Quantum metrology gets real

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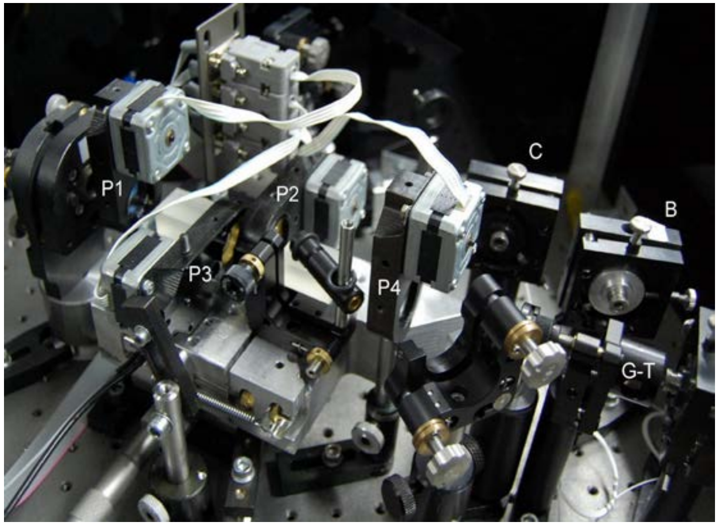
Faculty of Physics, University of Warsaw, Poland



All-Ireland Conference
on Quantum Technologies
Maynooth University
1 June 2016

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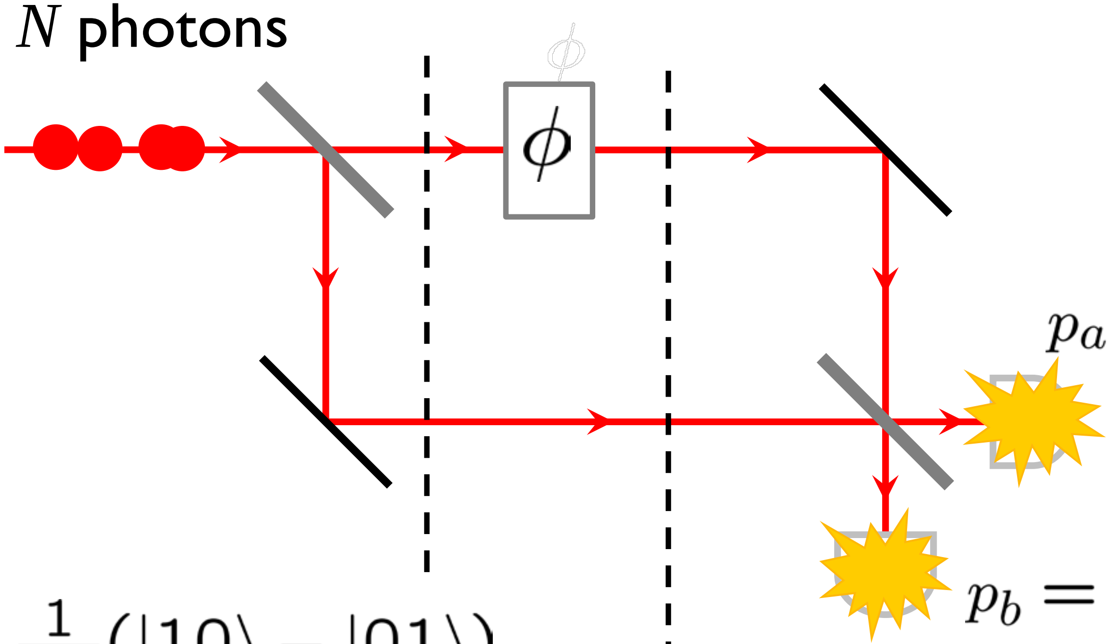
Phase measurement

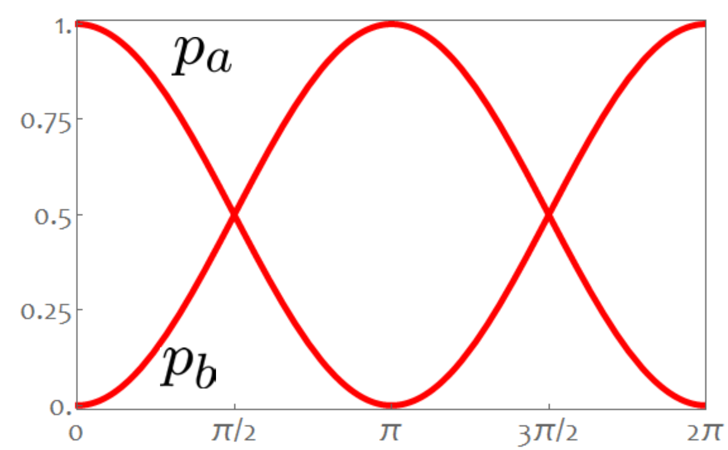


$$p_a = \frac{1}{2}(1 + \cos\phi)$$

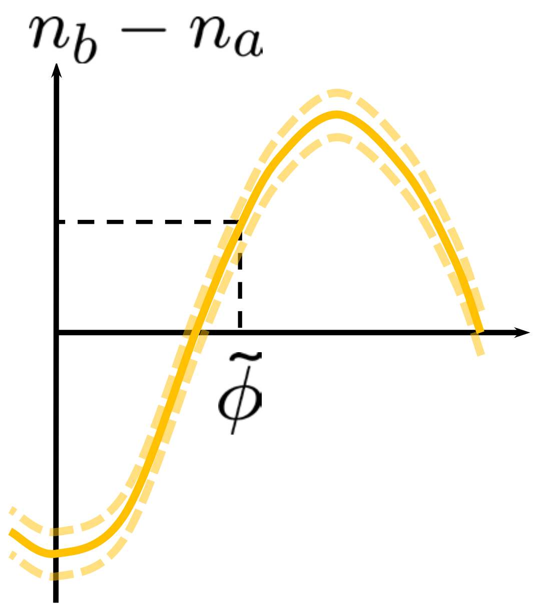
$$p_b = \frac{1}{2}(1 - \cos\phi)$$

$$\frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$$

$$\frac{1}{\sqrt{2}}(e^{i\phi}|10\rangle - |01\rangle)$$



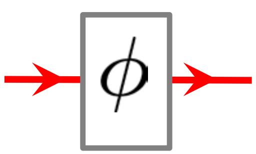
Estimation procedure

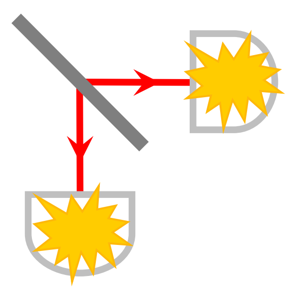


Example: around $\pi/2$ operating point:

$$\tilde{\phi} = \frac{\pi}{2} + \frac{n_b - n_a}{N}$$

Actual value ϕ → $p(r|\phi)$ → Measurement result r → $\Phi(r)$ → Estimate $\tilde{\phi} = \Phi(r)$





$$r \equiv (n_a, n_b)$$

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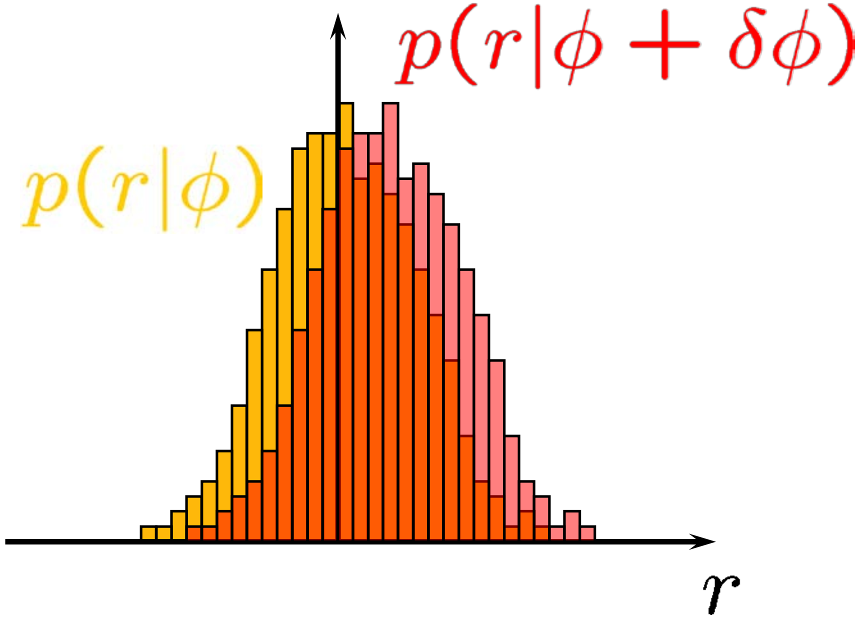
Fisher information

$$F(\phi) = \sum_r p(r|\phi) \left(\frac{\partial}{\partial \phi} \ln p(r|\phi) \right)^2$$



Cramér-Rao bound:
for unbiased estimators

$$\Delta\tilde{\phi} \geq \frac{1}{\sqrt{F(\phi)}}$$

Shot noise limit: for N independently used photons

$$F(\phi) = N$$

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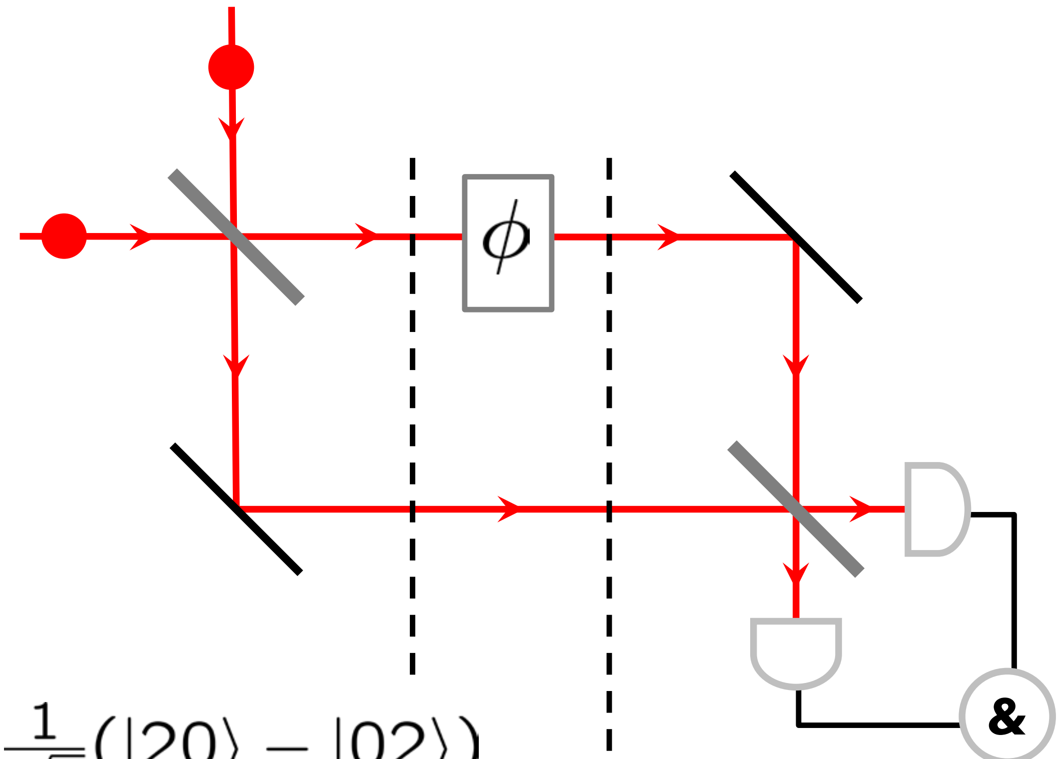
Two-photon interferometry



$$\frac{1}{\sqrt{2}}(|20\rangle - |02\rangle)$$

$$\frac{1}{\sqrt{2}}(e^{2i\phi}|20\rangle - |02\rangle)$$

Coincidence between ports:

$$p_c = \frac{1}{2}(1 + \cos 2\phi)$$

Double count on one port:

$$p_d = \frac{1}{2}(1 - \cos 2\phi)$$

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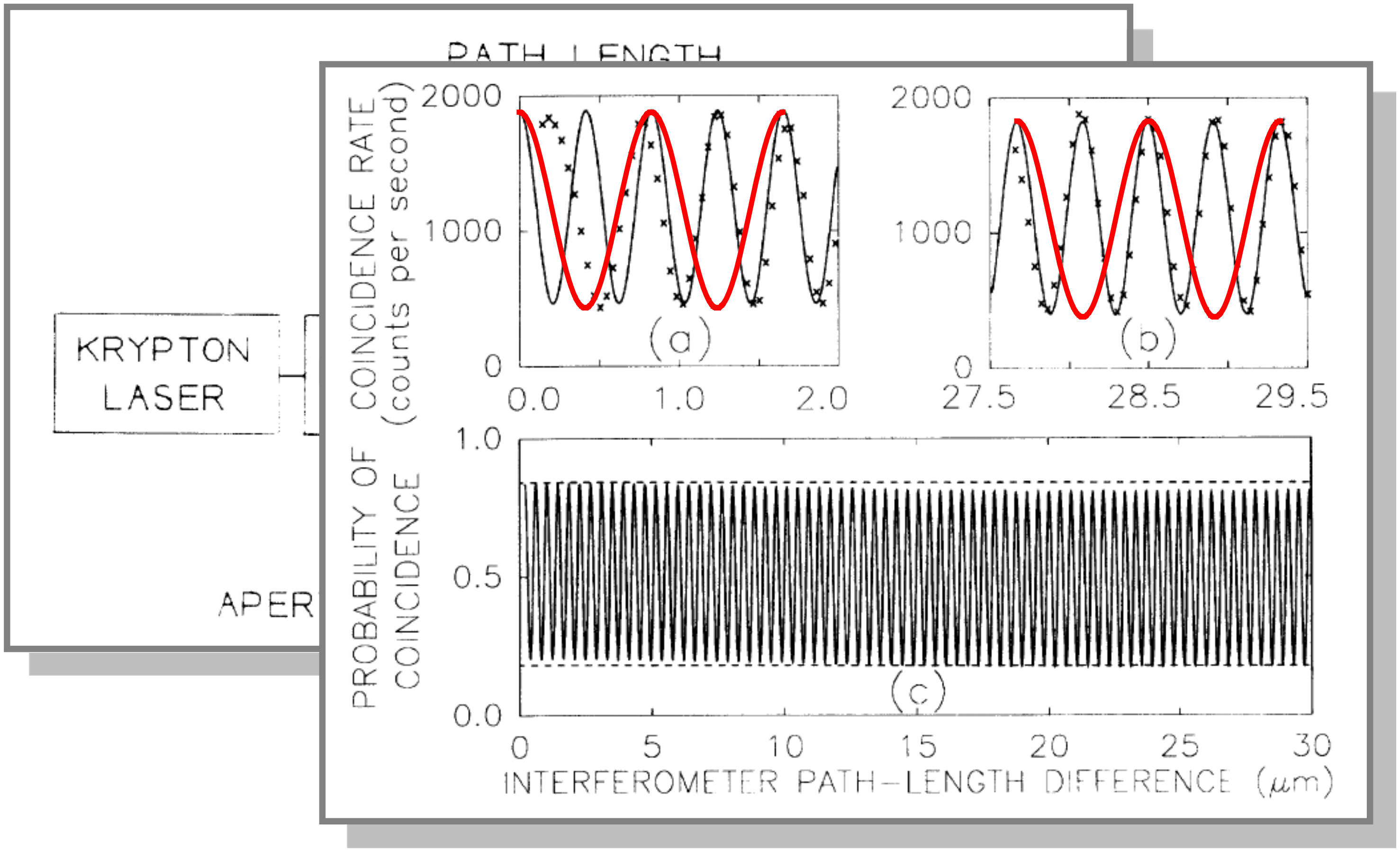
Experiment

J. G. Rarity *et al.*, Phys. Rev. Lett. **65**, 1348 (1990)



Two photons sent one-by-one (shot noise limit): $F = 2$

Two-photon interference: $F = 4$

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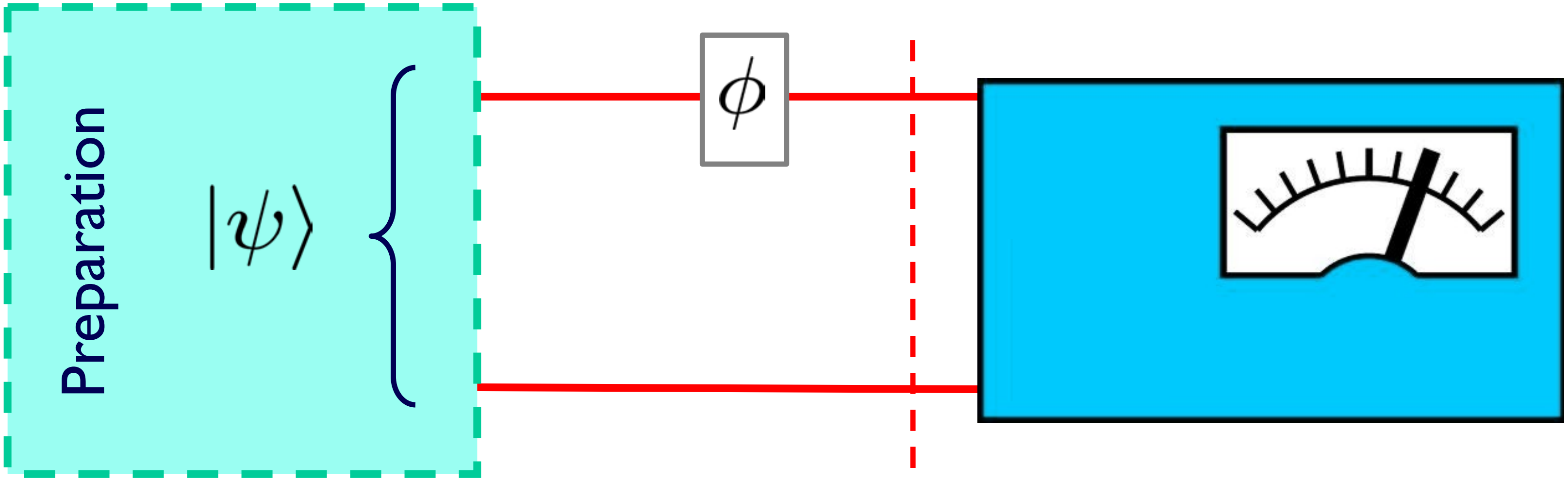
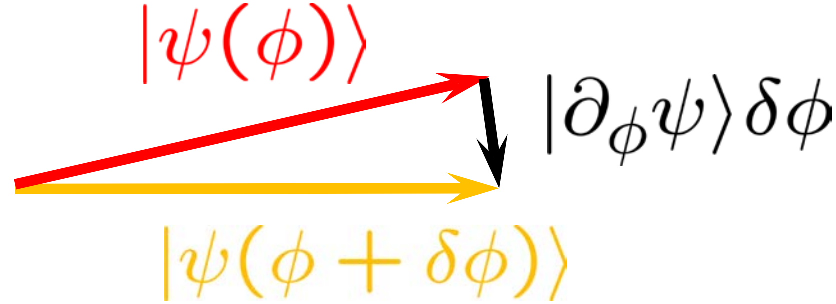
General picture





For any measurement

$$\mathsf{F}(\phi) \leq \mathsf{F}_Q(\phi)$$

where *Quantum Fisher information* reads

$$\mathsf{F}_Q(\phi) = 4\Big(\langle \partial_\phi \psi | \partial_\phi \psi \rangle - |\langle \psi(\phi) | \partial_\phi \psi \rangle|^2\Big)$$

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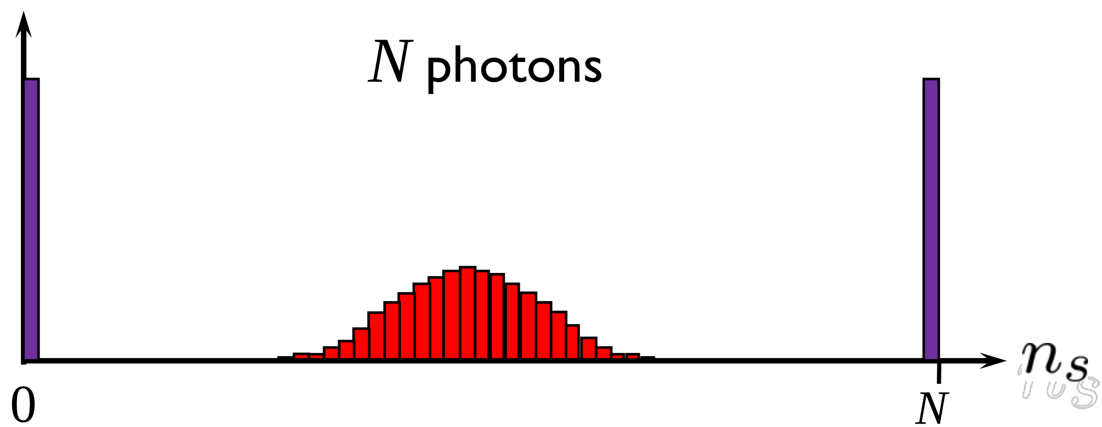
Heisenberg limit

$$\Delta\tilde{\phi}\cdot\Delta n_s \geq \frac{1}{2}$$

Δn_s – photon number uncertainty in the sensing arm

$\Delta\tilde{\phi}$ – precision of phase estimation



N independently used photons (shot noise limit):

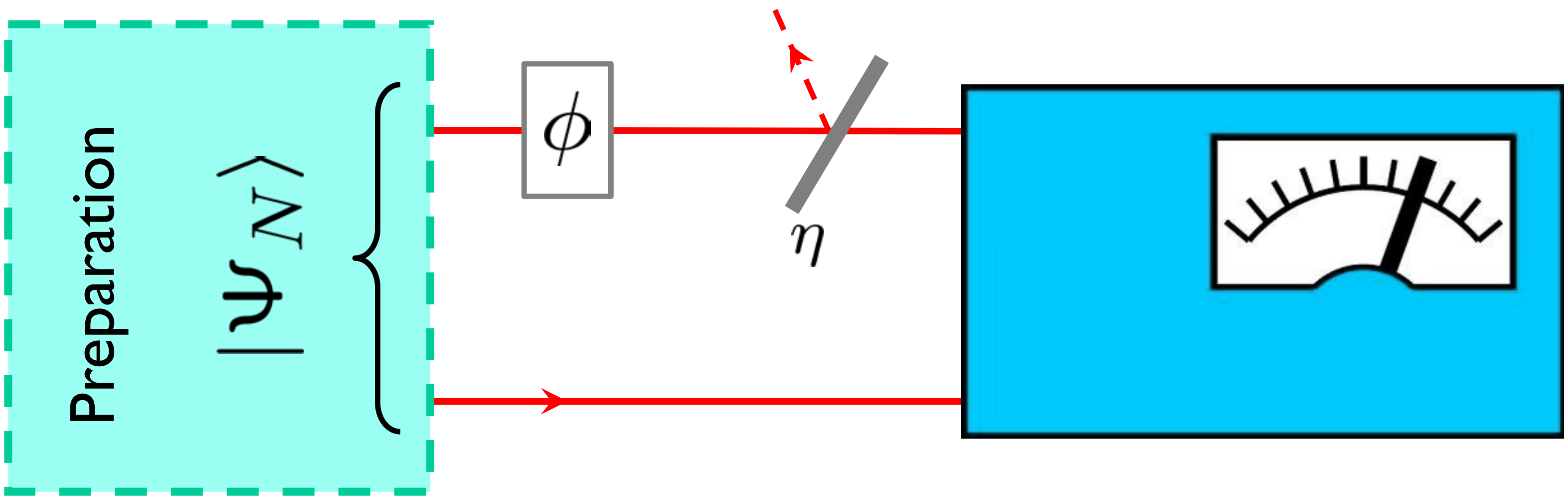
$$\Delta\tilde{\phi} = \frac{1}{\sqrt{N}}$$

Maximum possible Δn_s defines the *Heisenberg limit*:

$$\Delta\tilde{\phi} = \frac{1}{N}$$

J. J. Bollinger *et al.*, Phys. Rev. A **54**, R4649(R) (1996)
J. P. Dowling, Phys. Rev. A **57**, 4736 (1998)

N00N state



$$|\Psi_N(\phi)\rangle = \frac{1}{\sqrt{2}}(\mathrm{e}^{\mathrm{i}N\phi}|N0\rangle - |0N\rangle)$$

No photon lost: $\sqrt{\eta}^N \mathrm{e}^{\mathrm{i}N\phi}|N0\rangle - |0N\rangle$

One photon lost: $\mathrm{e}^{\mathrm{i}N\phi}|N-1,0\rangle$

More photons…

M.A. Rubin and S. Kaushik, Phys. Rev. A **75**, 053805 (2007)
G. Gilbert, M. Hamrick, and Y.S. Weinstein, J. Opt. Soc. Am. B **25**, 1336 (2008)

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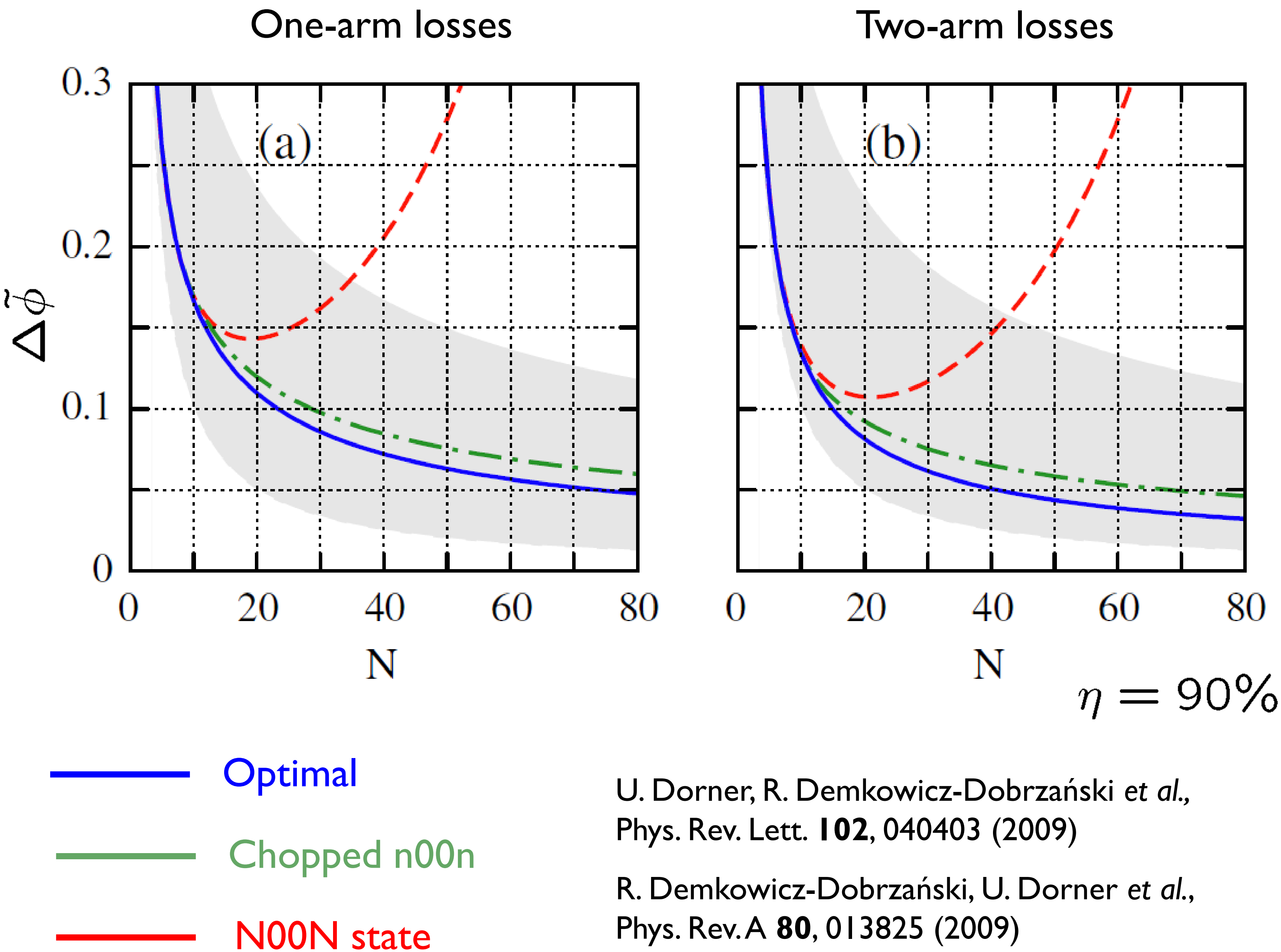
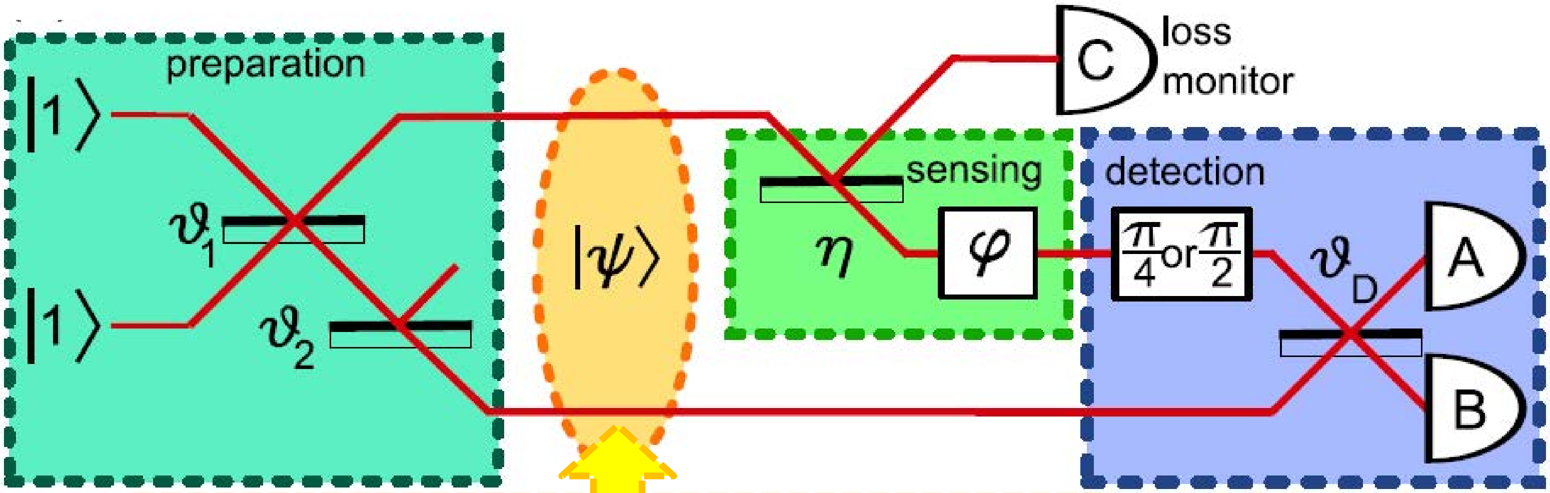
Numerical optimisation



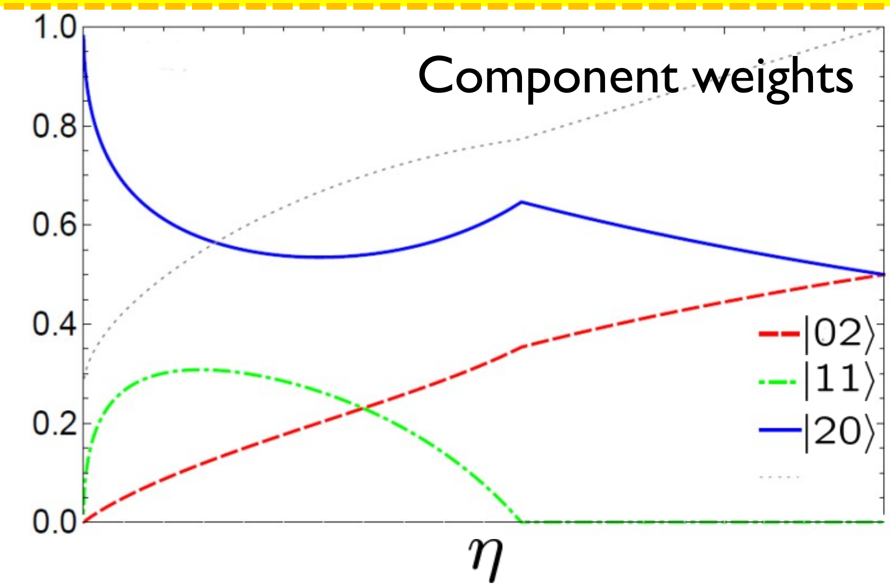
U. Dorner, R. Demkowicz-Dobrzański *et al.*, Phys. Rev. Lett. **102**, 040403 (2009)

R. Demkowicz-Dobrzański, U. Dorner *et al.*, Phys. Rev. A **80**, 013825 (2009)

Two-photon experiment

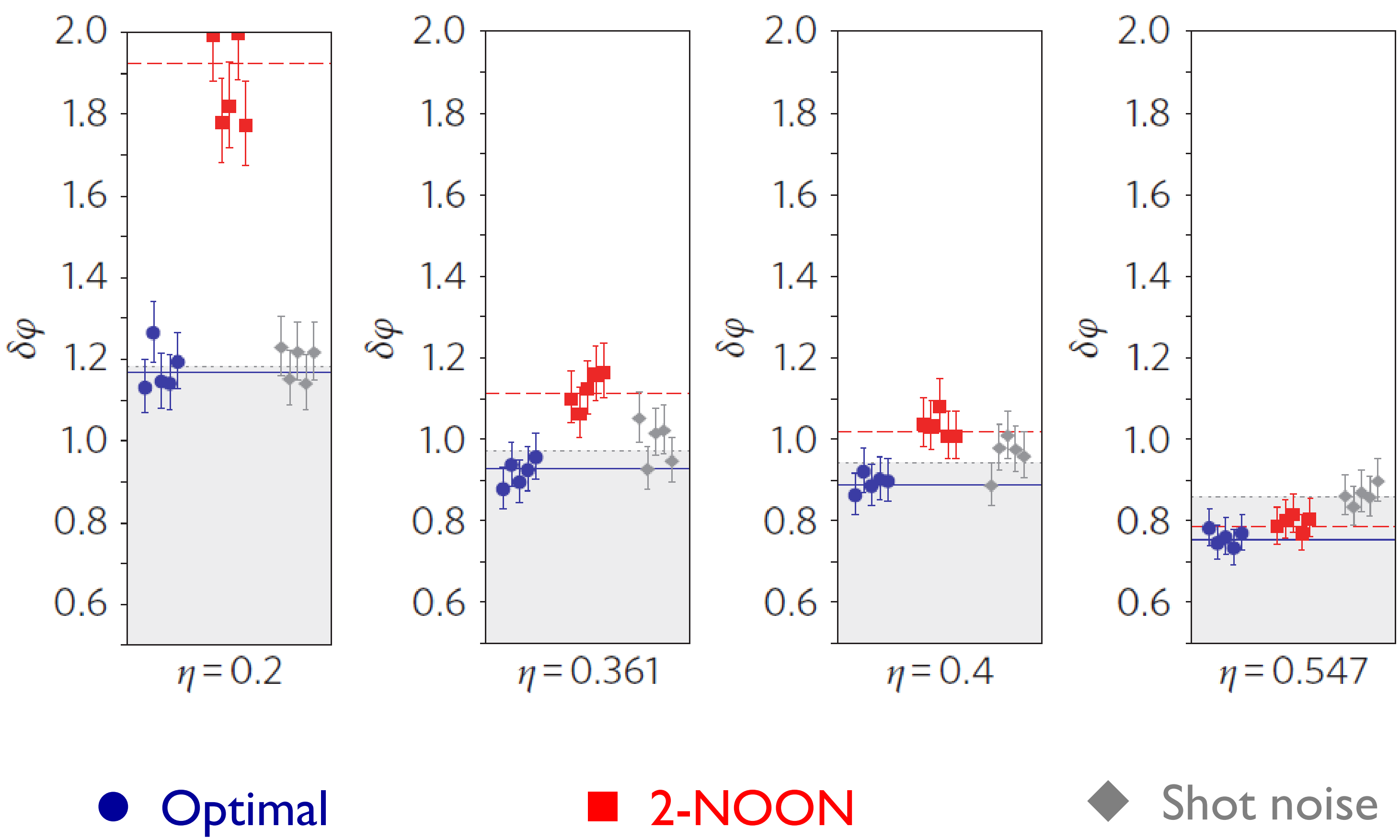


$$\alpha|20\rangle + \beta|11\rangle + \gamma|02\rangle$$

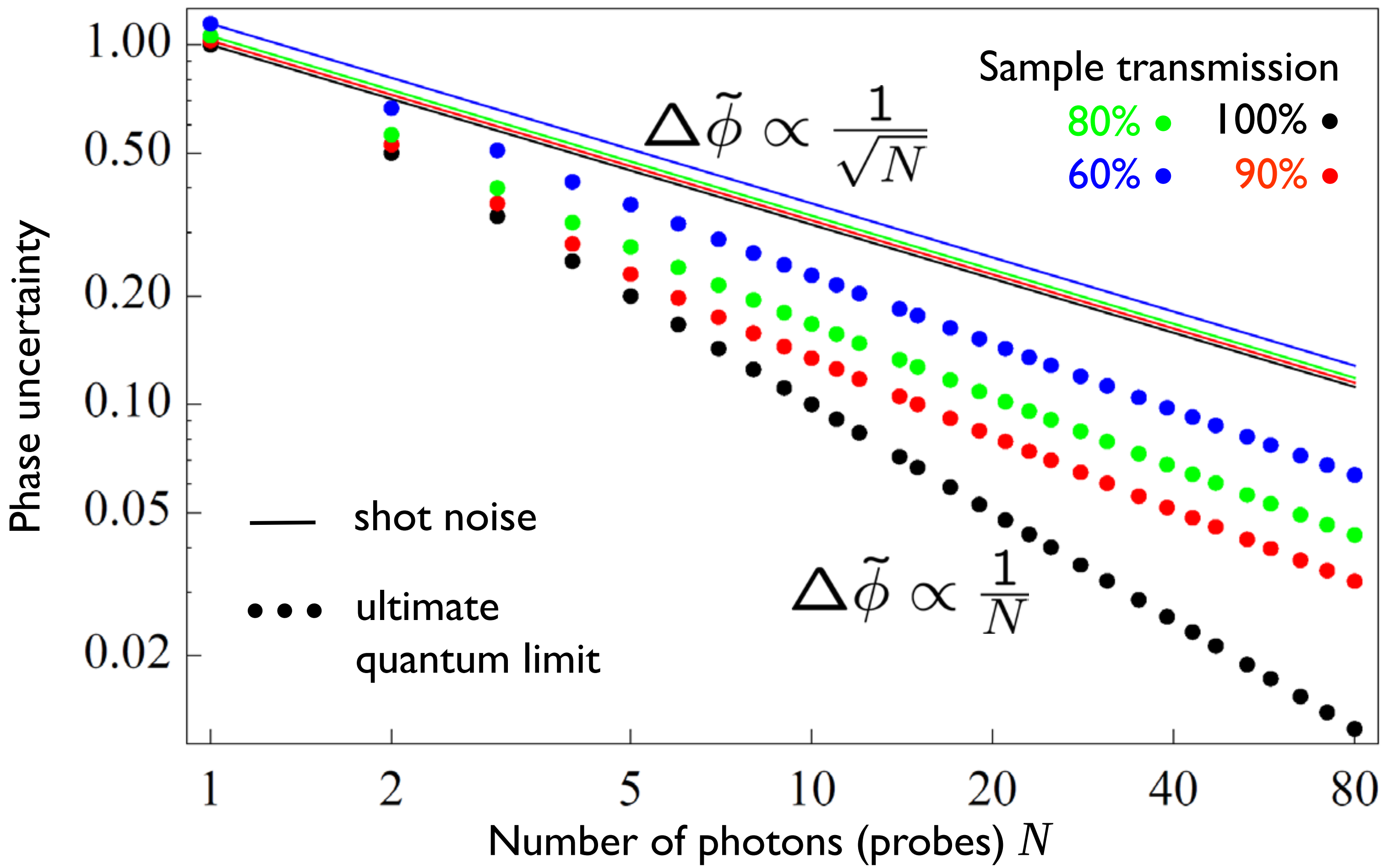


Phase uncertainty

M. Kacprowicz et al., Nature Photon. **4**, 357 (2010)



Scaling



K.Banaszek, R. Demkowicz-Dobrzański, and I. Walmsley,
Nature Photon. **3**, 673 (2009)

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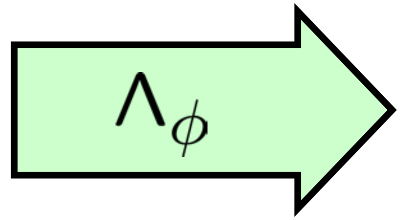
General picture

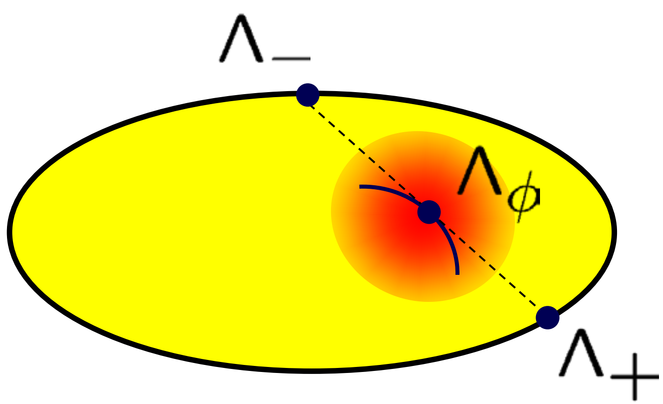
R. Demkowicz-Dobrzański, J. Kołodyński, and M. Guţă, Nature Commun. **3**, 1063 (2012)

Actual value ϕ



$$\hat{\varrho}_\phi = \Lambda_\phi\big(|\psi\rangle\langle\psi|\big)$$

$$\Delta\tilde{\phi} \geq \frac{\text{const}}{\sqrt{N}}$$



$$\Lambda_\phi \approx p_+(\phi)\Lambda_+ + p_-(\phi)\Lambda_-$$

Table 1 | Precision bounds of the most relevant models in quantum-enhanced metrology.

Channel considered	Classical simulation	Channel extension
Depolarisation	$\sqrt{(1-\eta)(1+3\eta)/4\eta^2}$	$\sqrt{(1-\eta)(1+2\eta)/2\eta^2}$
Dephasing	$\sqrt{1-\eta^2}/\eta$	$\sqrt{1-\eta^2}/\eta$
Spontaneous emission	NA	$(1/2)\sqrt{1-\eta}/\eta$
Lossy interferometer	NA	$\sqrt{1-\eta}/\eta$

NA, not available.
The bounds are derived using the two methods discussed in the paper. All the bounds are of the form $\Delta\varphi_N \geq (\text{const}/\sqrt{N})$, where constant factors are given in the table. Classical simulation method does not provide bounds for spontaneous emission and lossy interferometer, as these channels are φ-extremal. For the dephasing model, it surprisingly yields an equally tight bound as the more powerful channel extension method.

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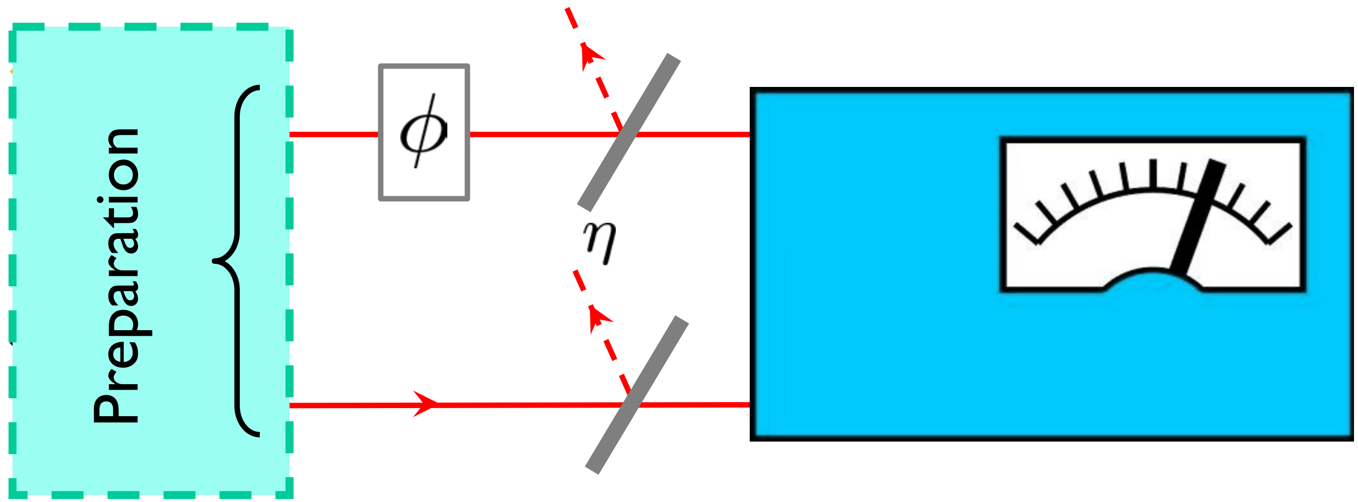
Two-arm losses



For a quantum state with $\langle N \rangle$ average photon number

Shot noise limit

$$\Delta\tilde{\phi} \geq \frac{1}{\sqrt{\eta\langle N\rangle}}$$

Ultimate quantum limit

$$\Delta\tilde{\phi} \geq \sqrt{\frac{1-\eta}{\eta\langle N\rangle}}$$

*Assuming no external phase reference is available:

M. Jarzyna and R. Demkowicz-Dobrzański, Phys. Rev. A **85**, 011801(R) (2012)

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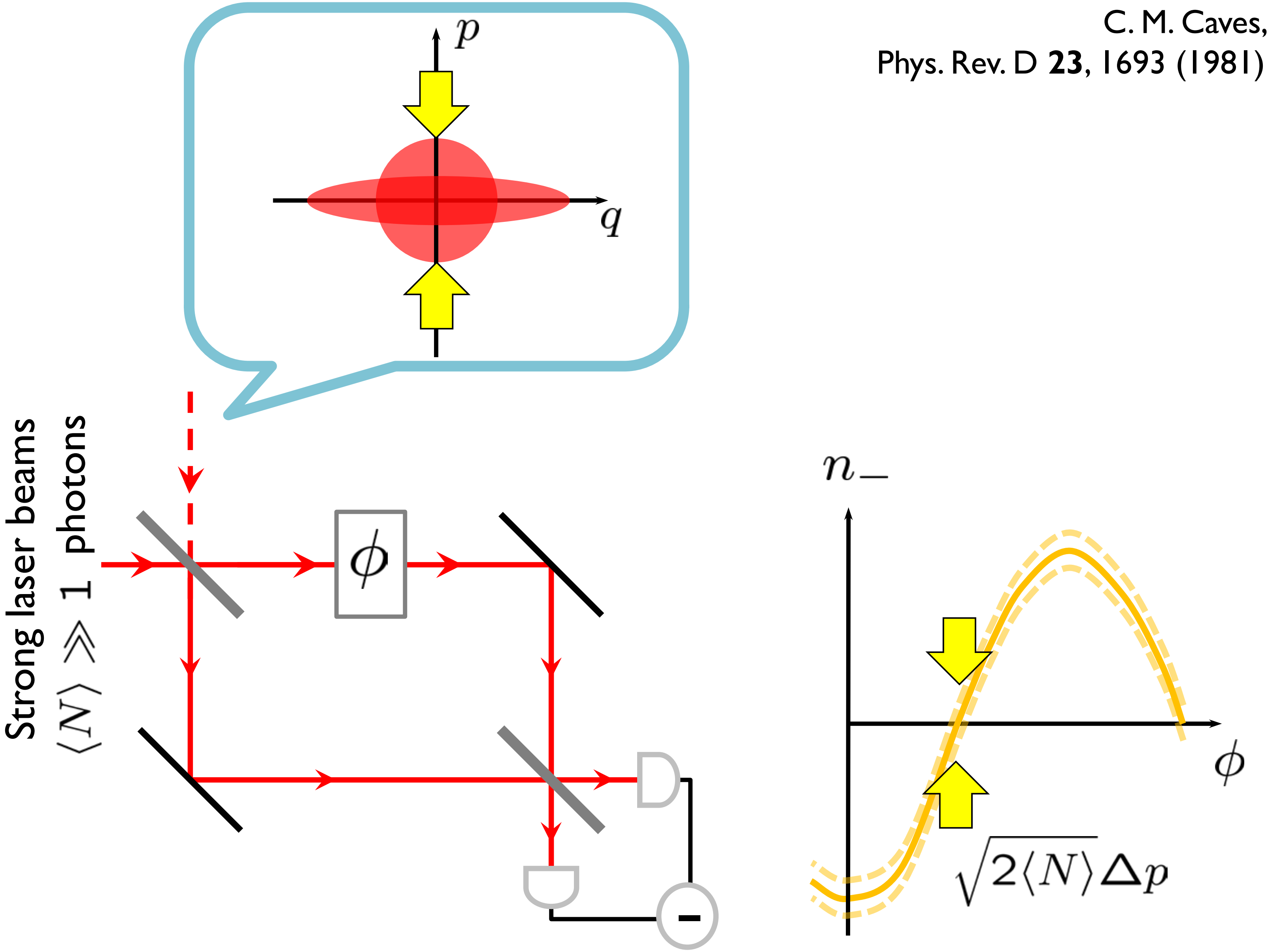
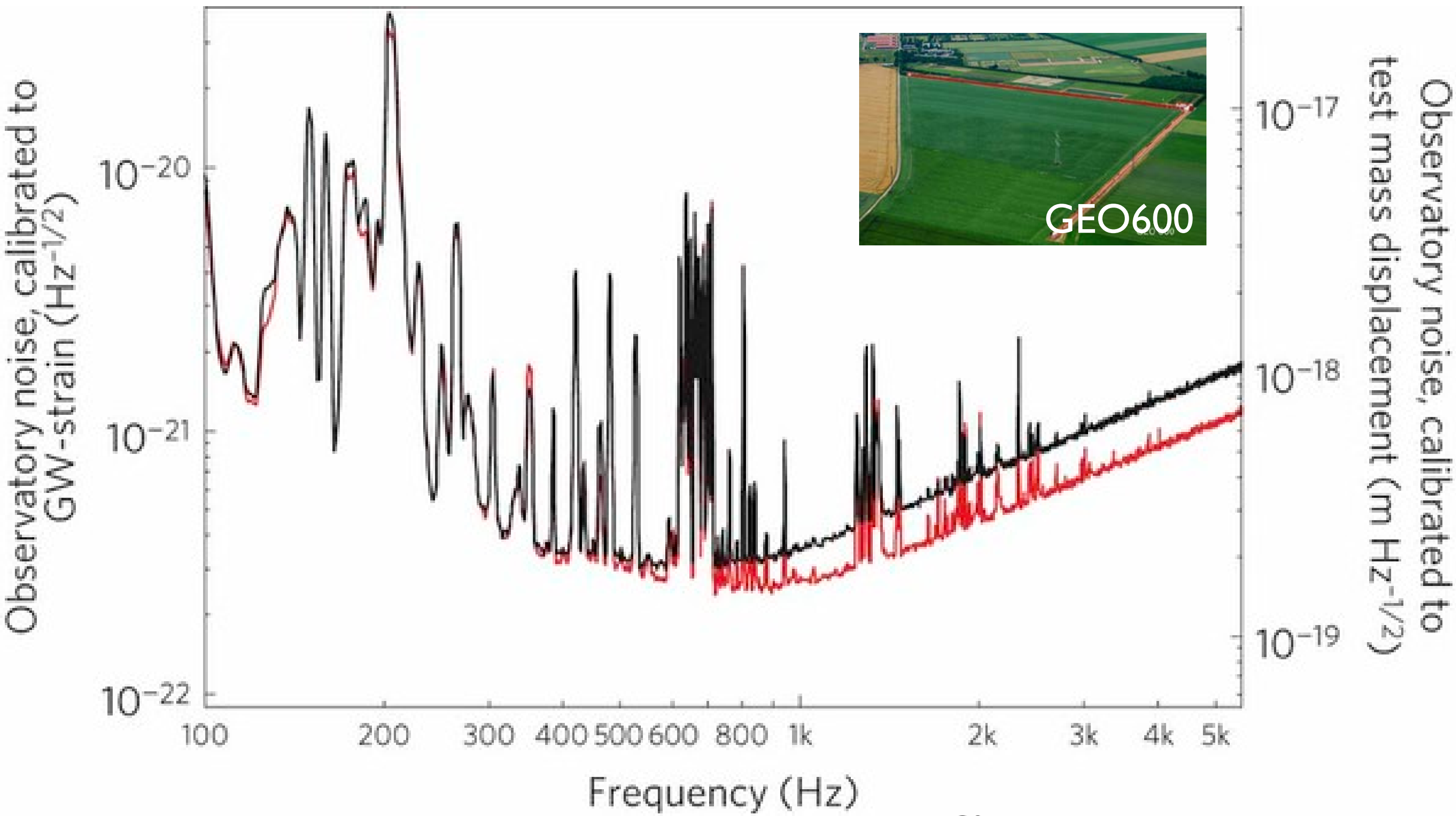
Shot noise revisited

C. M. Caves,
Phys. Rev. D **23**, 1693 (1981)



Gravitational wave detection

J. Abadie *et al.* (The LIGO Scientific Collaboration), Nature Phys. **7**, 962 (2011)



$$\frac{\Delta\tilde{\phi}_{\text{squeezed}}}{\Delta\tilde{\phi}_{\text{shot noise}}} \approx 0.66$$

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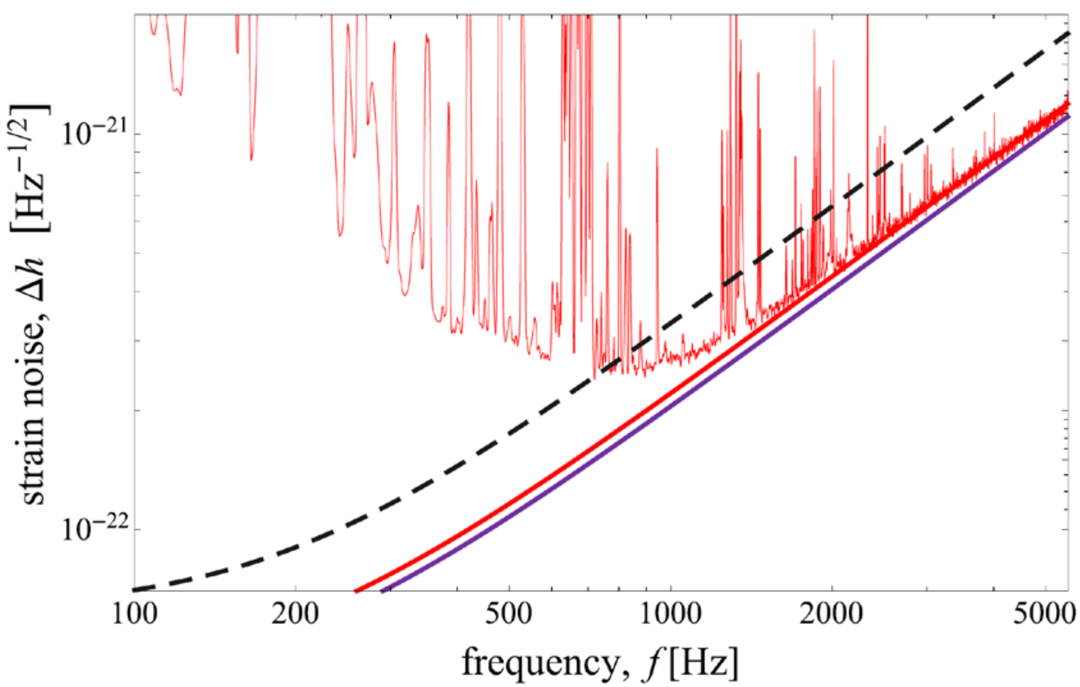
Noise analysis

R. Demkowicz-Dobrzański, K. Banaszek, and R. Schnabel, Phys. Rev. A **88**, 041802(R) (2013)

When most power comes from the laser beam

$$\Delta\tilde{\phi} \approx \sqrt{\frac{1-\eta+2\eta(\Delta p)^2}{\eta\langle N\rangle}}$$



Shot noise limit

10dB squeezing (implemented)

16dB squeezing and ultimate bound

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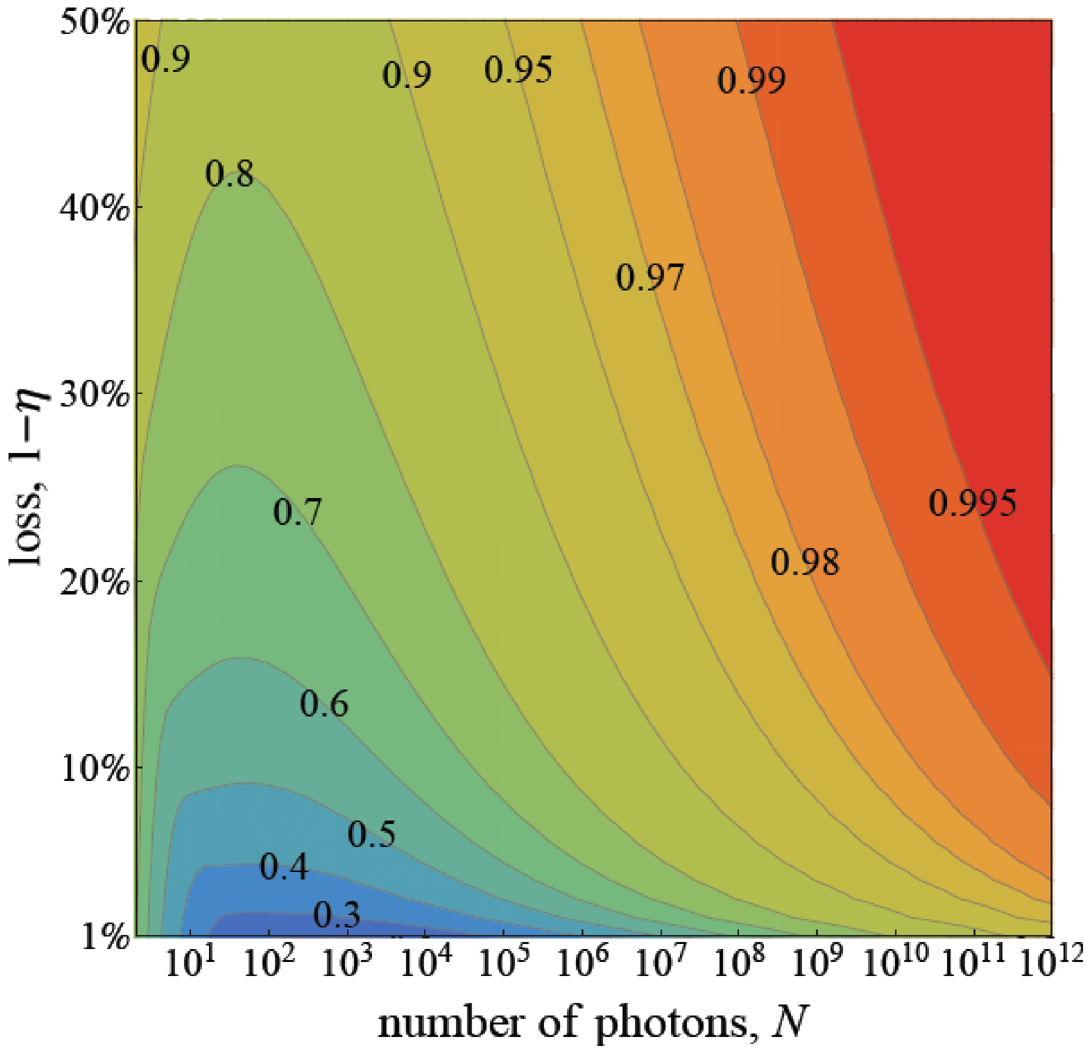
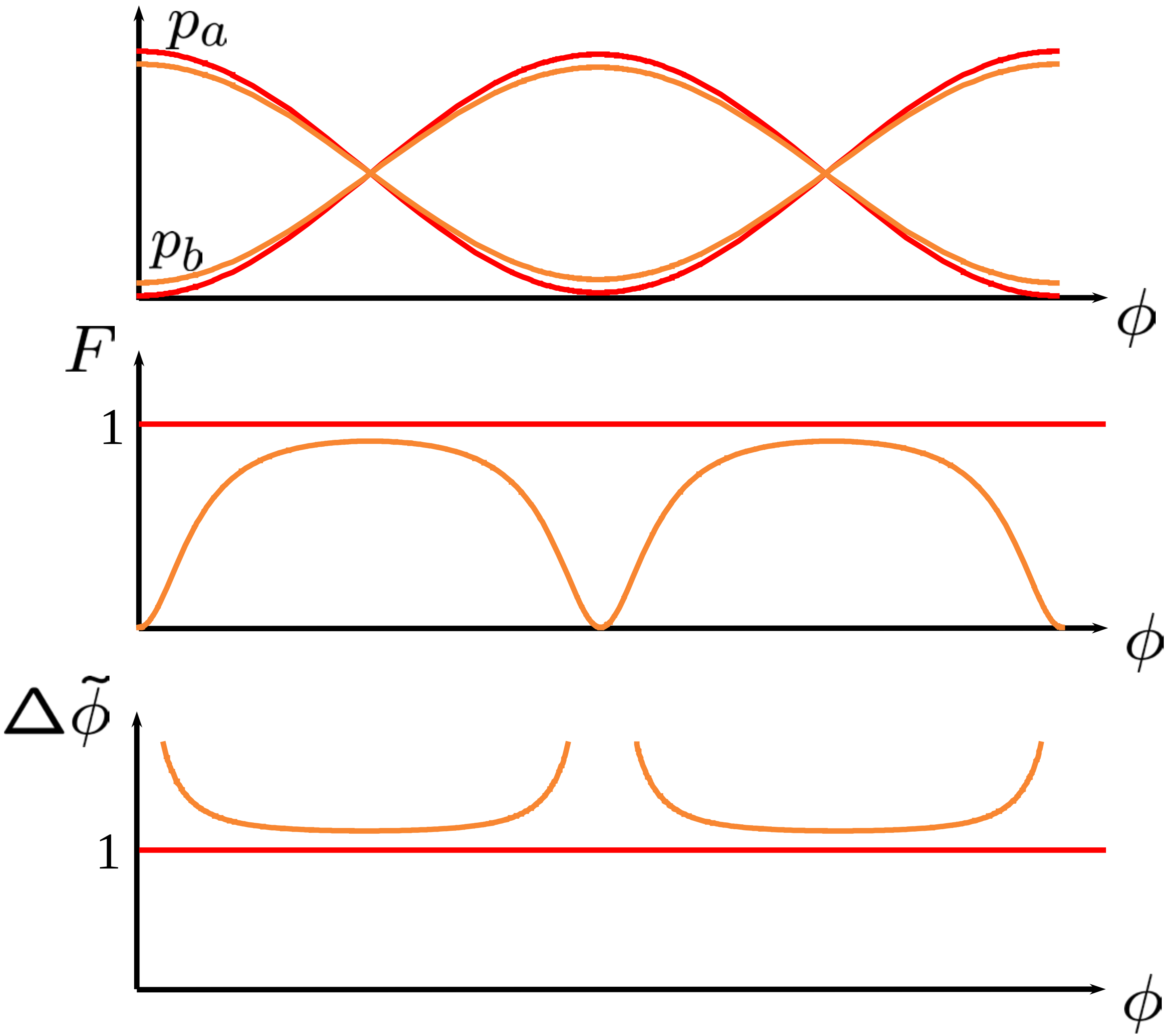
Optimality of squeezed states

R. Demkowicz-Dobrzański, K. Banaszek, and R. Schnabel, Phys. Rev. A **88**, 041802(R) (2013)



$$\frac{\Delta\tilde{\phi}_{\text{optimal}}}{\Delta\tilde{\phi}_{\text{squeezed}}}$$

Operating point



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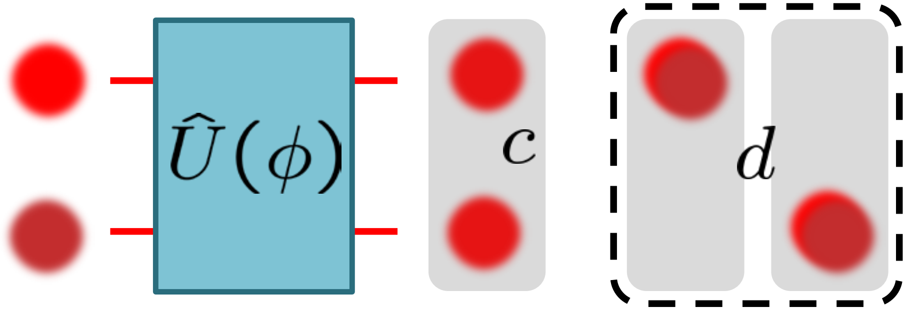
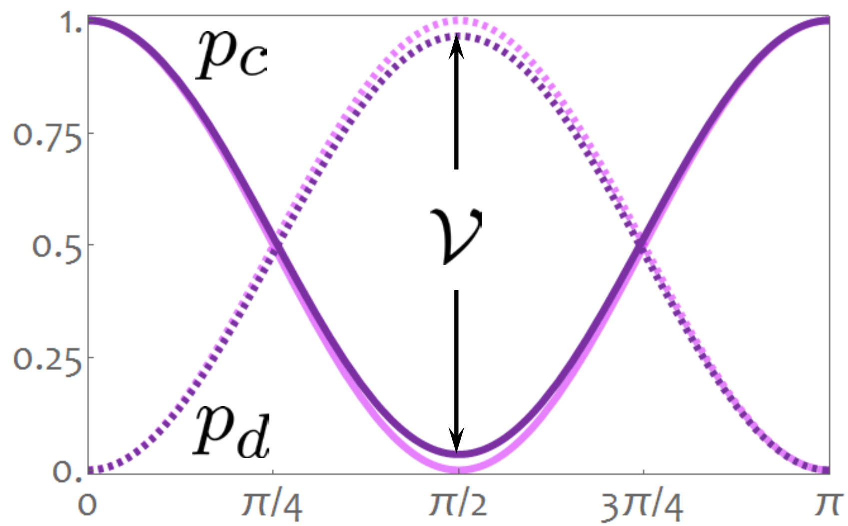
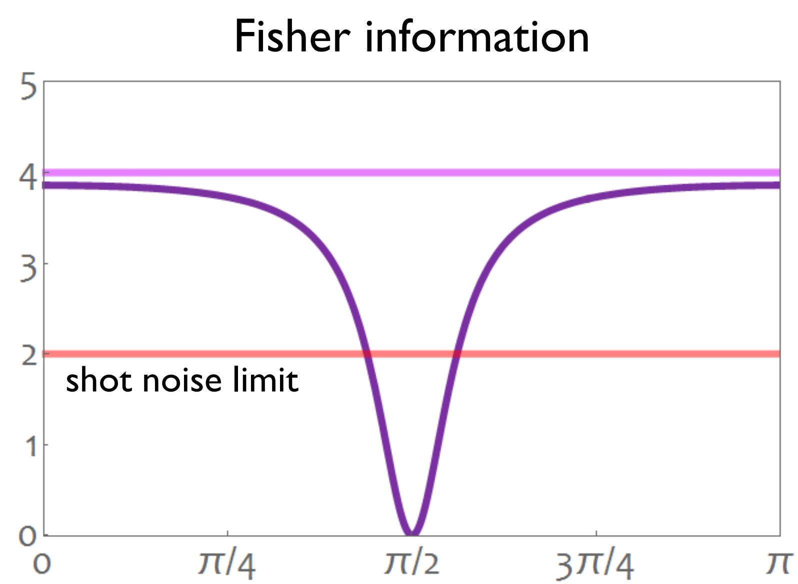
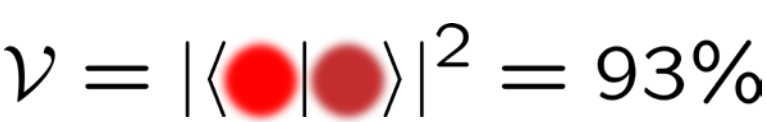
Partial spectral distinguishability









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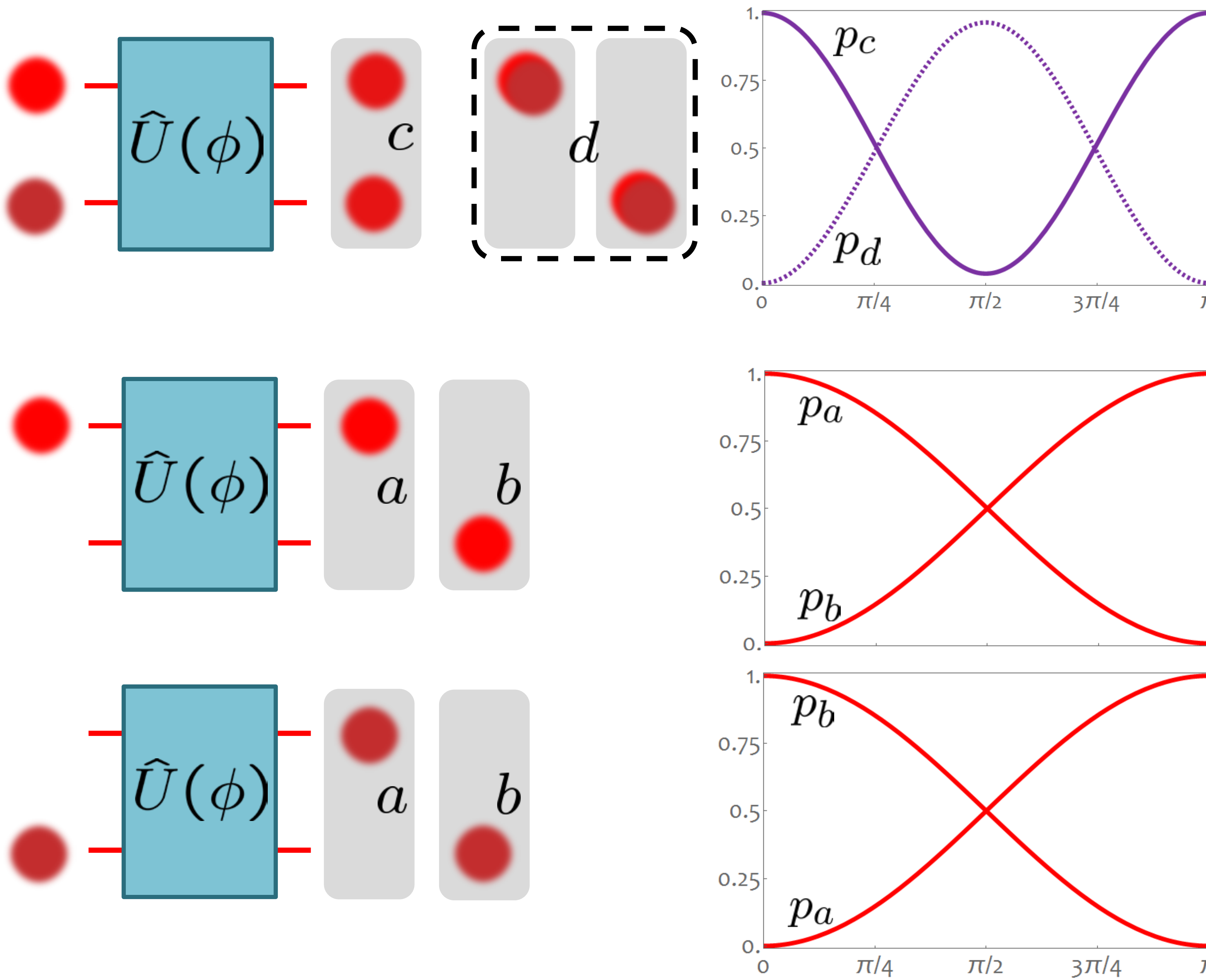
One- and two-photon interference



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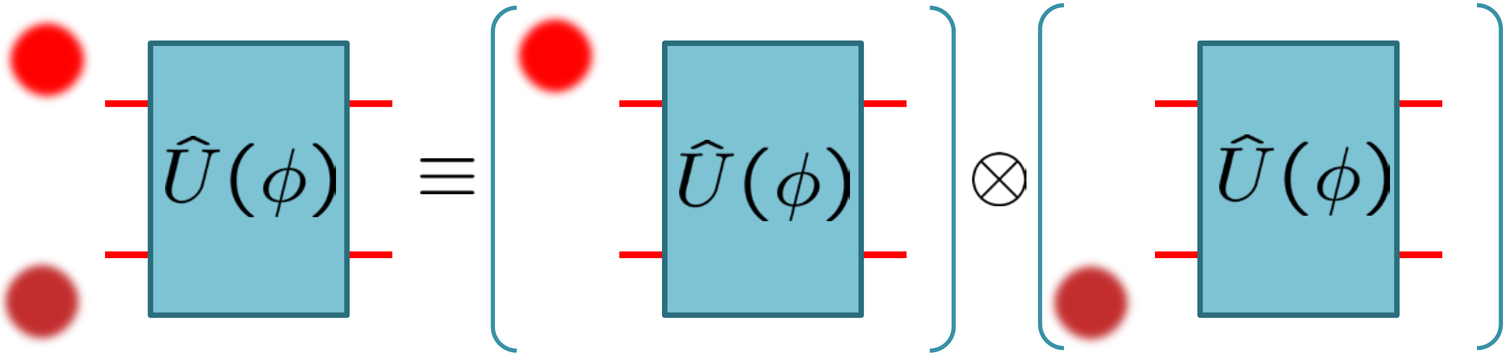
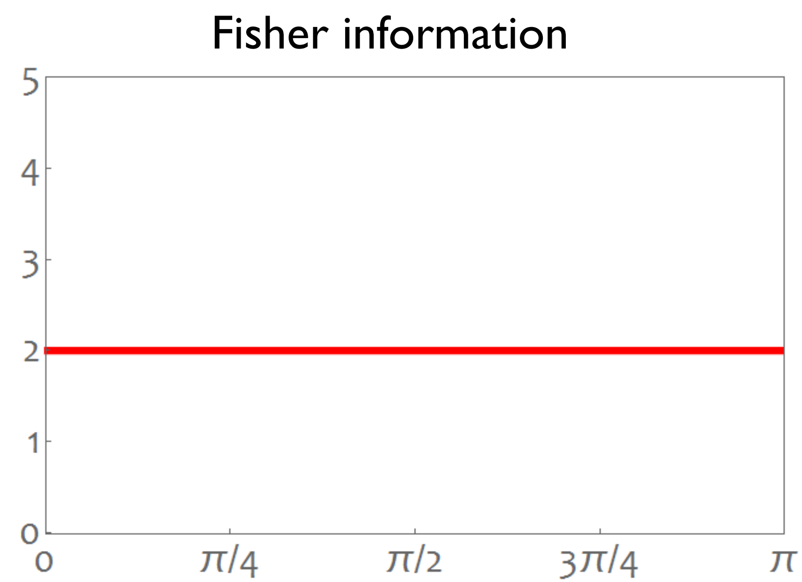
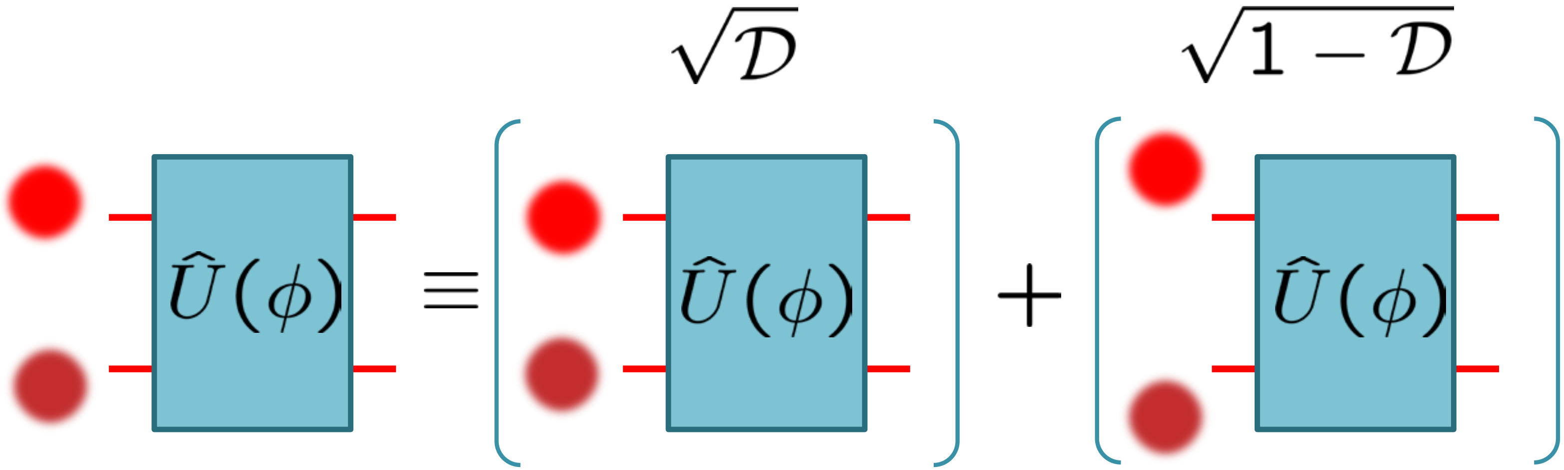
Transverse displacement



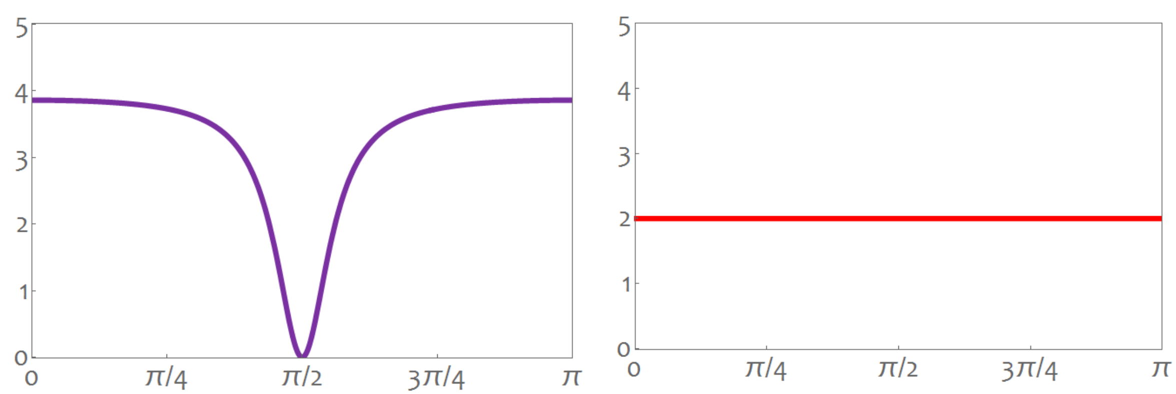


Partial transverse overlap

Coherent superposition



Fisher information



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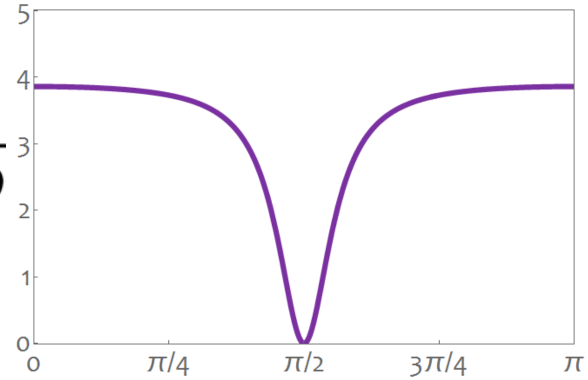
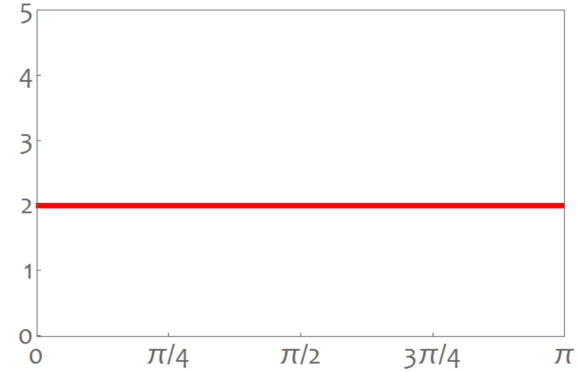
Coherent superposition

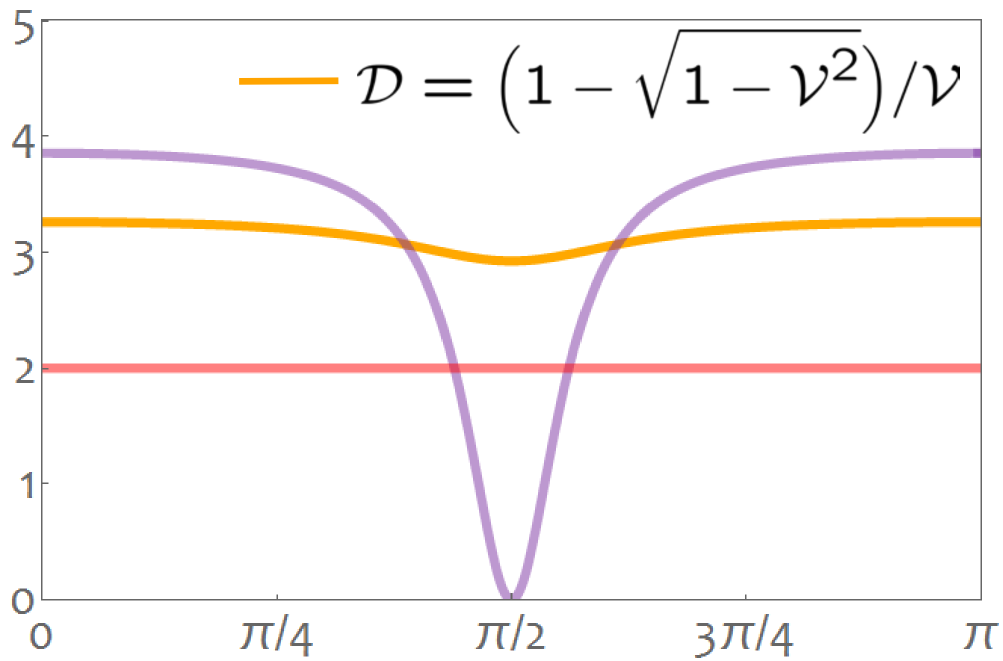
$\sqrt{\mathcal{D}}$



$+\sqrt{1-\mathcal{D}}$

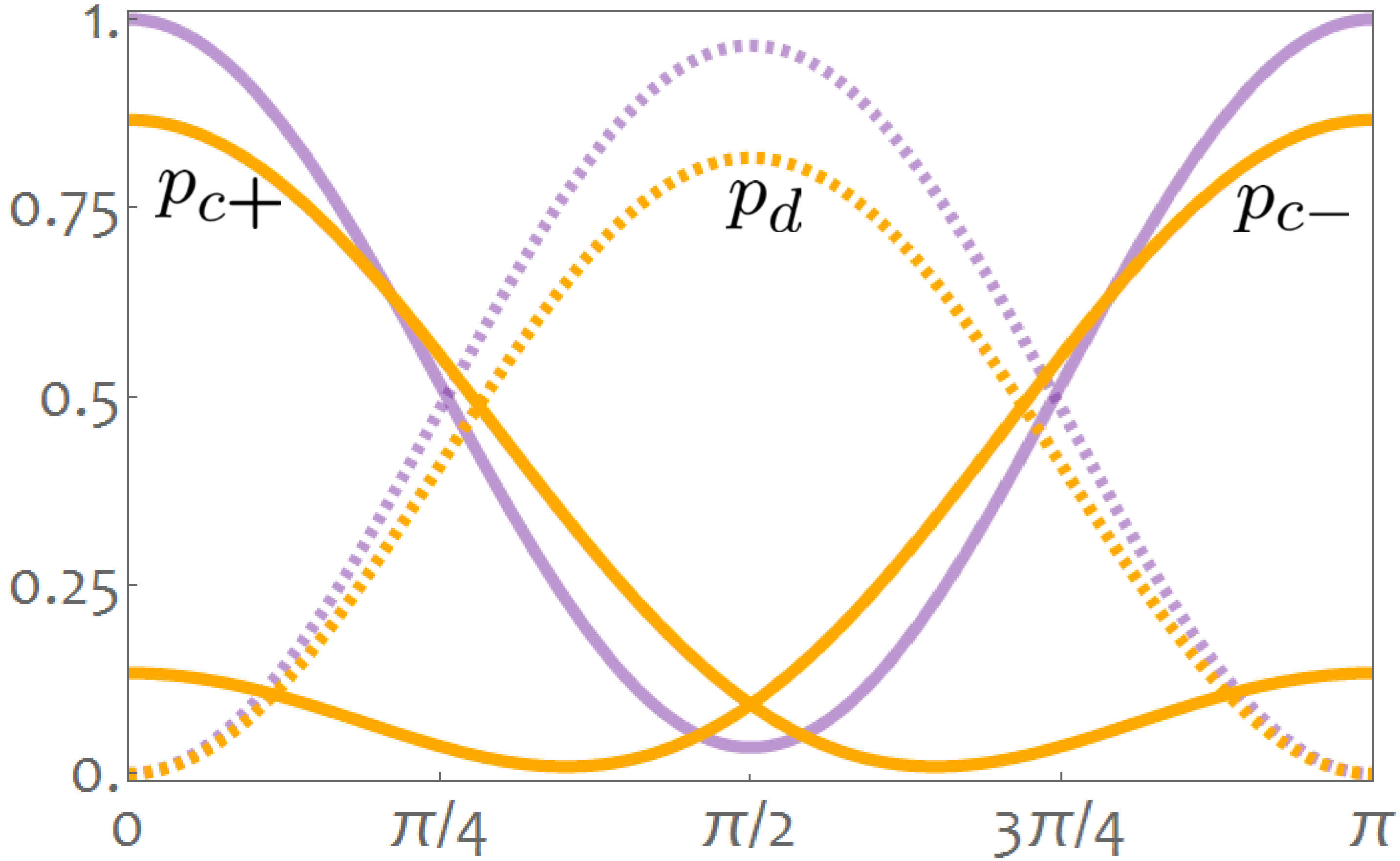


$=$



No postselection or any attempt to resolve the spectral degree of freedom inducing $\mathcal{V} < 1$!!!!!

Optimal measurement



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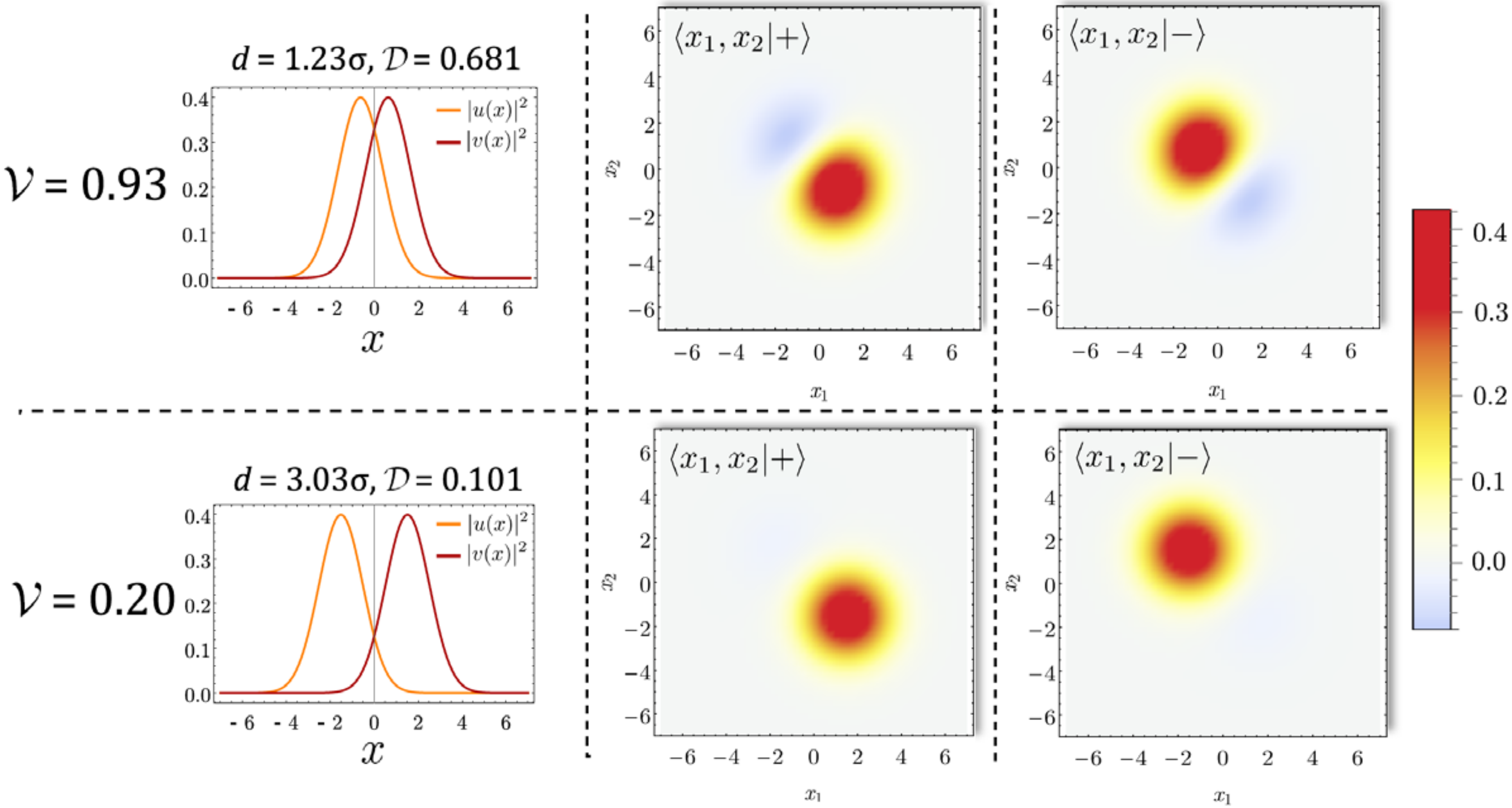
Projection basis

Spatial modes

Optimal two-photon projections $|\pm\rangle$



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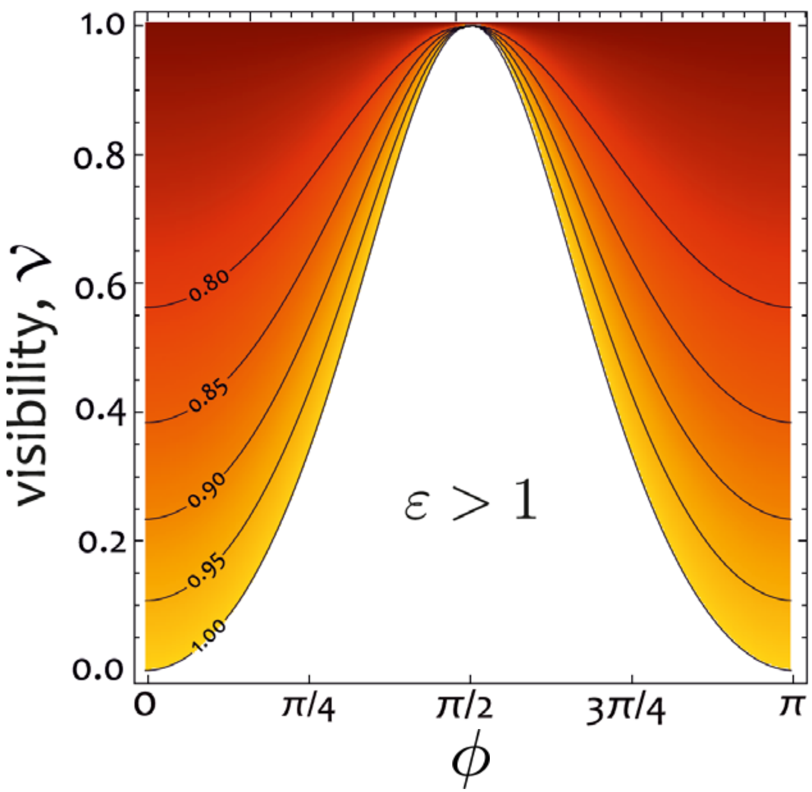
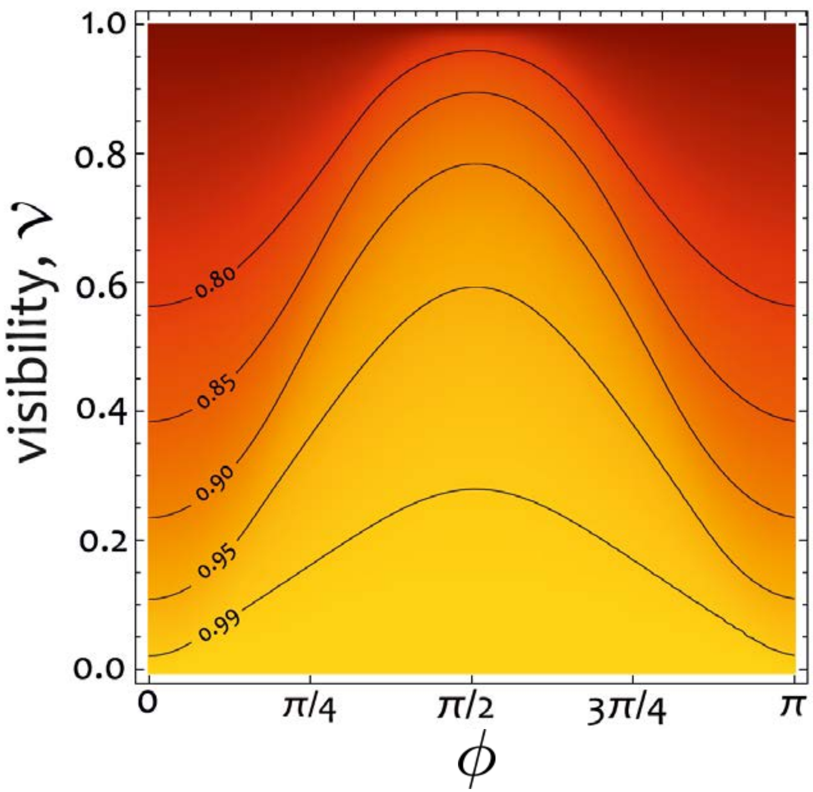
Enhancement

Relative uncertainty $\varepsilon = \Delta^{\text{pair}} / \Delta^{\text{shot}}$



No spatial displacement



Spatial overlap optimized for individual operating point

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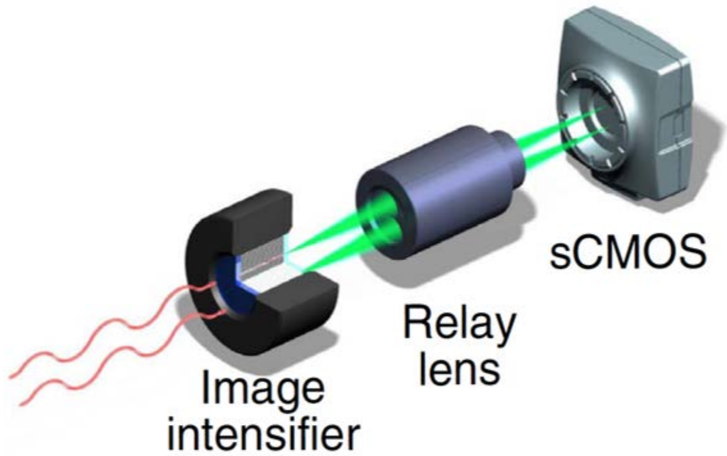
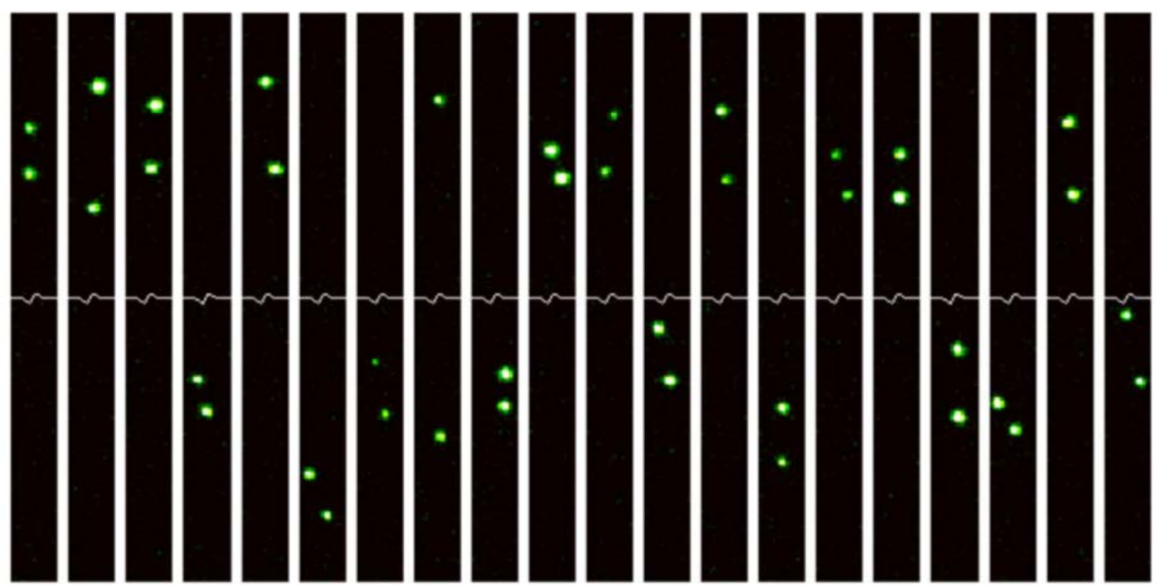
Shot-by-shot imaging



R. Chrapkiewicz, W. Wasilewski, and K.Banaszek, Opt. Lett. **39**, 5090 (2014)

M. Jachura and R. Chrapkiewicz, Opt. Lett. **40**, 1540 (2015)



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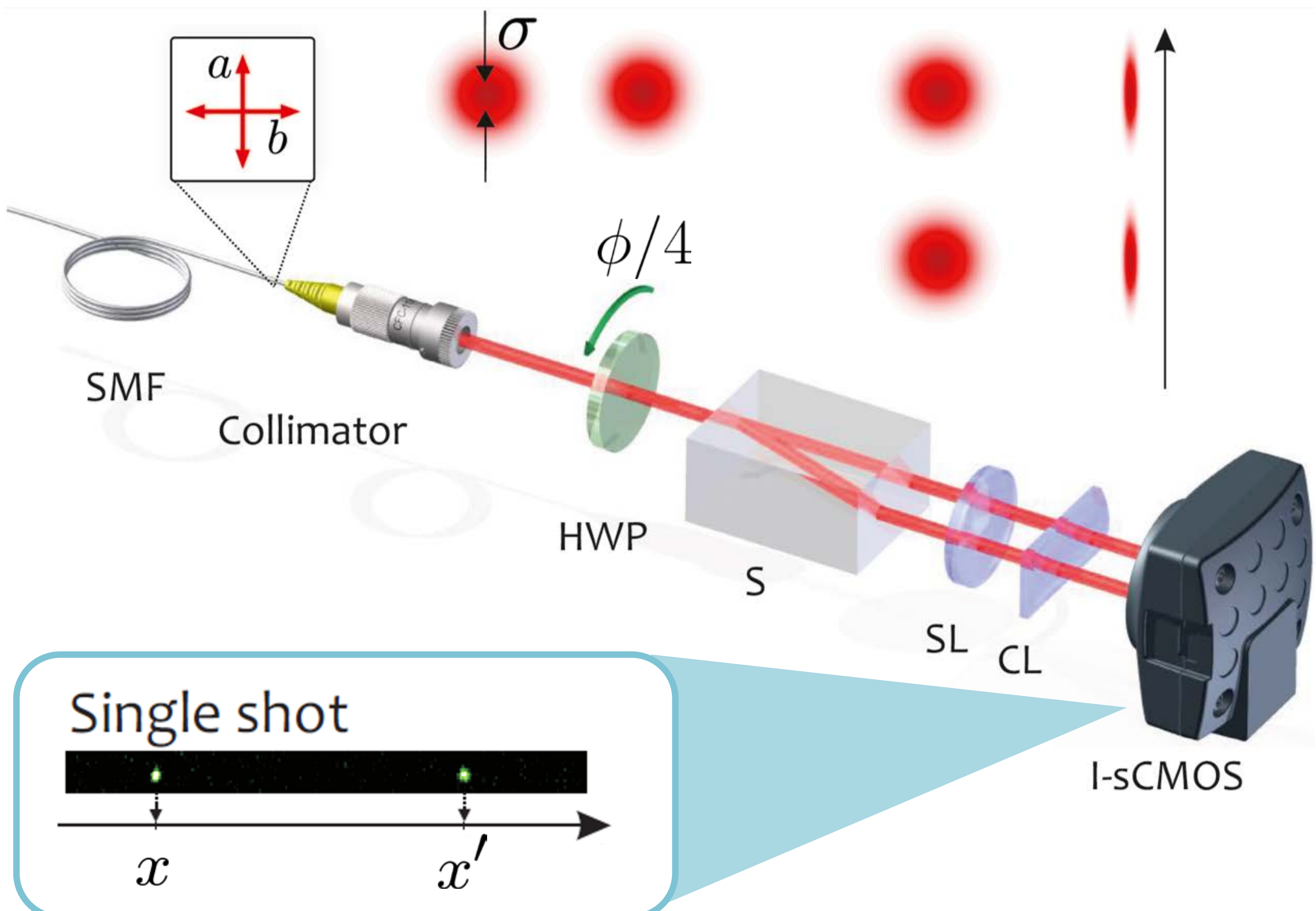
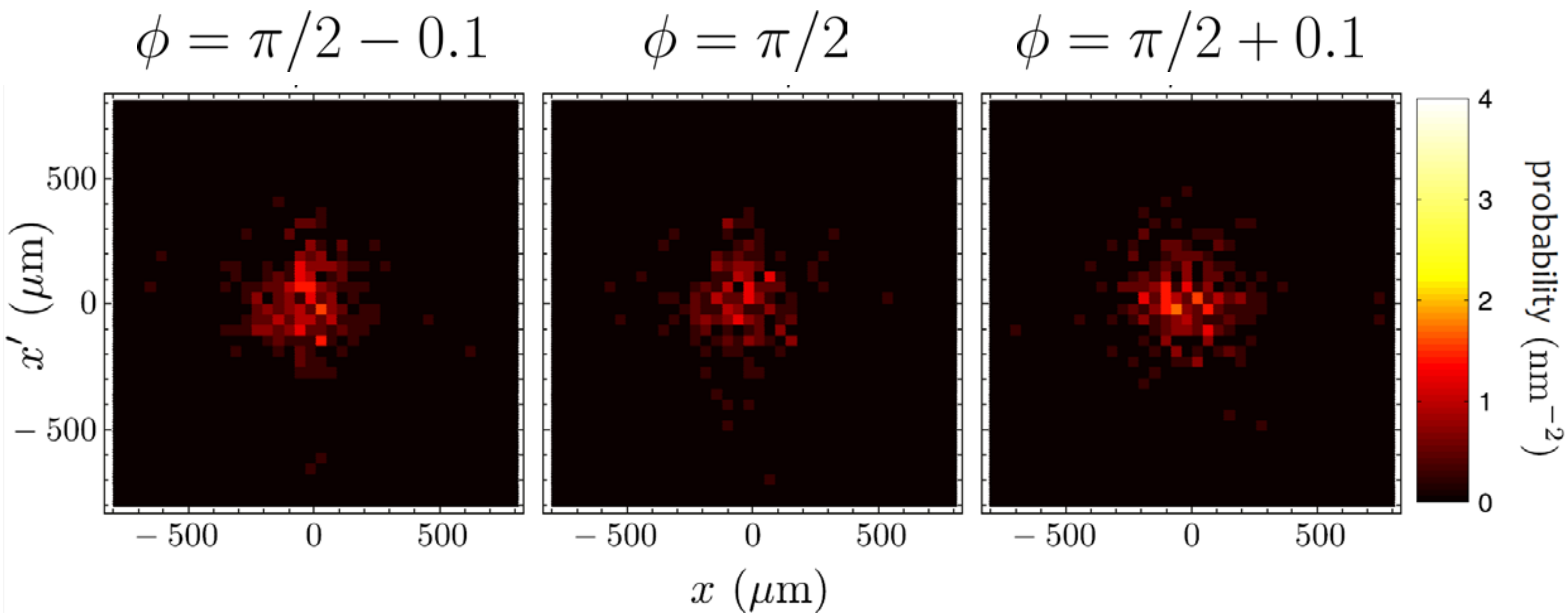
Imaging experiment



Coincidence events



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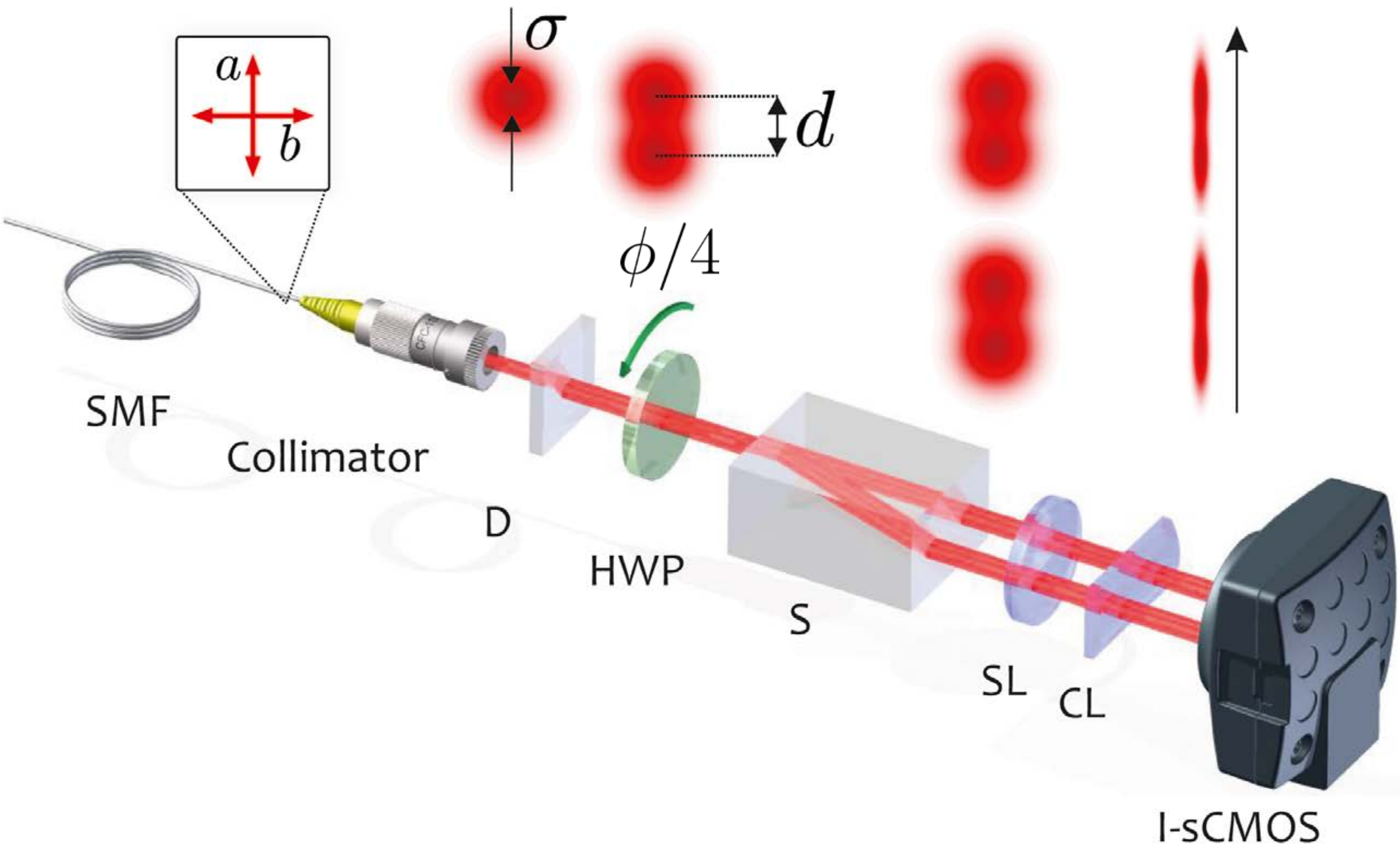
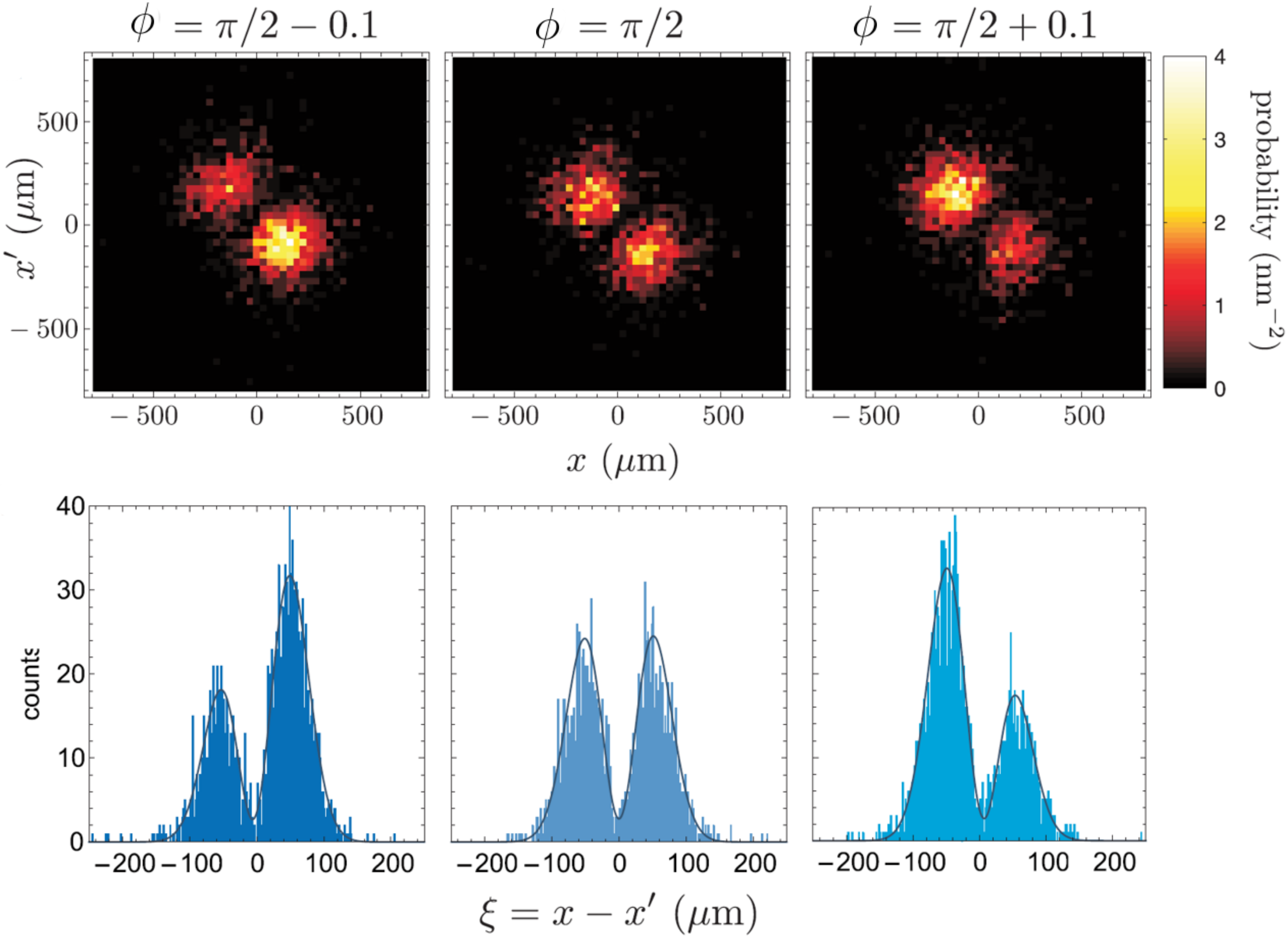
Transverse displacement



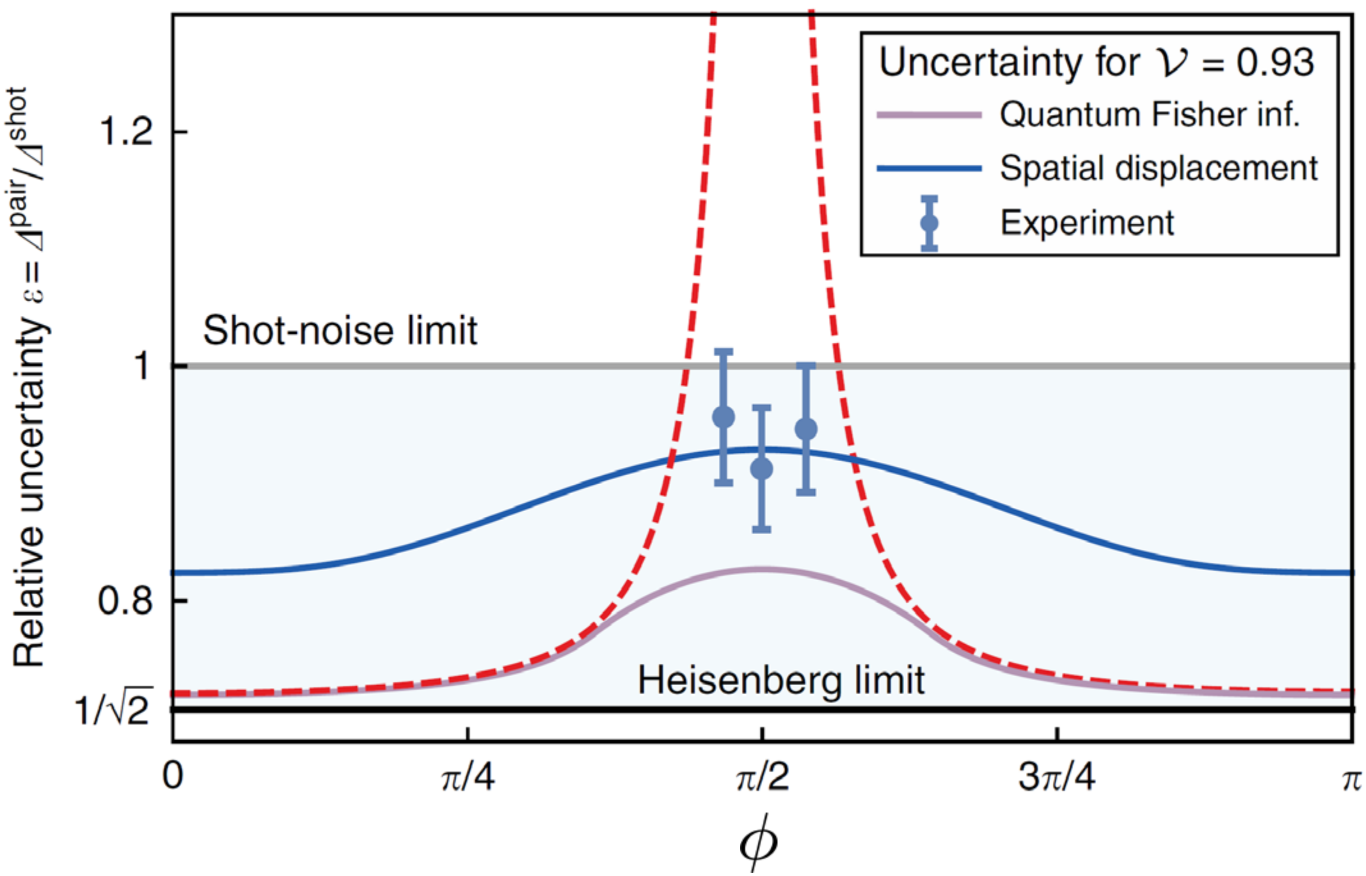
Coincidence events

M. Jachura *et al.*, Nature Commun. **7**, 11411 (2016)



Relative uncertainty

M. Jachura *et al.*, Nature Commun. **7**, 11411 (2016)



$d/\sigma = 1.64$

spatial displacement

locally optimized $\mathcal{D}$

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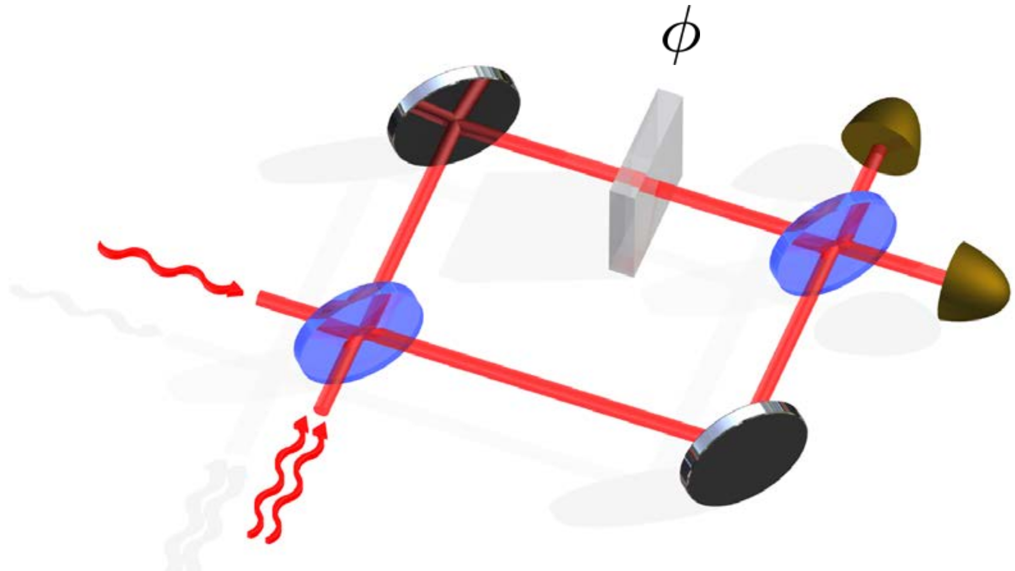
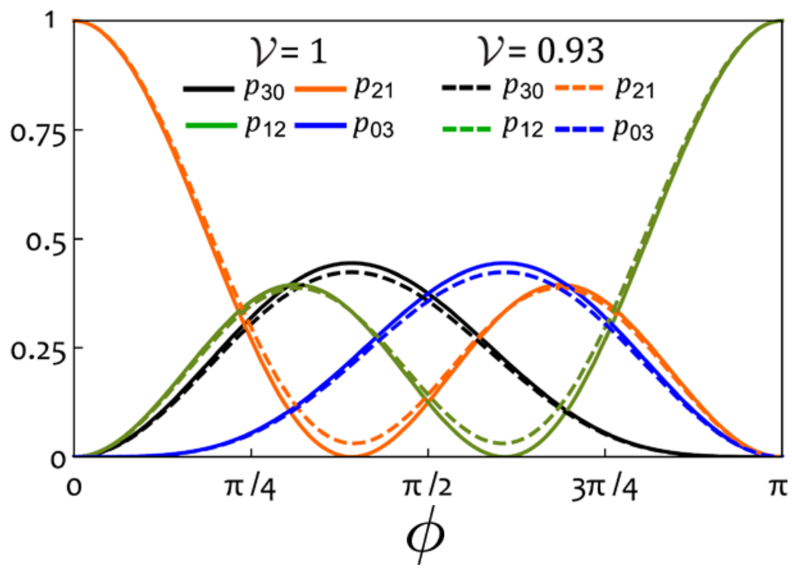
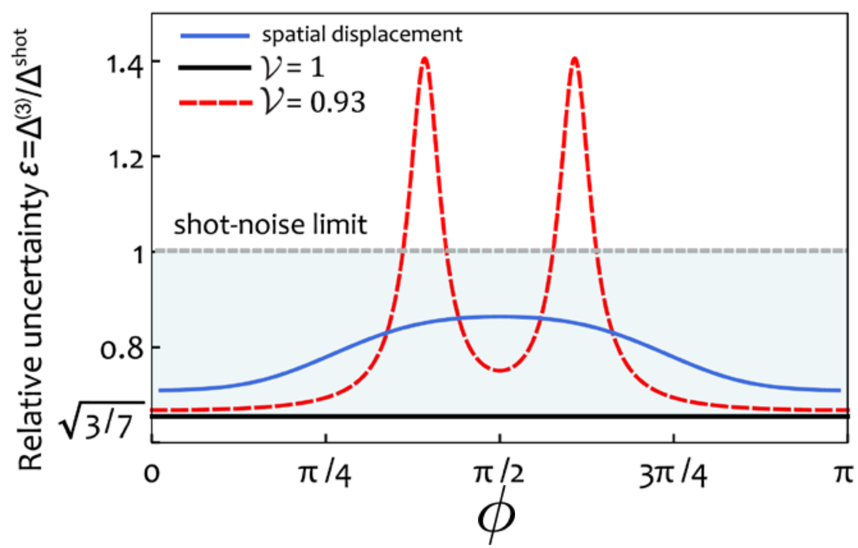
2 + 1 photons

M. Jachura *et al.*, Nature Commun. **7**, 11411 (2016)







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Conclusions

- Benefit analysis of quantum metrology needs to take into account noise and imperfections
- Even in noisy scenarios quantum enhancement is possible – and worthwhile!
- (Nearly) optimal operation can be achieved with (relatively) modest means
- Applications where fixed-scale enhancement is useful / critical
- Qubits live in a vast physical space – explore!

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Acknowledgements

Radosław Chrapkiewicz
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Michał Jachura
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Marcin Kacprowicz
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