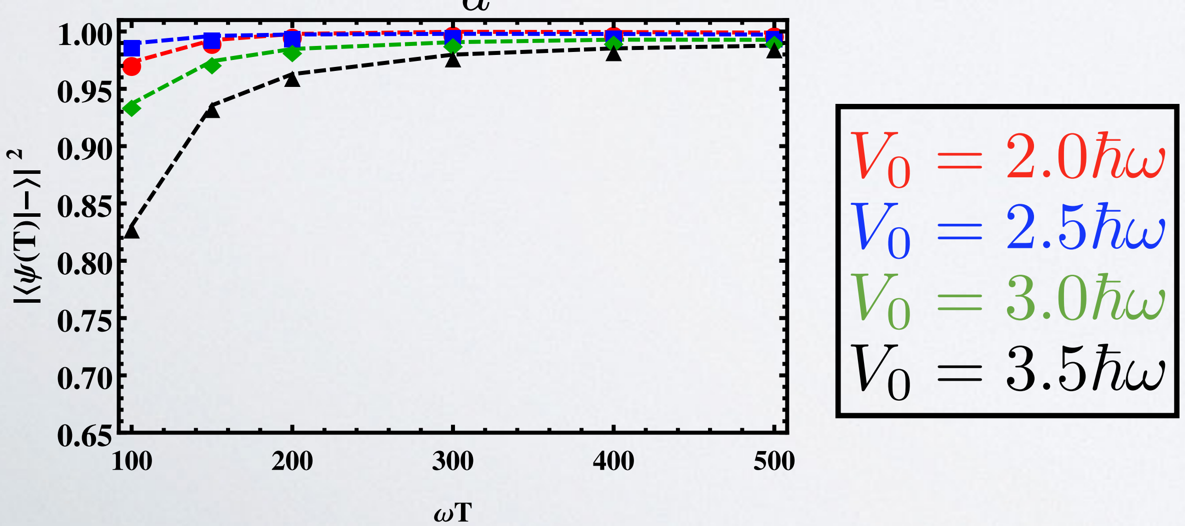


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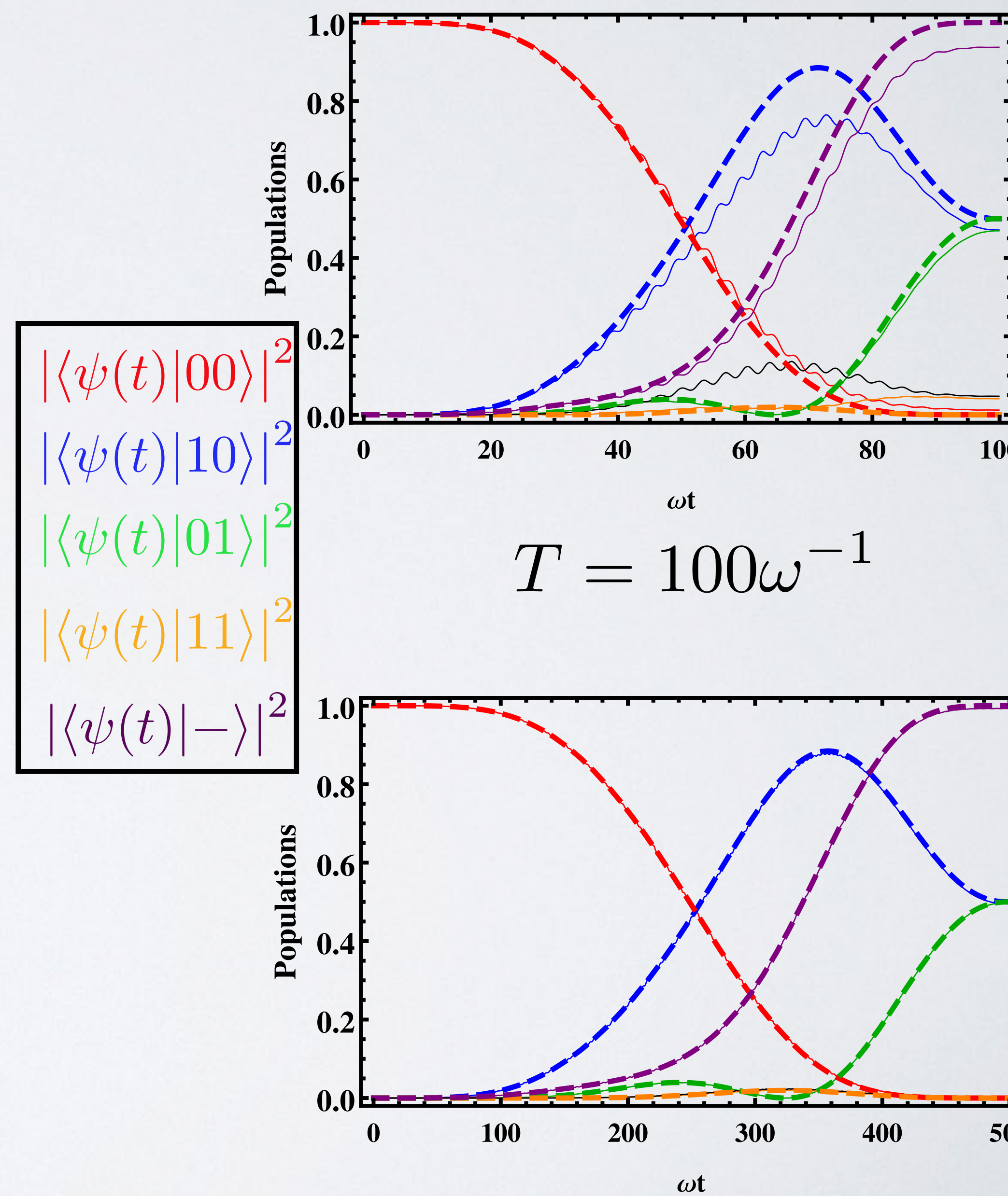



$T = 100\omega^{-1}$

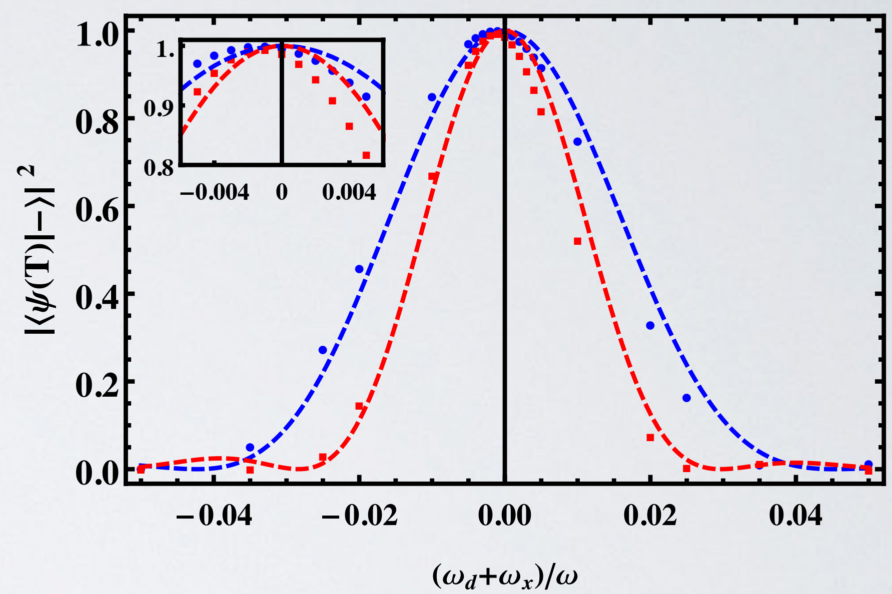
$T = 500\omega^{-1}$

# RESONANCE CONDITION?

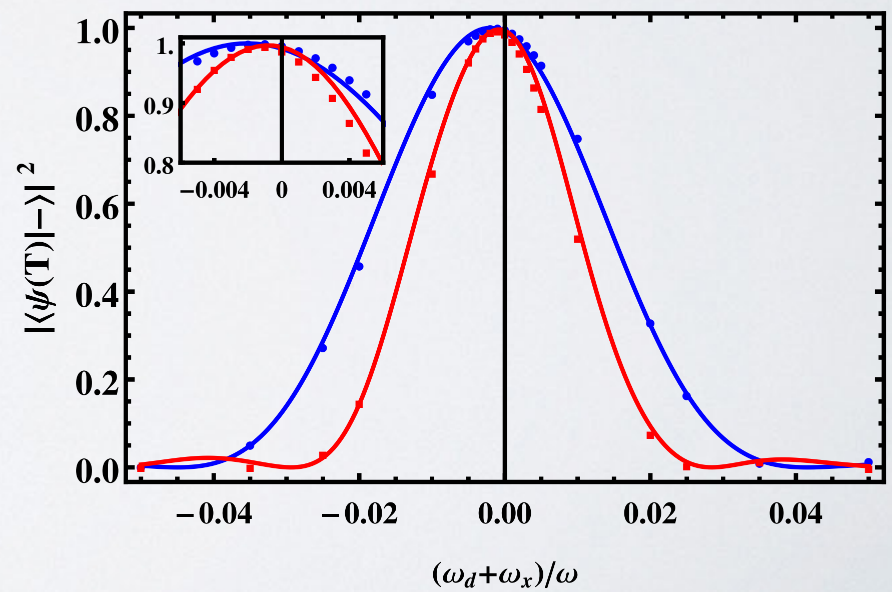
- Highest fidelity if one is slightly off resonance!
- Due to coupling to higher levels
- Shift inhibits population loss in $|10\rangle \leftrightarrow |20\rangle$ transition
- Six level model provides a good fit to the full dynamics (points)

$$H_{6L} = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_x\theta_x^- & \Omega_\rho & 0 & \delta_1 & 0 \\ \Omega_x\theta_x^+ & -2\left(\omega_d+\omega_x\right) & 0 & \Omega_\rho e^{i(\omega_x-\omega_d)t} & 0 & 0 \\ \Omega_\rho & 0 & 0 & \Omega_x\theta_x^+ & 0 & 0 \\ 0 & \Omega_\rho e^{-i(\omega_x-\omega_d)t} & \Omega_x\theta_x^- & 0 & \delta_2 & \delta_2 \\ \delta_1^* & 0 & 0 & \delta_2^* & 0 & 0 \\ 0 & 0 & 0 & \delta_2^* & 0 & 0 \end{pmatrix}$$

Four-level model

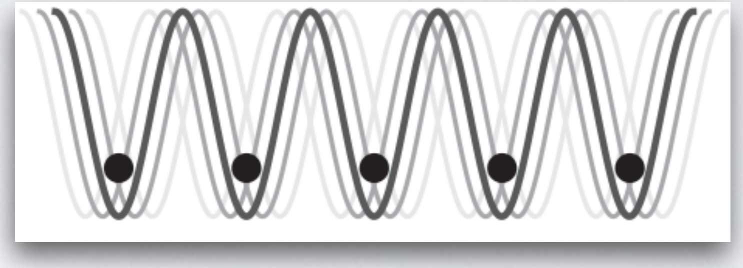



Six-level Model



# SUMMARY

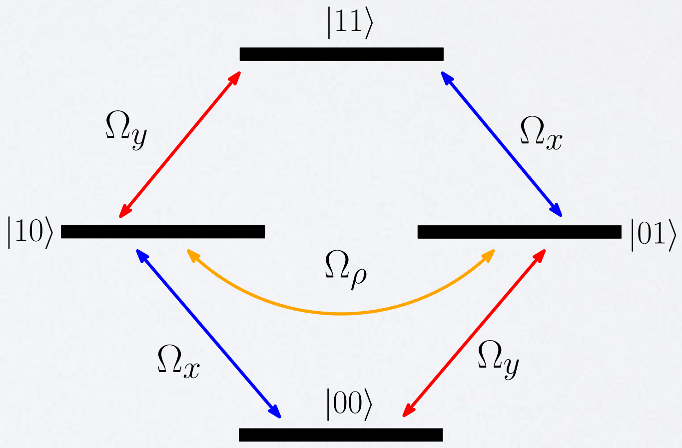
- Using 'shaking' and the interference term provides a lot of control.



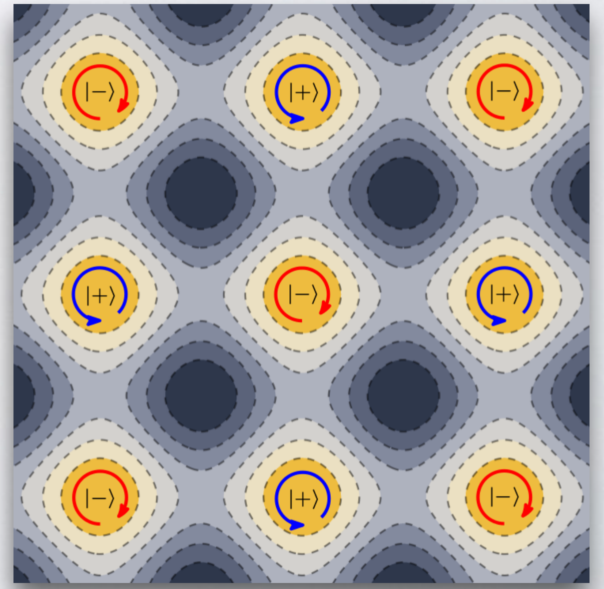
- The four-level model is a good approximation

$$\frac{\partial I}{\partial t} + \frac{i}{\hbar}[H, I] = 0$$

- The existence of Lewis-Riesenfeld Invariants allows for optimisation against noise and errors




- Higher orbital states in optical lattices can be created in a fast way



# QUANTUM OPTICS UCC

# QUESTIONS?

**Come see my poster!**