Quantum Computation
with mechanical cluster states

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QTeQ QUANTUM TECHNOLOGY at QUEEN'S

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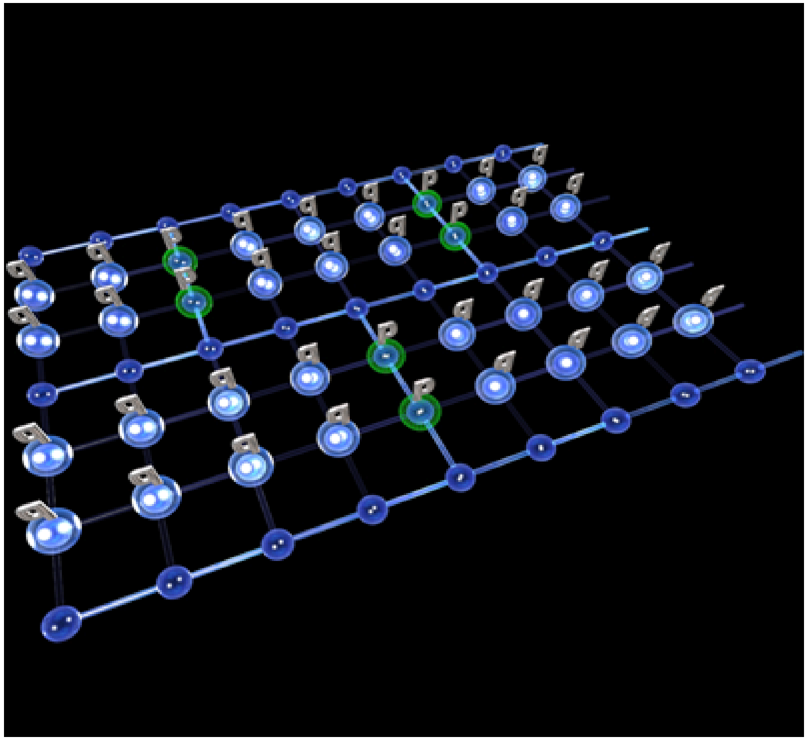
Distinguishable bosons
[Continuous Variables (CVs), qumodes]

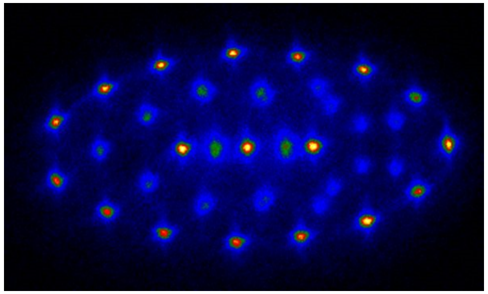
What can we do with many qumodes?

Quantum computation over CVs

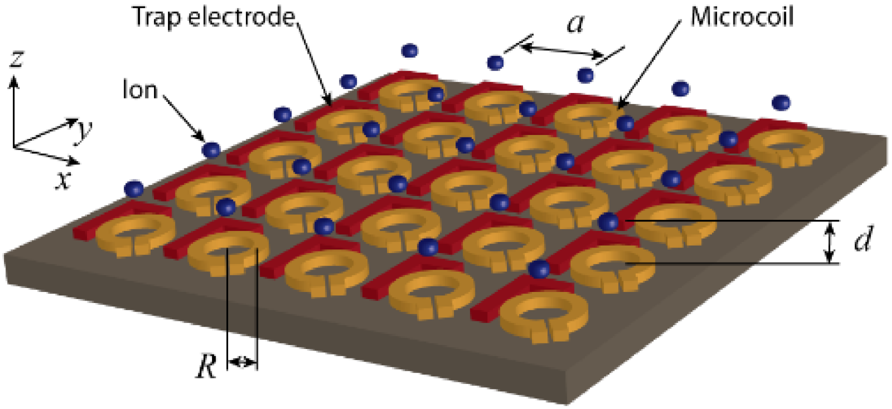


Gu et al., PRA (2009)

Quantum simulators over CVs

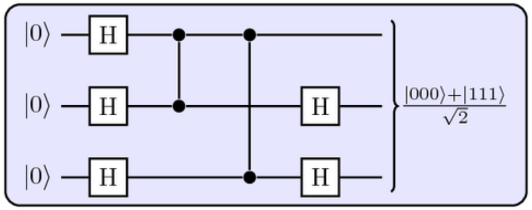


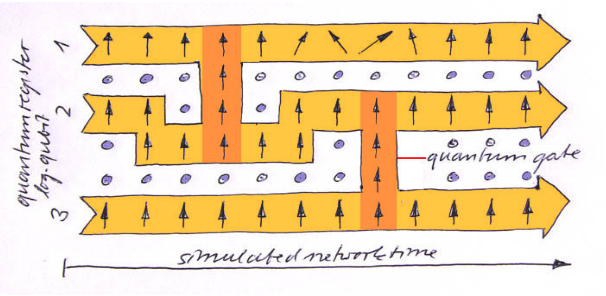
Freidenauer at al, Nat. Phys (2008)



Chiaverini et al., PRA (2008)

Models of computation





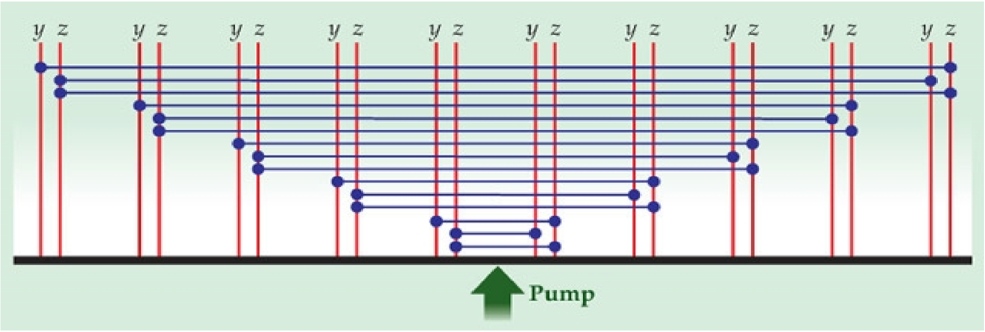
	Circuit-Based Quantum Computation	**Measurement-Based Quantum Computation (MBQC) (cluster states)**
Continuous Variables	Lloyd & Braunstein PRL (1999)	Menicucci et al. PRL (2006)
Fault tolerant (with finite energy)	Gottesman, Kitaev, Preskill PRA (2001) Lund, Ralph, Haselgrove, PRL (2008)	Menicucci PRL (2014)

Cluster states with traveling light modes: recent experimental progresses

60-mode graph states

Frequency encoding

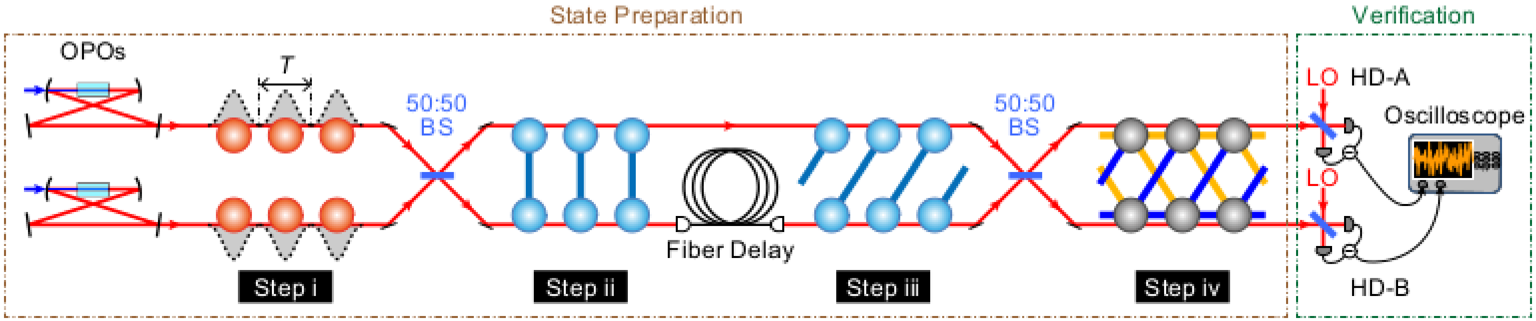
Single crystal & freq comb
[Chen et al., PRL (2014)]



10,000-mode graph states

Temporal encoding

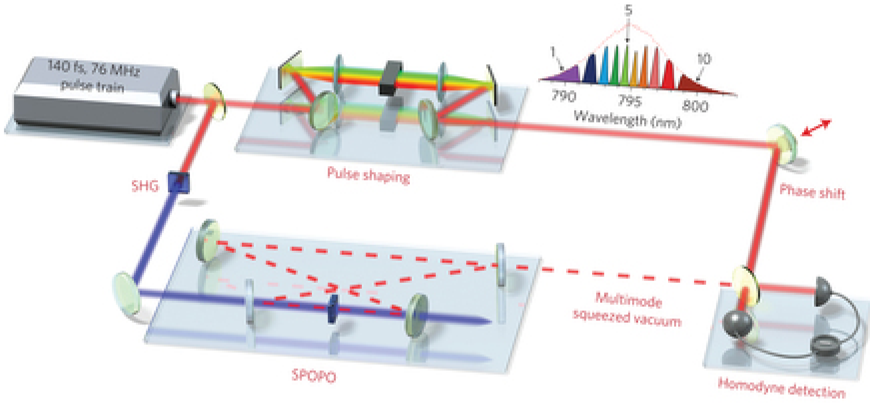
Pulsed squeezed states
[Yokoyama et al., Nature Photonics (2013)]



500+ entangled partitions

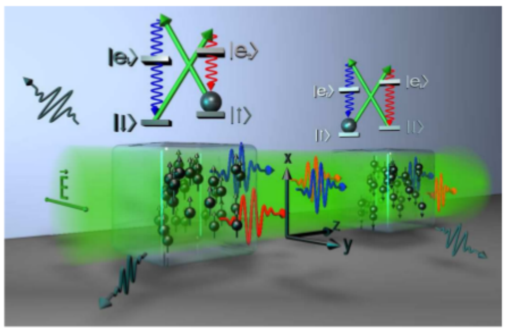
Frequency encoding

Single crystal & freq comb
[Roslund et al., Nature Photonics (2014)]

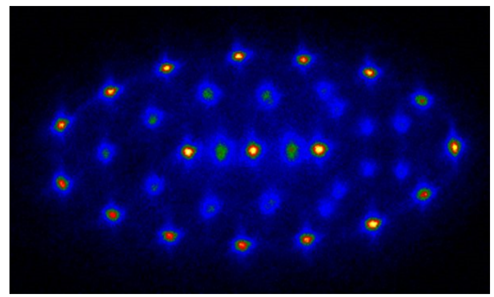


Also interesting alternative platforms: confined/massive continuous variables

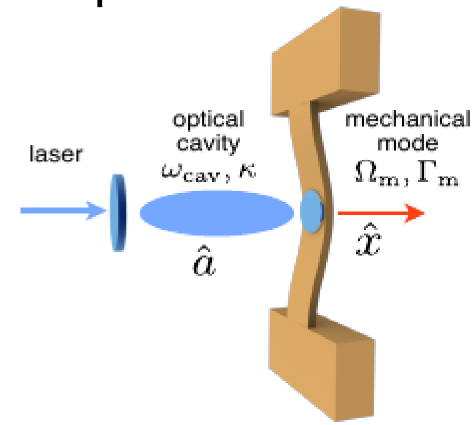
Atomic ensembles



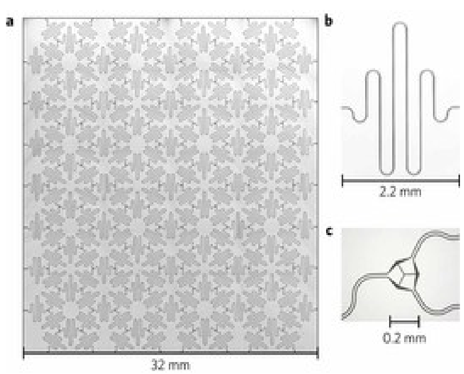
Trapped Ions



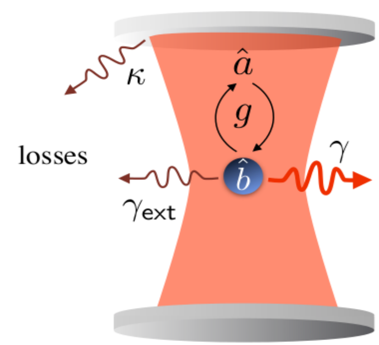
Optomechanics



Circuit-QED



Cavity-QED



Why interesting?

Confined systems can be scaled/integrated more easily

Outline

- Measurement-based quantum computation with CVs

- Generation of universal resources for CV quantum computation:
  - Adiabatic generation of cluster states
  - Optomechanical cluster-state generation via reservoir engineering

- Quantum tomography for confined CVs
  - A single qubit to read them all
  - A single qumode to read them all

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Continuous Variables (distinguishable bosons)

Position and momentum operators

$$\mathsf{q_j} = \frac{1}{\sqrt{2}}(\mathsf{b_j} + \mathsf{b_j^\dagger}) \qquad \mathsf{p_j} = \frac{1}{\mathsf{i}\sqrt{2}}(\mathsf{b_j} - \mathsf{b_j^\dagger}) \qquad [\mathsf{q_j}, \mathsf{p_k}] = \mathsf{i}\delta_{\mathsf{j,k}}$$

Computational basis

$$|0\rangle, |1\rangle \ \mapsto \ |\mathsf{v}\rangle_\mathsf{q} \qquad (\mathsf{q}|\mathsf{v}\rangle_\mathsf{q} = \mathsf{v}|\mathsf{v}\rangle_\mathsf{q}\ , \quad \mathsf{v} \in \mathbb{R})$$

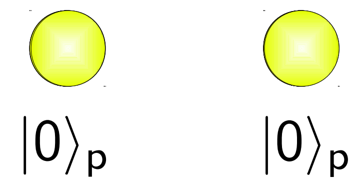
Entangling gate

$$\mathsf{CZ_{jk}} \equiv \exp[\mathsf{i}\mathsf{q_j} \otimes \mathsf{q_k}]$$

Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation

- Prepare each node in zero-momentum eigenstate



[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

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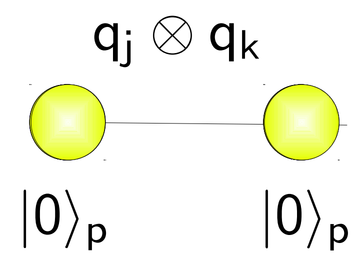

- Prepare each node in zero-momentum eigenstate

- Entangle connected nodes with

$$\mathsf{CZ}_{jk} \equiv \exp[i q_j \otimes q_k]$$

[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

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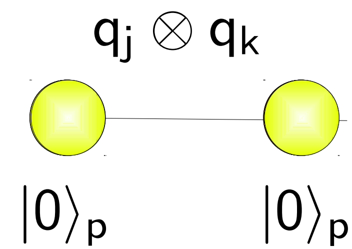

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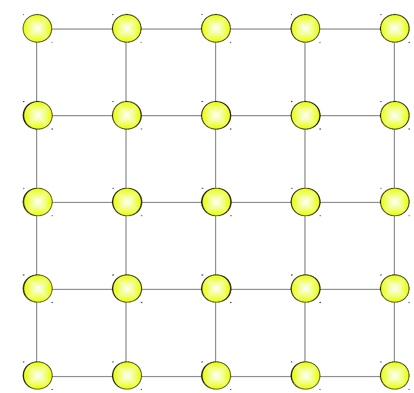
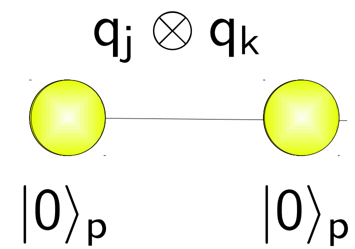


CV cluster state

[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation



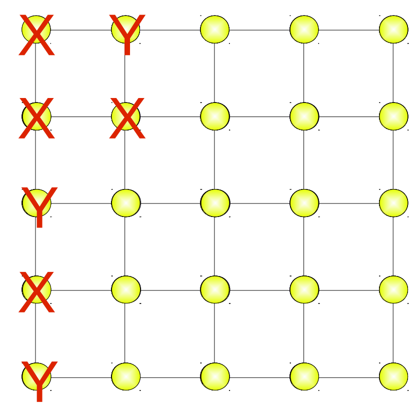
- Prepare each node in zero-momentum eigenstate

- Entangle connected nodes with

$$CZ_{jk} \equiv \exp[i q_j \otimes q_k]$$

- Measure each node locally

- Arbitrary (non-Gaussian) measurements plus feed forward in a lattice guarantee universality



CV cluster state

[Zhang and Braunstein, PRA (2006); Menicucci et al., PRL (2006)]

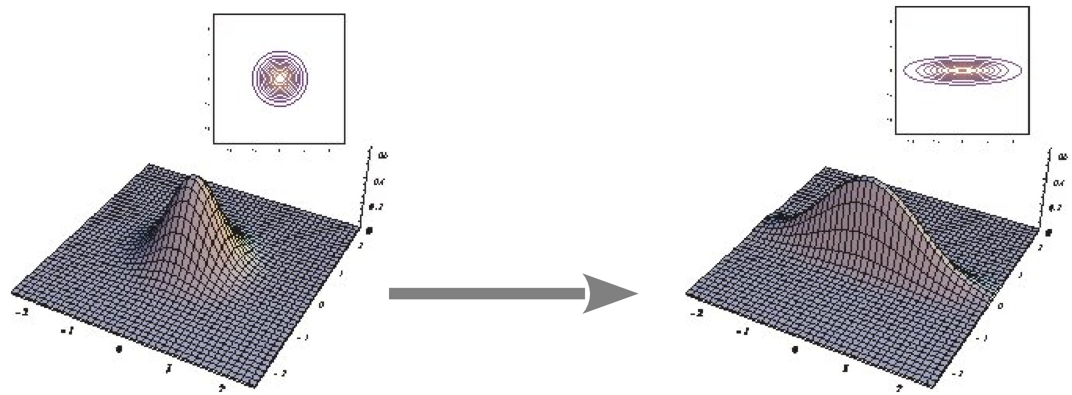
Continuous Variables (with finite energy)

Squeezing operator

$$\mathsf{T}(\mathsf{r}) \equiv \exp[\mathsf{i}\ln(\mathsf{r})(\mathsf{qp}+\mathsf{pq})/2]$$

$$\mathsf{T}^\dagger(\mathsf{r})\mathsf{q}\mathsf{T}(\mathsf{r}) = \mathsf{rq}$$

$$\mathsf{T}^\dagger(\mathsf{r})\mathsf{p}\mathsf{T}(\mathsf{r}) = \mathsf{p}/\mathsf{r}$$



Position and momentum basis are infinitely squeezed:

$$\lim_{\mathsf{r}\to\infty} \mathsf{T}(\mathsf{r})|0\rangle \equiv |0\rangle_\mathsf{p}$$

The physically relevant states are finitely squeezed ones

Fault tolerance is guaranteed for large enough squeezing

Gaussian states

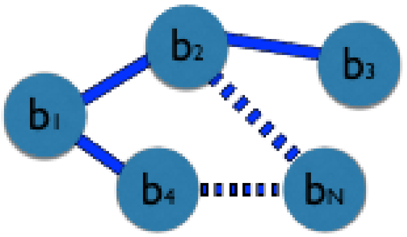
Restricting to quadratic operations (CZ)
and finite energy (squeezed states)

	Full quantum mechanics	Gaussian world
States	Density operator ρ	First and second moments $R = (q_1, \ldots, q_N, p_1, \ldots, p_N)$ $\langle R \rangle$ $[V]_{kl} = \langle R_k R_l + R_l R_k \rangle / 2$
Closed Dynamics	Unitaries W $\rho' = W \rho W^\dagger$	Symplectic S $V' = S V S^T$

Finite energy CV graph states are Gaussian

Consider the union

$$G_{(\mathcal{V},\mathcal{E})} \equiv \{\mathcal{V},\mathcal{E}\}$$



of vertices $\mathcal{V}$ and edges $\mathcal{E}$ with associated

adjacency matrix A: $A_{j,k} \begin{cases} 1 & \forall j,k \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}$

Associated **ideal** graph state (infinite energy):

$$|G\rangle = CZ|\mathbf{0}\rangle_p \qquad |\mathbf{0}\rangle_p = \bigotimes_{j\in\mathcal{V}} |0\rangle_{p_j} \qquad CZ = \prod_{\{j,k\}\in\mathcal{E}} CZ_{jk}$$

Associated **finite-energy** graph state:

$$|G_r\rangle \equiv CZ\, T(r)|\mathbf{0}\rangle \qquad V = \frac{1}{2} S^T \begin{pmatrix} \frac{1-r}{1+r} I_N & 0_N \\ 0_N & \frac{1+r}{1-r} I_N \end{pmatrix} S$$

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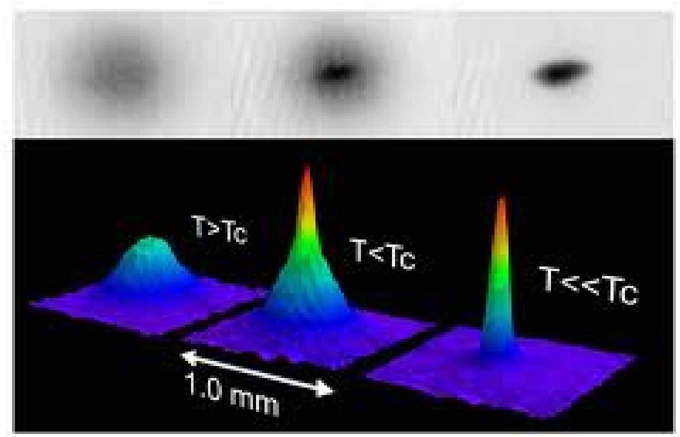
For confined CVs it would be convenient to have an alternative way to generate large graph states:

a Hamiltonian system whose ground state is the desired graph state

Ex: generation by cooling of a Bose-Einstein condensate by cooling to the ground state.



Desiderata

- Two-body interactions (easier to find in “natural” systems)
- Local interactions (experimental compactness)
- Gapped Hamiltonian (adiabatic cooling)
- Frustration Free (the ground state minimize each local term; robustness against local perturbation)

Desiderata

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- Frustration Free (the ground state minimize each local term; robustness against local perturbation)

Discrete variables (qubits):

No-go result
“There is no two-body frustration-free Hamiltonian with genuinely entangled non-degenerate ground state”

[Nielsen, quant-ph/0504097;
Bartlett & Rudolph, PRA ('06);
Van den Nest et al., PRA ('08);
X. Chen et al. PRL ('09);
J. Cai et al. PRA ('10);
J. Chen et al., PRA ('11)]

A CV Hamiltonian with all the desiderata

$$H_G(r) \equiv \sum_{k\in\mathcal{V}} \frac{\omega_k}{2}(q_k^2/r^4 + N_k^2)$$

The ground state is the CV graph state (with squeezing r)

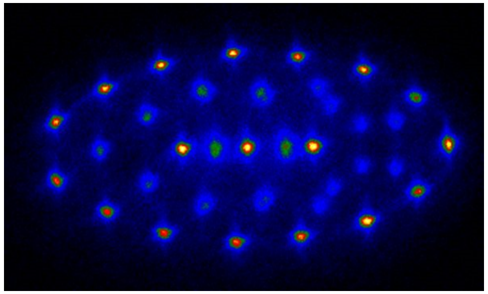
$$N_k \equiv p_k - \sum_{j\in\mathcal{N}_k} q_j \qquad (\mathcal{N}_k \text{ are the neighbour nodes of } k)$$

- Two-body interactions (quadratic, the graph state is Gaussian)
- Local interactions (nearest- and next-to-nearest-neighbours)
- Frustration Free (local terms commute)
- Gapped Hamiltonian

Note: mixed momentum/position interaction

[Aolita, Roncaglia, AF, Acin, PRL '11]

Possible experimental platforms



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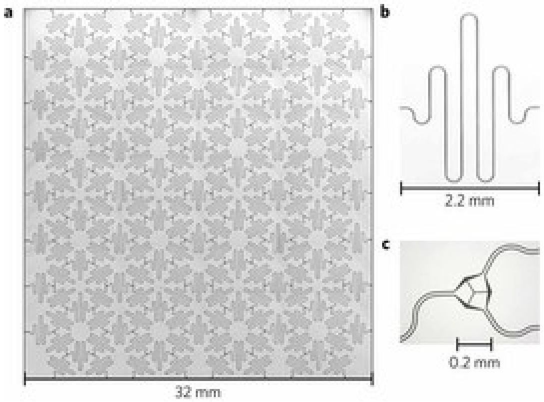

Circuit-QED

Natural interactions

$$q_j \otimes q_k \qquad b_j^\dagger b_k + b_j b_k^\dagger$$

The challenge

How to implement also

$$q_j \otimes p_k \qquad p_j \otimes p_k$$

between the desired modes (n-neighbours and n-n-neighbours)?

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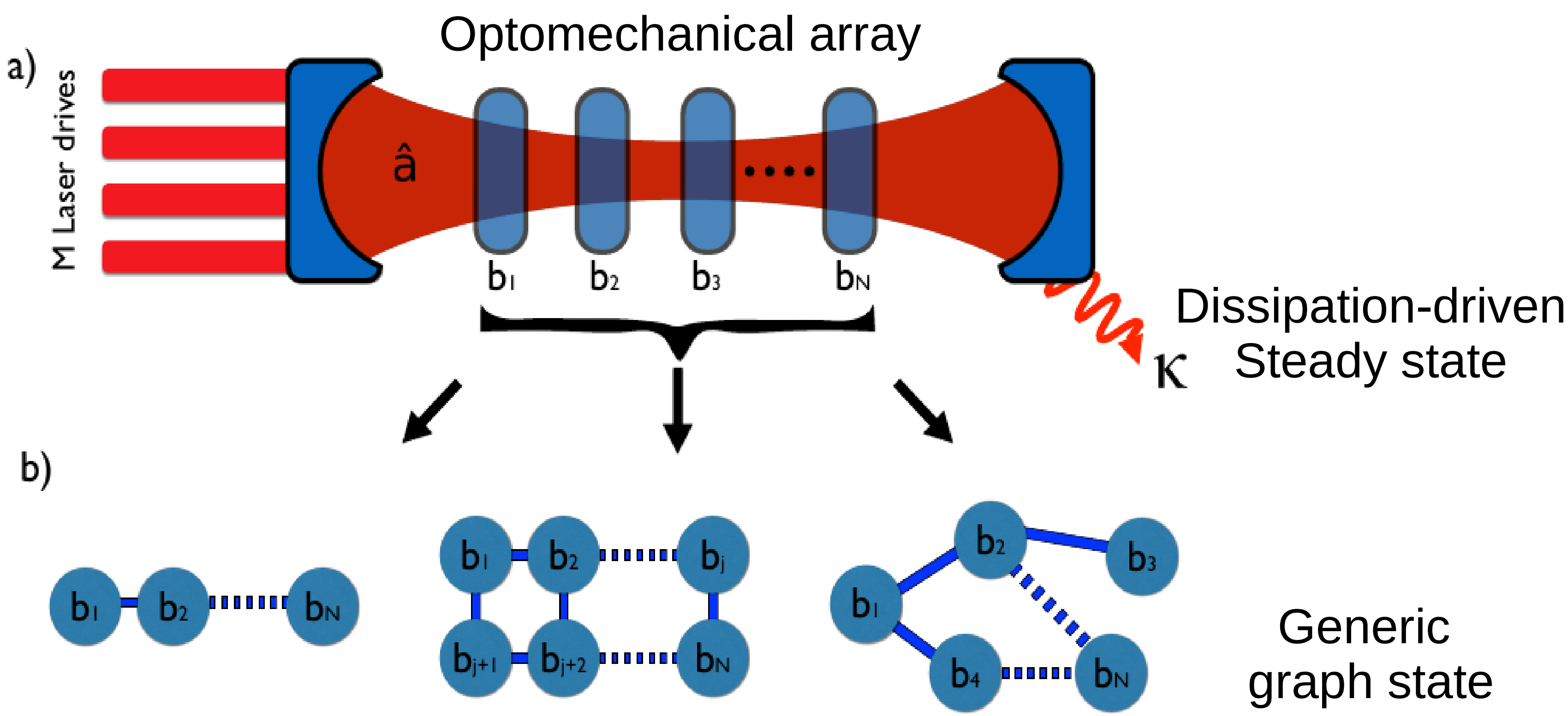
Generate arbitrary graph states of mechanical oscillators exploiting the open dynamics of optomechanical systems



[Houhou, Aissaoui, AF, PRA '15]

Exploiting the open-system dynamics

Assume the two-mode Hamiltonian system

$$H = \beta a^\dagger (c + r\, c^\dagger) + \mathrm{H.c.}$$

with losses on mode a only

$$\frac{d\rho}{dt} = -i[H, \rho] + \kappa\left(a\rho a^\dagger - \frac{1}{2}a^\dagger a \rho - \frac{1}{2}\rho a^\dagger a\right)$$

The dynamics preserves Gaussianity:

$$\frac{d\langle R\rangle}{dt} = D\langle R\rangle \qquad \frac{dV}{dt} = DV + VD^T + B$$

The system is dissipatively driven to a **unique and squeezed steady state**

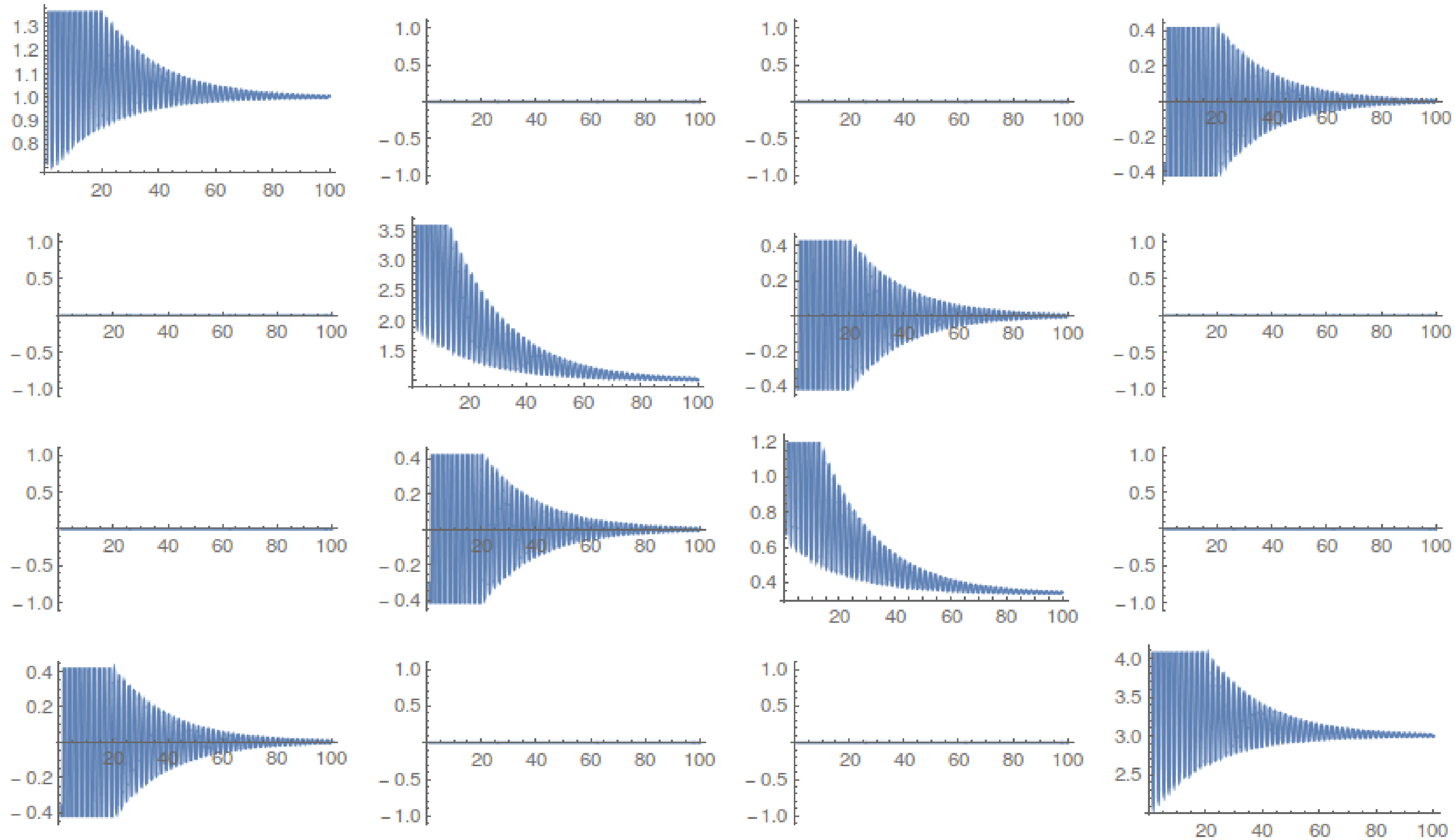
$$V_{ss} = \frac{1}{2}I_2 \oplus \begin{pmatrix} \frac{1-r}{1+r} & 0 \\ 0 & \frac{1+r}{1-r} \end{pmatrix}$$

$$10\,\mathrm{Log}_{10}\left(\frac{1-r}{1+r}\right)\,\mathrm{dB}$$

of squeezing

Exploiting the open-system dynamics

$$\frac{\mathrm{d}V}{\mathrm{d}t} = DV + VD^T + B \longrightarrow V_{ss} = \frac{1}{2} I_2 \oplus \begin{pmatrix} \frac{1-r}{1+r} & 0 \\ 0 & \frac{1+r}{1-r} \end{pmatrix}$$

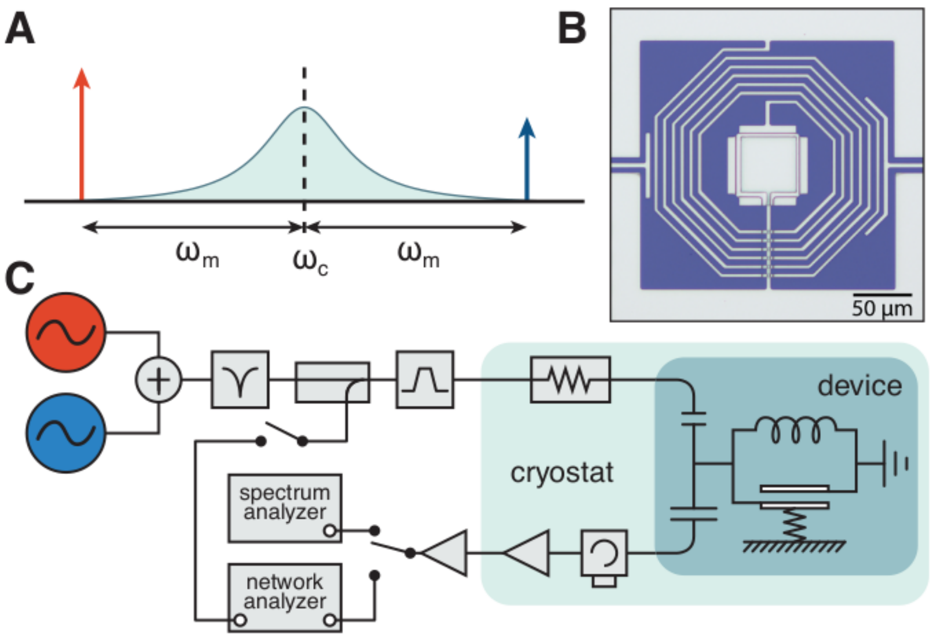


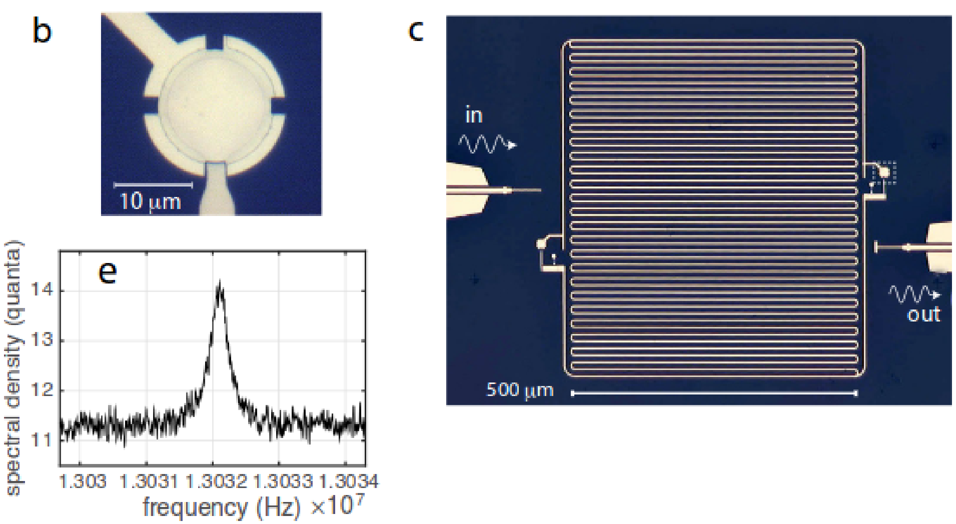
$(r = 0.5,\ k = 0.1,\ \beta = 2)$

For $\beta \geq \kappa/(4\sqrt{1-r^2}) \Rightarrow \tau_{ss} \geq \frac{4}{\kappa}$

Exploiting the open-system dynamics

$$\frac{dV}{dt} = DV + VD^T + B \longrightarrow V_{ss} = \frac{1}{2} I_2 \oplus \begin{pmatrix} \frac{1-r}{1+r} & 0 \\ 0 & \frac{1+r}{1-r} \end{pmatrix}$$





Woolman et al.,

Science 349, 952 (2015)

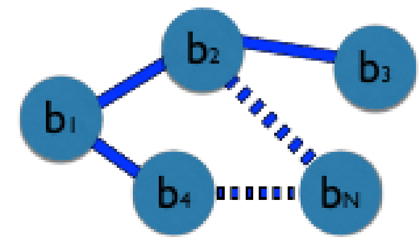
Pirkkalainen et al.,

PRL 115, 243601 (2015)

Exploiting the open-system dynamics (graph)

Consider an arbitrary N-mode graph state (with finite squeezing)

$$V = \frac{1}{2} S^T \begin{pmatrix} \frac{1-r}{1+r} I_N & 0_N \\ 0_N & \frac{1+r}{1-r} I_N \end{pmatrix} S$$



$$S \longleftrightarrow U \quad \Longrightarrow \quad c = U\, b$$

$b = (b_1, \ldots, b_N)$ **local**

$c = (c_1, \ldots, c_N)$ **collective**

where U is given by the polar decomposition (given adjacency matrix A):

$$-(i\, I_N + A) = R\, U$$

With N **Hamiltonian switching** steps, one can exploiting the dissipation to drive each collective mode at a time into a squeezed state:

$$H^{(k)} = \beta a^\dagger (c_k + r\, c_k^\dagger) + \mathrm{H.c.} \qquad (k = 1, \ldots, N)$$

Hence the local modes will be in the desired graph state!

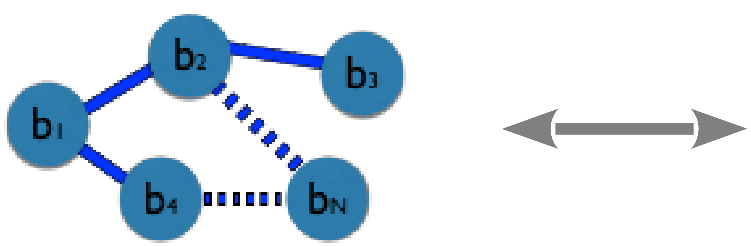
[Li, Ke, and Ficek, PRA (2009); Ikeda & Yamamoto, PRA (2013)]

How can we implement the Hamiltonian switch?

Consider the set of Hamiltonians with free parameters $\alpha_j^{\pm}$, $\phi_j^{\pm}$:

$$H = a^\dagger \sum_{j=1}^{N} g_j \left(\alpha_j^+ e^{i\phi_j^+} b_j^\dagger + \alpha_j^- e^{i\phi_j^-} b_j \right) + \mathrm{H.c.}$$

arbitrary graph



S , U

$b = (b_1, \ldots, b_N)$ **local**

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At each step k set the free parameters as follows:

$$\begin{cases} \alpha_j^- &= \dfrac{\beta}{g_j} |U_{kj}| \\ \alpha_j^+ &= r\alpha_j^- \\ \phi_j^- &= -\phi_j^+ = \arg(U_{kj}) \end{cases}$$

$$\Downarrow$$

$$H \equiv H^{(k)} = \beta a^\dagger (c_k + r c_k^\dagger) + \mathrm{H.c.}$$

Example: 4-mode linear graph

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

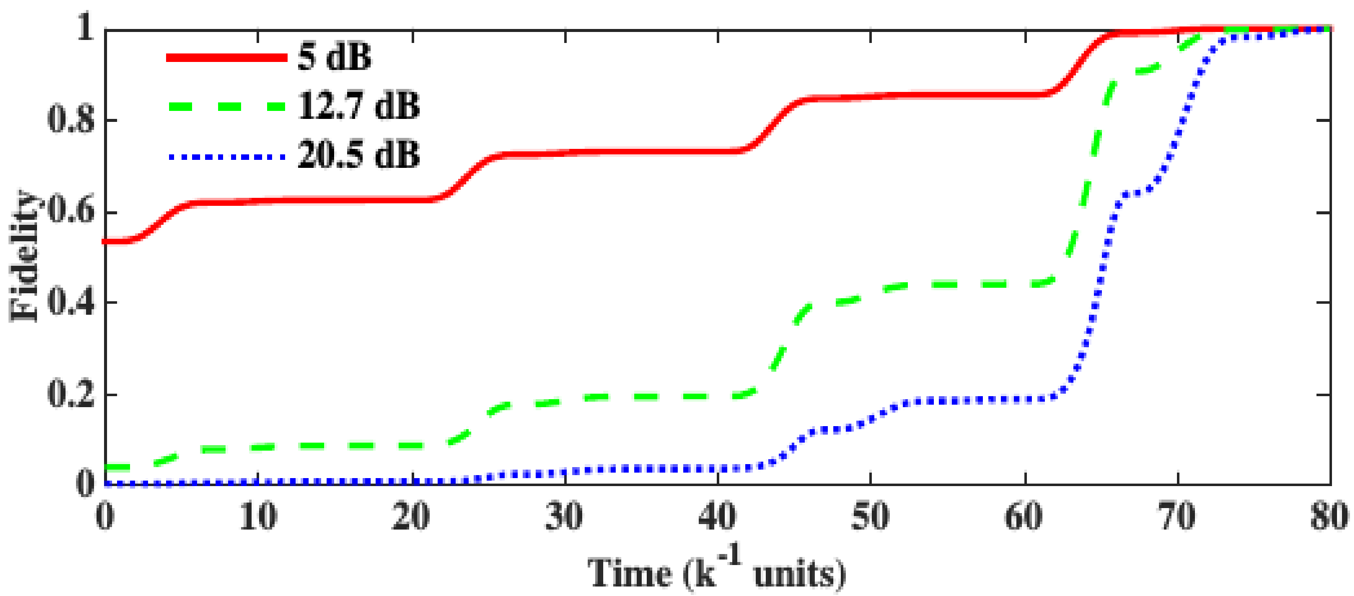
$$-\left(\mathrm{i}\,\mathrm{I_N} + \mathrm{A}\right) = \mathrm{R\,U}$$

Step	α_1^-	α_2^-	α_3^-	α_4^-	ϕ_1^-	ϕ_2^-	ϕ_3^-	ϕ_4^-
1	$\frac{\sqrt{2(5+\sqrt{5})}}{5}$	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\frac{\sqrt{5-2\sqrt{5}}}{5}$	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$3\pi/2$	π	$\pi/2$	0
2	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\frac{\sqrt{2(5-\sqrt{5})}}{5}$	$\frac{\sqrt{5-2\sqrt{5}}}{5}$	π	$3\pi/2$	π	$\pi/2$
3	$\frac{\sqrt{5-2\sqrt{5}}}{5}$	$\frac{\sqrt{2(5-\sqrt{5})}}{5}$	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\pi/2$	π	$3\pi/2$	π
4	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\frac{\sqrt{5+2\sqrt{5}}}{5}$	$\frac{\sqrt{2(5-\sqrt{5})}}{5}$	$\frac{\sqrt{5-2\sqrt{5}}}{5}$	0	$\pi/2$	π	$3\pi/2$

Example: 4-mode linear graph

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \longrightarrow -(\mathrm{i}\,\mathrm{I}_{\mathrm{N}} + \mathrm{A}) = \mathrm{R}\,\mathrm{U}$$

Real time evolution of the fidelity:
(fixed switching time $t_s = 20/\kappa$)

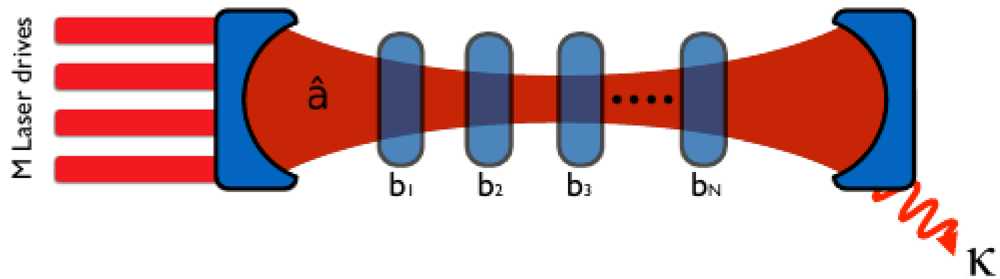


Finite-time evolution is enough to reach the target state

Hamiltonian engineering in optomechanics

Inspired by 1- and 2-mode schemes [Clerck, Hartmann, Marquardt, Meystre, Vitali,...]

$$\epsilon(t) = \sum_{k=1}^{M} \epsilon_k e^{-i\omega_k t} e^{i\Phi_k}$$



$$\mathcal{H} = \omega_c a^\dagger a + \sum_{j=1}^{N} \left[\Omega_j b_j^\dagger b_j + g_j a^\dagger a (b_j^\dagger + b_j) \right] + \epsilon(t) a^\dagger + \epsilon^*(t) a$$

$$g_j \ll \omega_c, \Omega_j$$
$$a \rightarrow a + \alpha$$
$$b_j \rightarrow b_j + \beta_j$$

- Linearizing $\alpha = \sum_{k=1}^{M} \frac{-i\epsilon_k}{\kappa/2 + i(\omega_c - \omega_k)} e^{-i\omega_k t} e^{i\Phi_k}$

Two drives per mechanical mode

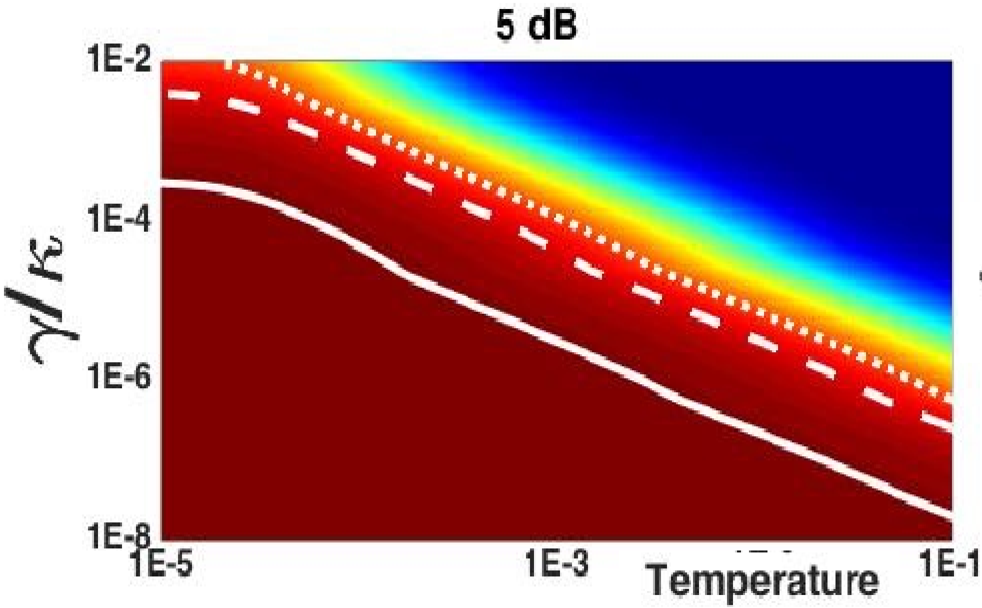
$$\omega_j^\pm = \omega_c \pm \Omega_j$$

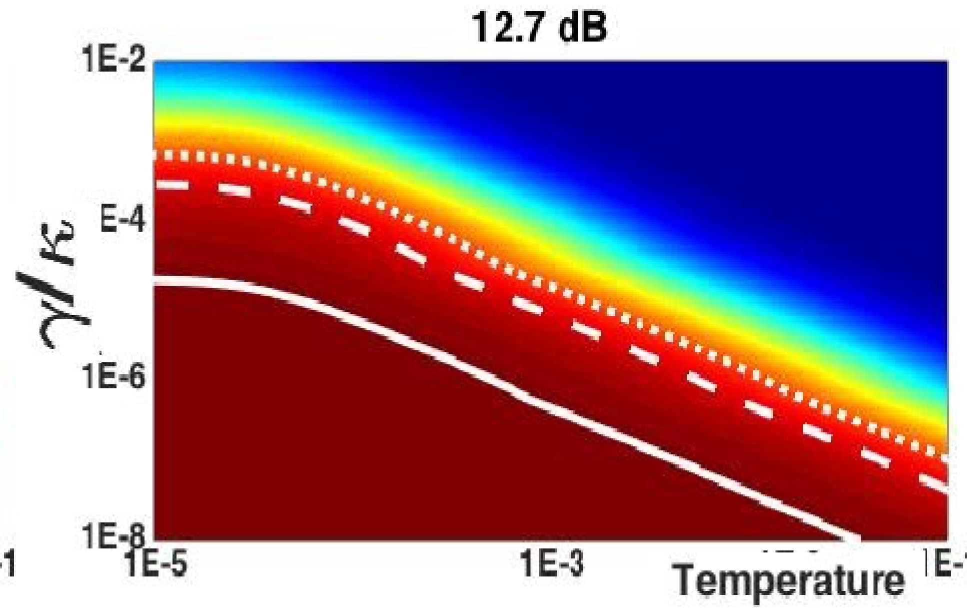
- Non-overlapping mechanical frequencies
- Rotating wave approximation

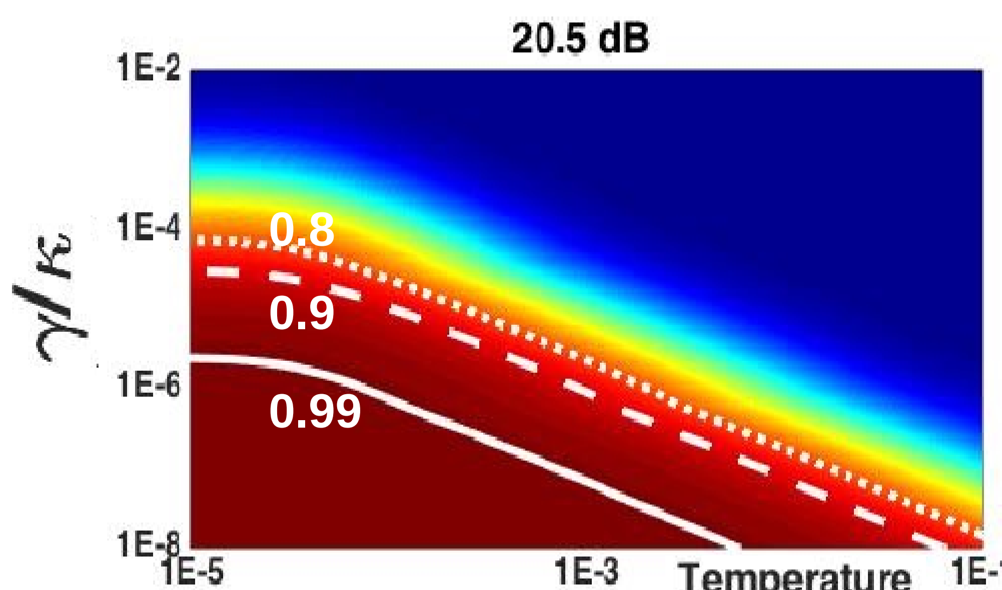
$$\alpha_j^\pm g_j \ll \Omega_j \Rightarrow \kappa \ll \Omega_j$$

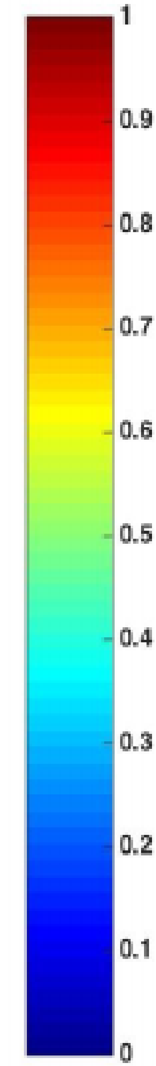
$$H = a^\dagger \sum_{j=1}^{N} g_j \left(\alpha_j^+ e^{i\phi_j^+} b_j^\dagger + \alpha_j^- e^{i\phi_j^-} b_j \right) + \mathrm{H.c.}$$

Effects of mechanical noise: examples









The higher the target squeezing the less the tolerable noise

Fidelity

$\Omega_j/2\pi = j \text{ MHz}$

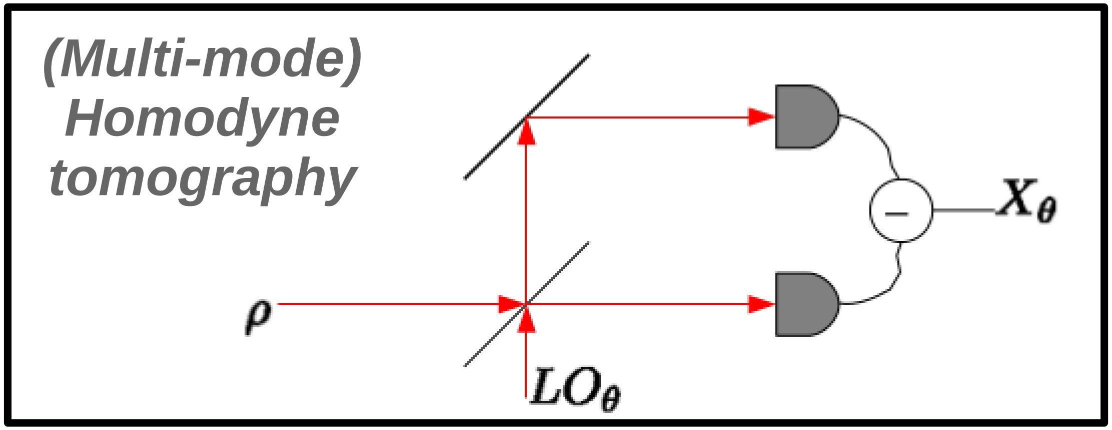
Outline

- Measurement-based quantum computation with CVs
- Generation of universal resources for CV quantum computation:
  - Adiabatic generation of cluster states
  - Optomechanical cluster-state generation via reservoir engineering
- Quantum tomography for confined CVs
  - A single qubit to read them all
  - A single qumode to read them all

Quantum tomography for confined CVs

The problem

Tomography is a well established framework:



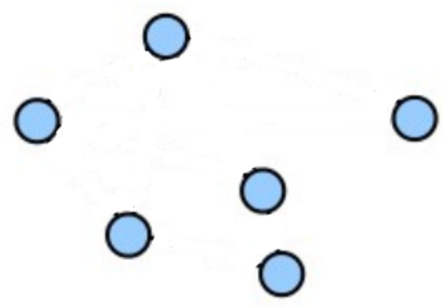
But how do we perform tomography on confined CVs – i.e., in the absence of optical homodyne?

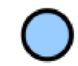
Our solution

Use a **single** qubit/qumode probe that **tunably** interacts with the confined system

The proposal

The confined CV system that we want to reconstruct:



 $= \omega_n b^\dagger b$

The proposal

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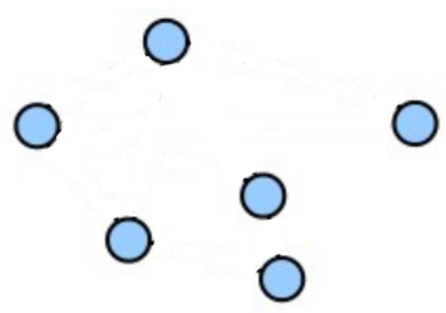
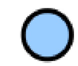
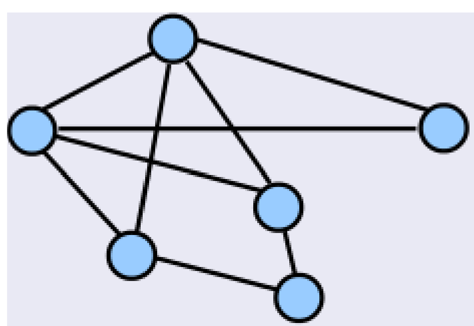


 $= \omega_n b^\dagger b$

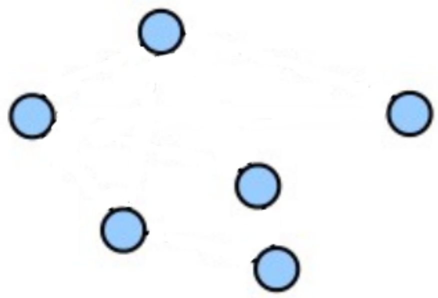
At t=0, “turn on” a **constant** harmonic interaction among the modes (typically available for confined CVs)

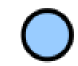


 $$= J_{nm}(b_n^\dagger b_m + b_n b_m^\dagger) + \\ + K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger)$$

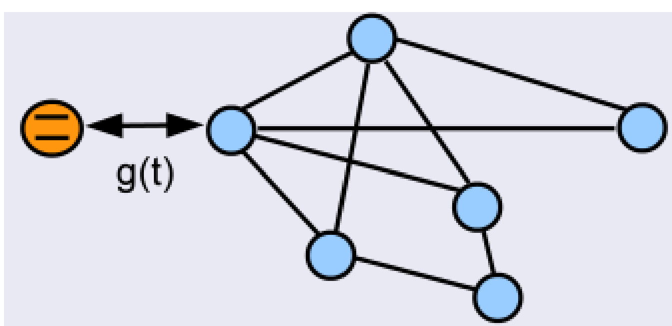
The proposal

The confined CV system that we want to reconstruct:



 $= \omega_n b^\dagger b$

At t=0, “turn on” a **constant** harmonic interaction among the modes (typically available for confined CVs) and a **tunable** interaction with a single qubit probe

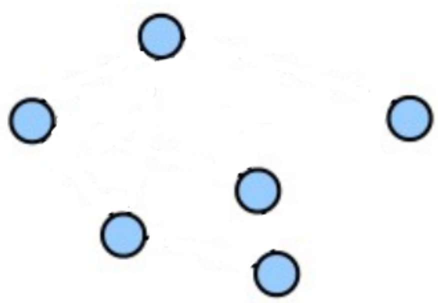


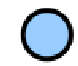
$$= J_{nm}(b_n^\dagger b_m + b_n b_m^\dagger) + \\ + K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger)$$

$$H_{\text{int}}(t) = g(t)\sigma_z(b_1 + b_1^\dagger)$$

The proposal

The confined CV system that we want to reconstruct:



 $= \omega_n b^\dagger b$

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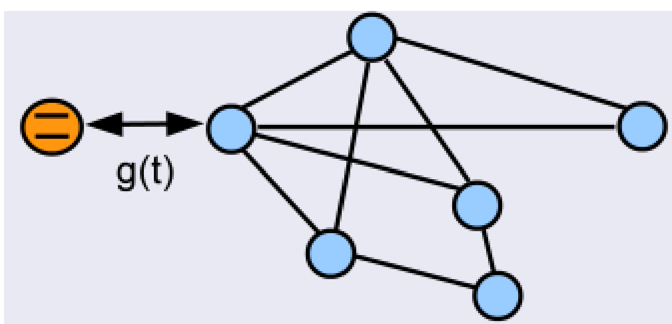


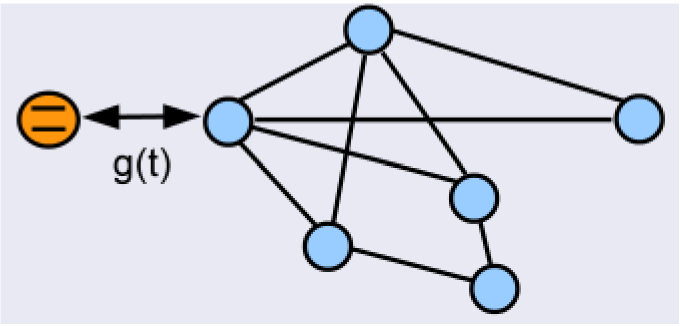
 $$= J_{nm}(b_n^\dagger b_m + b_n b_m^\dagger) + \\ + K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger)$$

$$H_{int}(t) = g(t)\sigma_z(b_1 + b_1^\dagger)$$

At t=T measure the qubit probe (and iterate the procedure).

Why should it work?

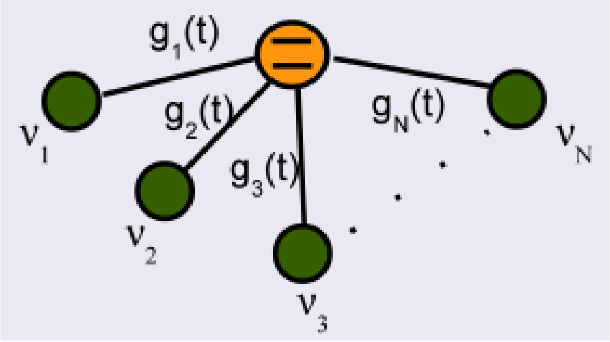
Local mode picture



- Nodes are mutually interacting
- The qubit interacts with a single node

$$H_{net} = \sum \omega_n b^\dagger b + \sum J_{nm}(b_n^\dagger b_m + b_n b_m^\dagger) + \sum K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger)$$

Normal mode picture

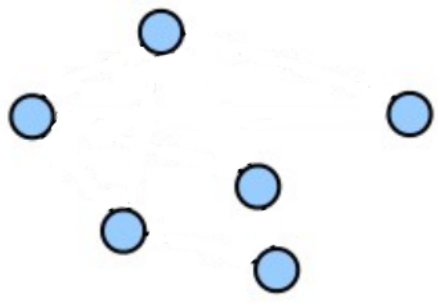


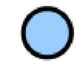
$$H_{net} = \sum \nu_n c^\dagger c$$

- Nodes are non-interacting
- The qubit interacts with all the nodes (*)
- Each node has a different frequency (**)

The proposal

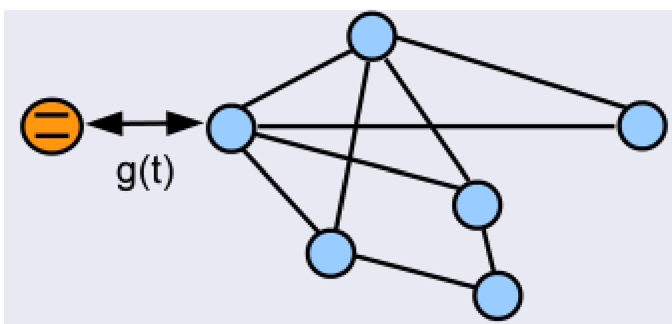
The confined CV system that we want to reconstruct:





$= \omega_n b^\dagger b$

At t=0, “turn on” a **constant** harmonic interaction among the modes (typically available for confined CVs) and a **tunable** interaction with a single **qubit** probe



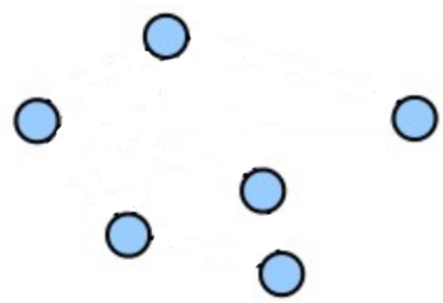
$$= J_{nm}(b_n^\dagger b_m + b_n b_m^\dagger) + \\ + K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger)$$

$$H_{\mathrm{int}}(t) = g(t)\sigma_z(b_1 + b_1^\dagger)$$

At t=T measure the qubit probe (and iterate the procedure).

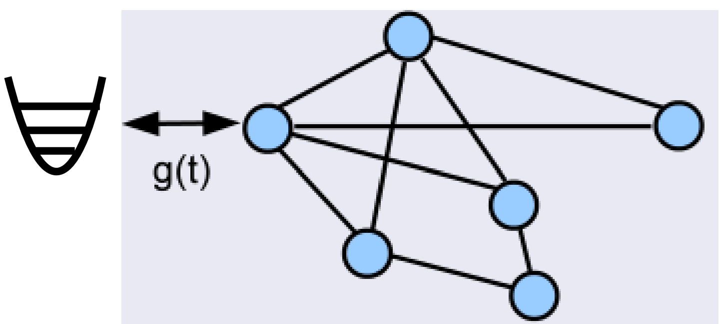
The proposal (qumode-probe)

The confined CV system that we want to reconstruct:



 $= \omega_n b^\dagger b$

At t=0, “turn on” a **constant** harmonic interaction among the modes (typically available for confined CVs) and a **tunable** interaction with a single **qumode** probe



$$= J_{nm}(b_n^\dagger b_m + b_n b_m^\dagger) + \\ + K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger)$$

$$H_{\rm int}(t) = g(t) X (b_1 + b_1^\dagger)$$

$$X = (a + a^\dagger)/\sqrt{2}$$

At t=T measure the qumode probe (and iterate the procedure).

Main result

$$H(t) = H_{\rm net} + H_{\rm int}(t),$$

$$\mathrm{V H_{net} V^\dagger} = \sum \nu_{\rm n} \mathrm{c^\dagger c}$$

$$\mathrm{V H_{int} V^\dagger} = \mathrm{g(t)} \sigma_{\rm z} \sum (\lambda_{\rm k} \mathrm{c_k} + \lambda_{\rm k}^* \mathrm{c_k^\dagger})$$

$$\mathrm{V H_{int} V^\dagger} = \mathrm{g(t) X} \sum (\lambda_{\rm k} \mathrm{c_k} + \lambda_{\rm k}^* \mathrm{c_k^\dagger})$$

The evolution is a conditional displacement in the phase space

There is a time t_0 such that for any $t > t_0$, for any complex vector $\boldsymbol{\beta} = (\beta_1, ..., \beta_N)$, one can find an interaction profile $g(s)$, yielding

$$\mathrm{U(t) = D(\sigma_z \beta)} \qquad \mathrm{U(t) = D(X\beta)}$$

$g(s)$ is linearly dependent on $\boldsymbol{\beta} \Rightarrow$ easy to compute!

[Tufarelli, AF, Kim, Bose, PRA '12]

[Moore, Tufarelli, Paternostro, AF, arXiv:1606XXX]

Explicit formula for the coupling *g(s)*

$$g(s) = \frac{i}{T}\sum_{i=1}^{N}\left(\frac{B_l}{\lambda_l^*}e^{-i\nu_l s} - \frac{B_l^*}{\lambda_l}e^{i\nu_l s}\right) + \left(he^{-i\omega s} + h^* e^{i\omega s}\right)$$

Only necessary for the qumode-probe case

$$\begin{pmatrix} -\mathbf{B}^* \\ \mathbf{B} \end{pmatrix} = (\mathcal{S}^\intercal \mathbf{M})^{-1} \begin{pmatrix} -\boldsymbol{\beta}^* \\ \boldsymbol{\beta} \end{pmatrix}$$

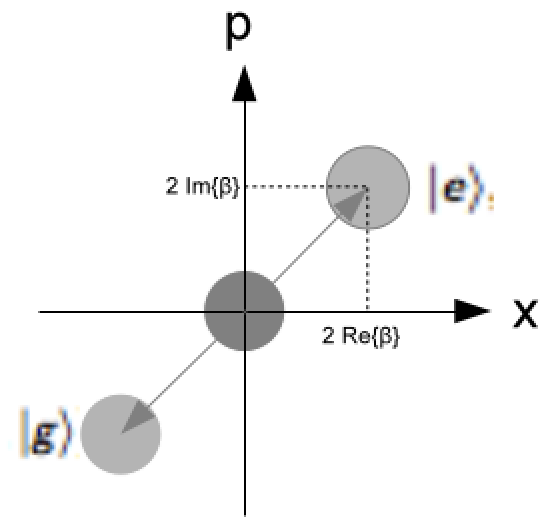
Where S and M depend on the the structure of the network only.

Phase space picture (qubit case)

Tunable coupling → **Qubit controlled displacements**

$$U(T) = D(\sigma_z \beta)$$

$$\beta = -2i \int_0^T ds\, g(s) e^{i\omega s}$$



Phase space picture (qubit case)

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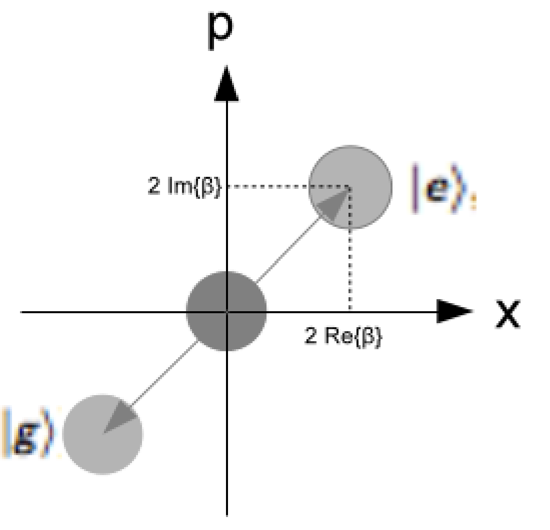

Preparing the qubit in state $|+\rangle$ one can measure **directly** the Characteristic Function:

$$\langle \sigma_x \rangle + i \langle \sigma_y \rangle = \chi(-2\beta)$$
$$\chi(\xi) = \mathrm{tr}\{\rho D(\xi)\}$$

Phase space picture (qubit case)

Tunable coupling → **Qubit controlled displacements**

$$\mathrm{U(T)} = \mathrm{D}(\sigma_{\mathrm{z}}\beta)$$

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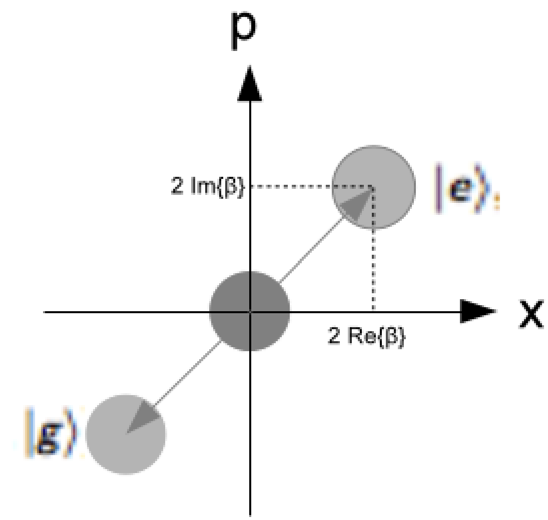

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$$\langle \sigma_x \rangle + i \langle \sigma_y \rangle = \chi(-2\beta)$$
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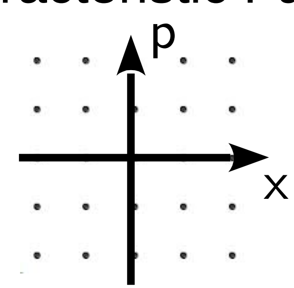
Varying g(s) one can sample the Characteristic Function:

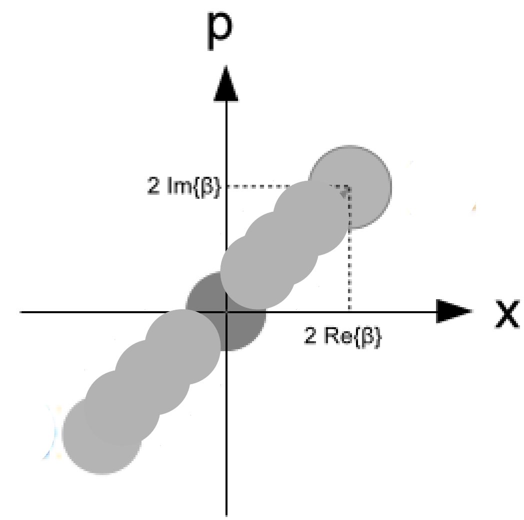


Phase space picture (qumode case)

Tunable coupling → **Qubit controlled displacements**

$$U(T) = D(X\beta)$$

$$\beta = -2i \int_0^T ds\, g(s) e^{i\omega s}$$



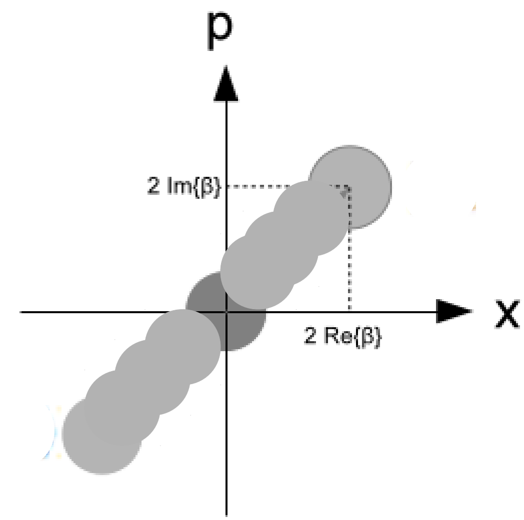
Phase space picture (qumode case)

Tunable coupling → **Qubit controlled displacements**

$$U(T) = D(X\beta)$$

$$\beta = -2i\int_0^T ds\, g(s)e^{i\omega s}$$

→



Preparing the qumode-probe in the vacuum state, its momentum at time T acquires information about any desired quadrature of the mechanical oscillator:

$$P(T) = P - \sqrt{2}|\beta|Q_\theta$$

$$Q_\theta = (be^{-i\theta} + b^\dagger e^{i\theta})/\sqrt{2}$$

$$\theta = \arg(\beta) + \pi/2$$

→ **Varying g(s) one can sample any mechanical quadrature**

Reconstruction algorithm

1. Choose a time $T > t_0$ and complex vector $\boldsymbol{\beta} = -\boldsymbol{\xi}/2$
2. Determine the corresponding profile $g(s)$
3. Prepare initial state $\boxed{\rho_{\text{tot}}(0) = |+\rangle\langle+| \otimes \rho}$
4. Evolve for a time T with coupling profile $g(s) \Rightarrow U(t) = D(\sigma_z \boldsymbol{\beta})$
5. Measure qubit observable σ_x or σ_y, go back to (3)
6. $\boxed{\langle\sigma_x\rangle + i\,\langle\sigma_y\rangle = \chi(\boldsymbol{\xi})}$
7. Repeat (1)-(6) for different values of $\boldsymbol{\beta}$

↓

Point-wise reconstruction of the multi-mode Characteristic Function

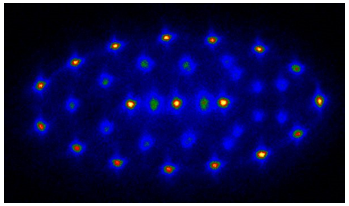
The tomographic protocol is minimal:

- Access to only one confined mode
- The probe is single qubit/qumode
- Tune only one parameter g(s)

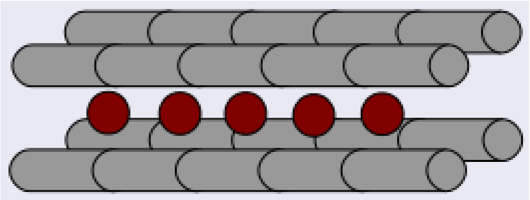
[Tufarelli, AF, Kim, Bose, PRA '12]

[Moore, Tufarelli, Paternostro, AF, arXiv:1606XXX]

Trapped ion implementation

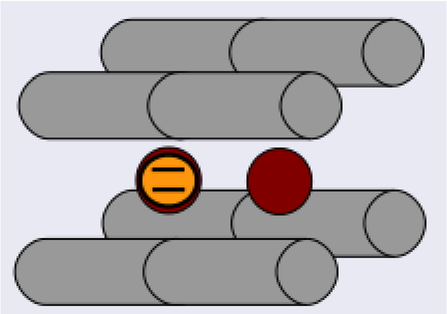


The network



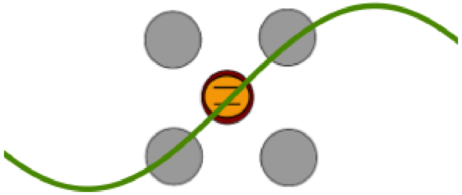
Motional state of the ions around equilibrium position plus Coulomb interaction

The qubit



Electronic transition of a chosen ion

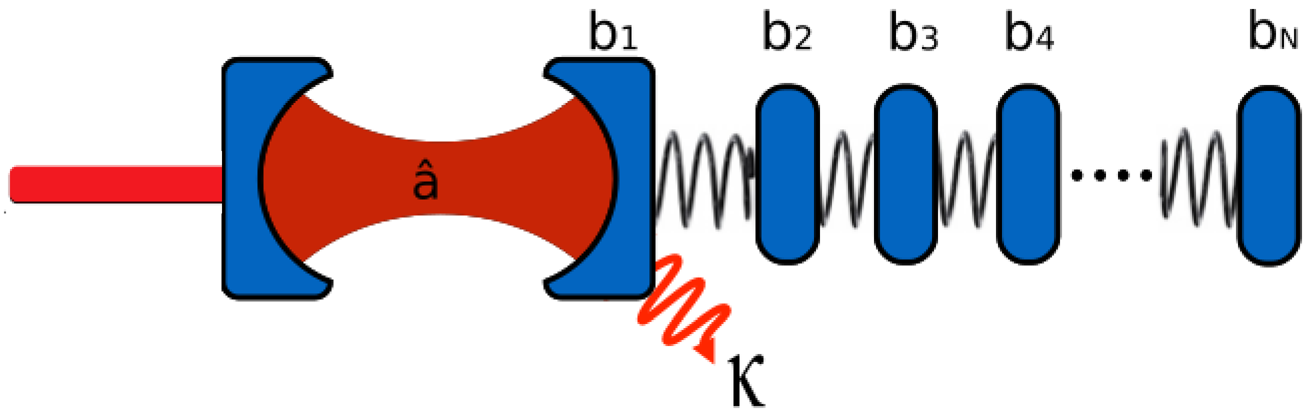
The tunable coupling



Place the chosen ion at the node of a resonant laser standing wave.

Laser power modulation determines g(t)

Opto-mechanical implementation



The probe

Cavity output mode

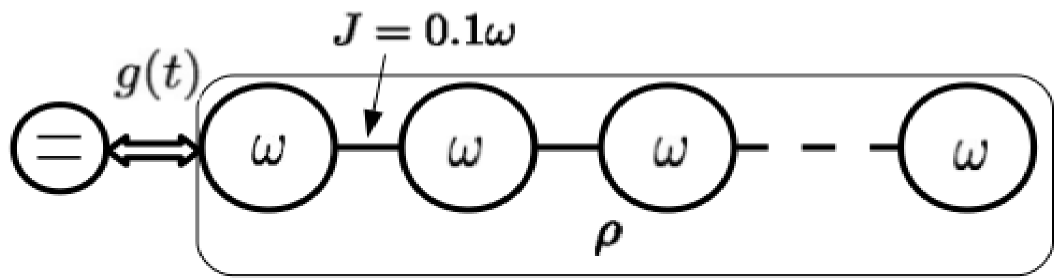
The network

Mechanical oscillators

The tunable coupling

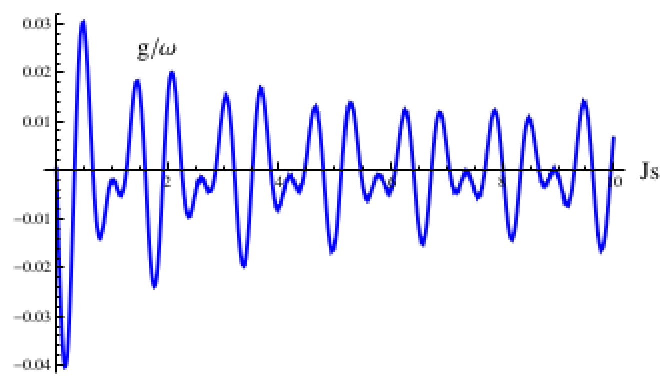
Laser power modulation determines g(t)

Qubit-probe example: linear chain (10 oscillators)



Suppose that we want to know $\chi(1, ..., 1)$

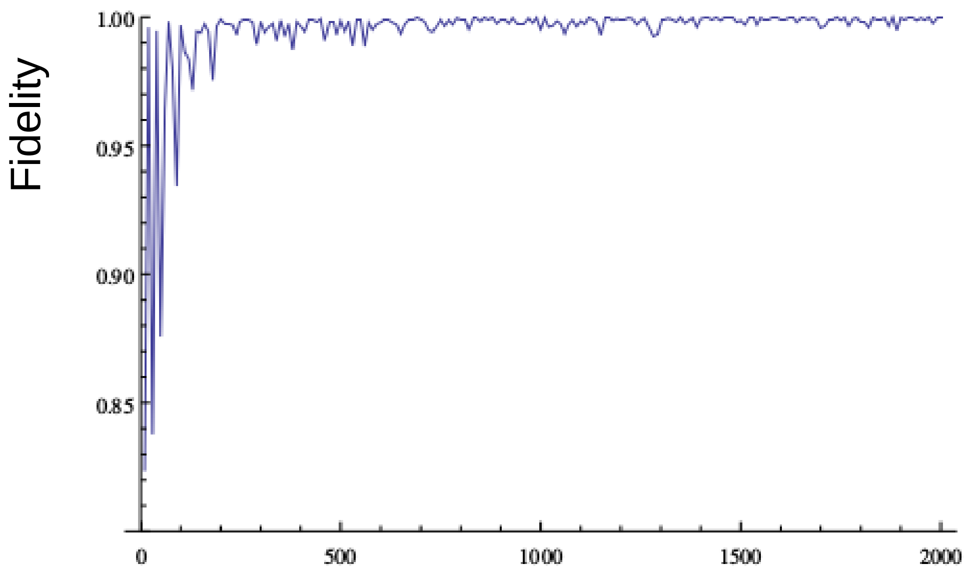
- Prepare $\rho_{\text{tot}} = |+\rangle\langle+| \otimes \rho$
- Evolve with g(s)=



- Measure either $\sigma_{\text{x}}, \sigma_{\text{y}}$ and repeat
- Statistics over many repetitions provides $\chi(1, ..., 1) = \langle\sigma_{\text{x}}\rangle + \text{i}\langle\sigma_{\text{y}}\rangle$

Qumode-probe example

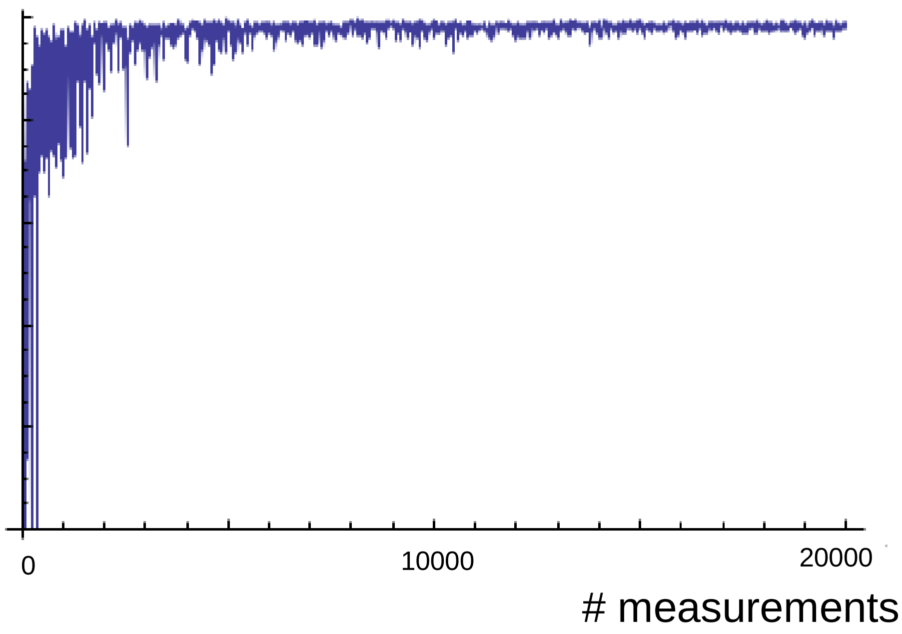
1 mechanical oscillator



Squeezed thermal state

$r = 0.2, \ T = 1$

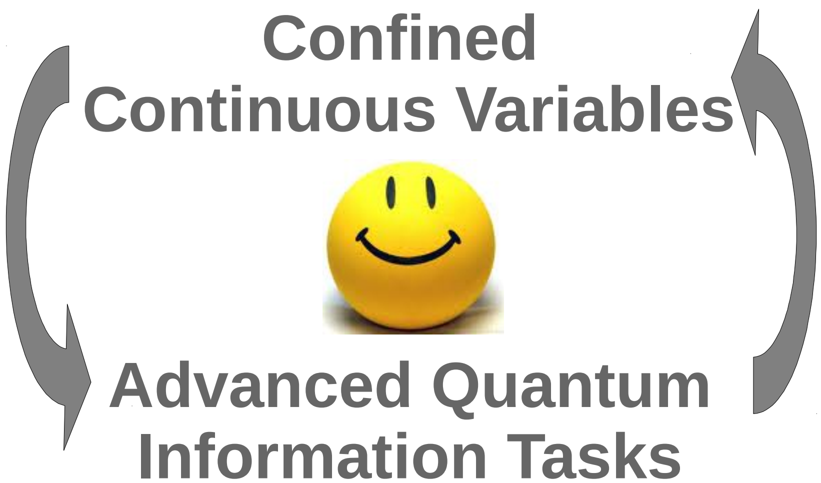
2 mechanical oscillators

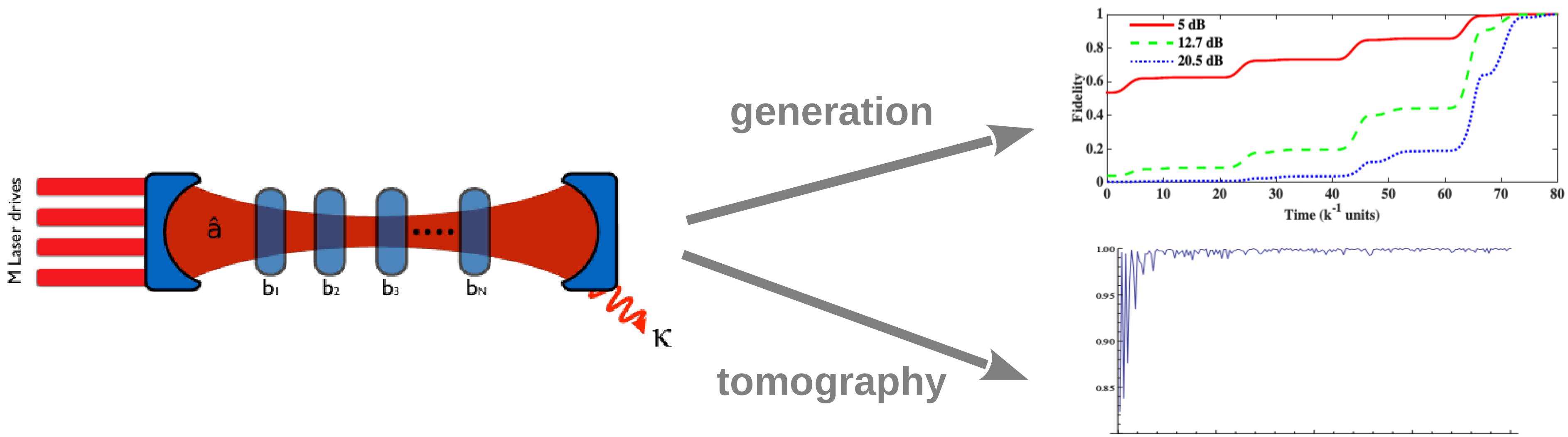


Thermal twin-beam state

$r = 0.2, \ T = 1.5$

To Conclude





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