Topological order in Mott insulators and dimer models

Grégoire Misguich
Institut de Physique Théorique
C.E.A. Saclay
France
http://ipht.cea.fr/Images/Pisp/gmisguich

Collaborators:

Shunsuke Furukawa	RIKEN, Tokyo
Frédéric Mila	EPFL, Lausanne
Masaki Oshikawa	ISSP, University of Tokyo
Vincent Pasquier	SPhT, CEA Saclay

Symposium on TQC, Dublin 15-17 Sept. 2008

Outline

- The Lieb-Shultz-Mattis (D=1, 1961)-Hastings (D>1, 2004) theorem:

  Mott insulators with a gap should be either
  - conventionally ordered,
  - or topologically ordered

- Heisenberg (« RVB ») spin liquids and Kitaev's toric code

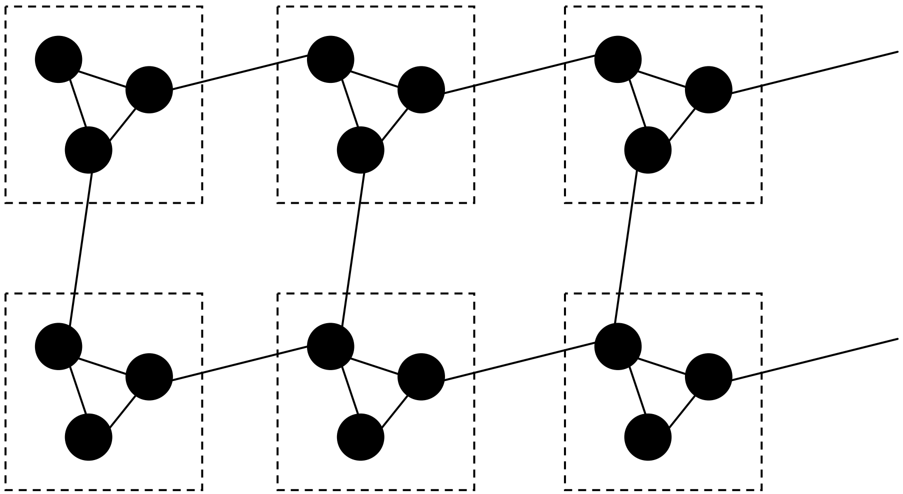
- Quantum dimer models as toy models for (Z2) topological order: Entanglement and vortex excitations

GM, "Les Houches" lectures notes, arXiv '08092257

Part I

The Lieb-Schultz-Mattis-Hastings theorem

What is a Mott insulator ?



- Electrons hopping on a lattice
  + **odd** number of electrons per unit cell.

(would be a metal according to non-interacting band theory)
Consider a *half-filled* system for simplicity.

$$H = \sum_{\substack{i<j \\ \sigma=\uparrow,\downarrow}} t_{ij} \left(c^{+}_{i\sigma} c_{j\sigma} + H.c. \right)$$

- Strong on-site electron-electron repulsion (=Hubbard model)

→ insulator (charge excitations are gapped)

$$+ U \sum_{i} c^{+}_{i\uparrow} c_{i\uparrow} c^{+}_{i\downarrow} c_{i\downarrow}$$

- In the limit of infinite repulsion

→ spin-1/2 Heisenberg model with an **odd** number of sites/cell

$$H = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad , \quad J_{ij} = 4 \frac{t_{ij}^{\;2}}{U}$$

The Lieb-Shutlz-Mattis theorem

D=1 Lieb-Schultz-Mattis (D=1: 1961)

D>1 Hastings [PRB 69, 104431 (2004)] (see also Affleck 1988; Bonesteel 1989; Oshikawa 2000, GM *et al.* 2002)

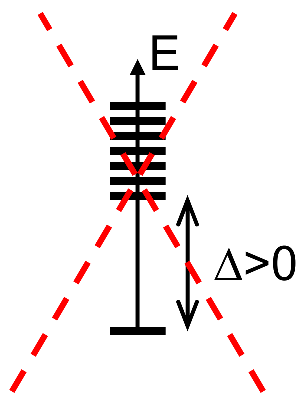
Nachtergaele & Sims, Com. Math. Phys. 276, 437 (2007).

"A system with a half-odd-integer spin in the unit cell

(+ periodic boundary conditions, $[S^z_{tot},H]=0$ & dimensions $L_1 \times L_2 \times \ldots \times L_D$ with $L_2 \times \ldots \times L_d$=odd)

cannot have simultaneously a gap and a unique ground-state

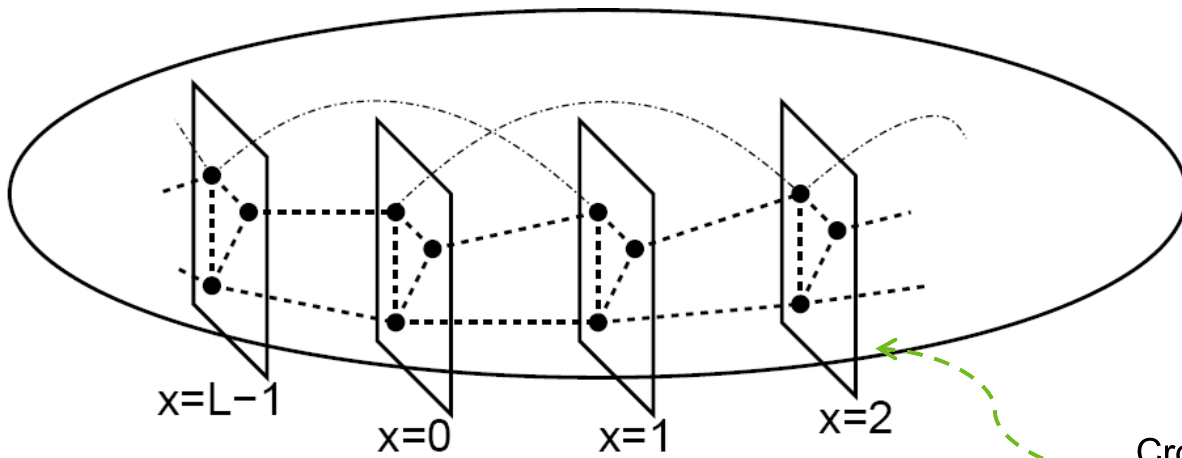
(in the thermodynamic limit)."



Gapped paramagnet
= forbidden

Oshikawa's argument (I)

Oshikawa, Phys. Rev. Lett. 84, 1535 (2000)



Represent the D-dimensional lattice as a periodic ring in the x direction

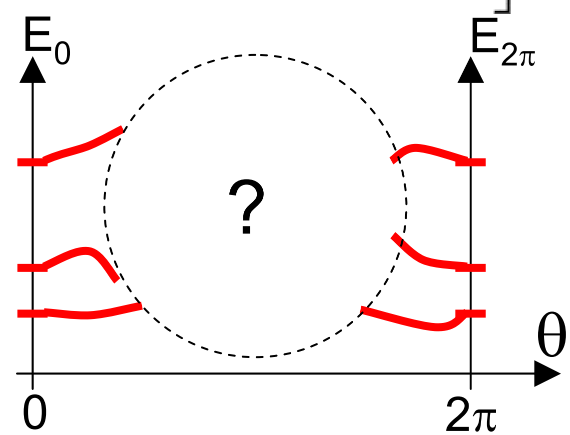
Cross section with C sites
Ex. in 2D: $C=n\,L_y$

Twisted Hamiltonian

$$H_\theta = \frac{1}{2}\sum_{ij} J_{ij}\left[S_i^z S_j^z + \frac{1}{2}\left(e^{i\theta(x_i-x_j)/L_x} S_i^+ S_j^- + \mathrm{H.c}\right)\right]$$

$$U = \prod_i \exp\left(2i\pi \frac{x_i}{L_x} S_i^z\right)$$

$$U H_0 U^{-1} = H_{2\pi}$$



Oshikawa's argument (II)

- H_θ is translation invariant $\Rightarrow$ k is well defined and indep. of θ
- U does generally not commute with translations :

$$U = \prod_i \exp\left(2i\pi \frac{x_i}{L_x} S_i^z\right)$$

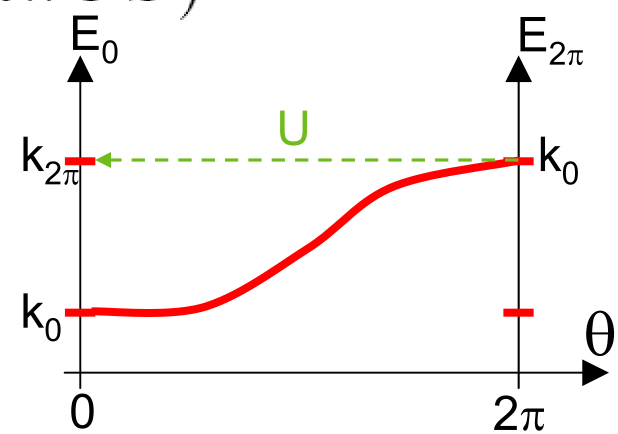
$$TU = UT \exp\left(2i\pi \frac{S^z_{\rm tot}}{L_x}\right) \exp\left(2i\pi C S\right)$$

x-translation

- Momentum shift :

Magnetization per site

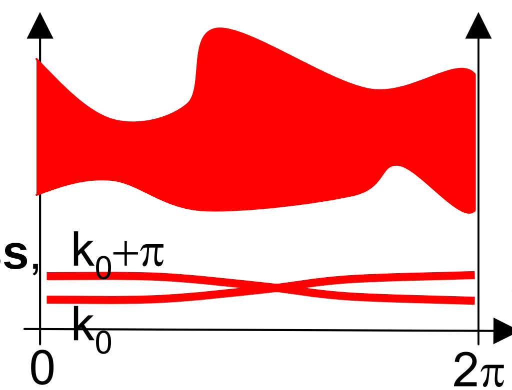
$$k_0 = k_{2\pi} + 2\pi C(S + m^z)$$

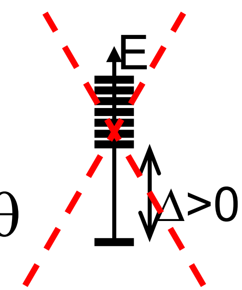


- Consider m^z=0, S=1/2 and C=odd (=n L_y in 2D):

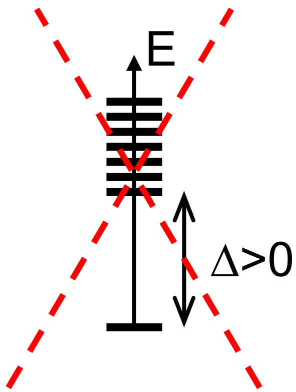
$k_0 = k_{2\pi} + \pi$

- If (we assume that) **the gap to excited states do not close during the adiabatic process**, *The ground state is (at least) two-fold degenerate*

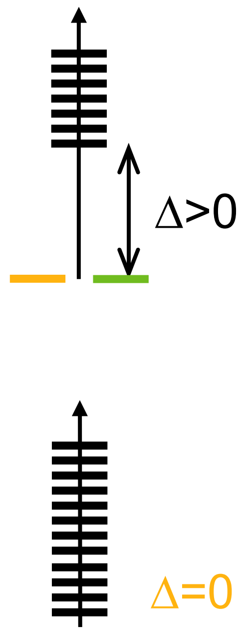




LSM-H, spontaneous symmetry breaking & topological order



Gapped paramagnet
= forbidden



1) Ground-state degeneracy

a- "Conventional" broken symmetry

b- Topological degeneracy

2) Gapless spectrum

a- Continuous broken sym. (Néel order)

b- Critical phase (or crit. point)

spin liquids with fractional excitations

Part II

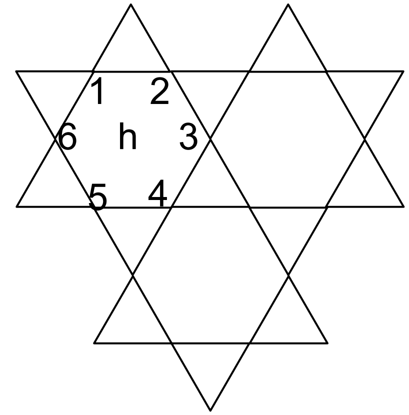
short-range resonating valence-bond (RVB) spin liquids & (Z_2) topological order

Some examples of topologically ordered spin model

Z_2 liquids

- Kitaev's "toric code" (but no conserved particle number/magnetization)
- Easy-axis kagome model

  Balents, Fisher & Girvin, Phys. Rev. B **65**, 224412 (2002)
  Sheng & Balents, Phys. Rev. Lett. **94**, 146805 (2005)



$$H = J_z \sum_{h \text{ hexagon}} \left(S^z_{h1} + \cdots + S^z_{h6}\right)^2$$

$$+ J_\perp \sum_{h \text{ hexagon}} \left(\left(S^x_{h1} + \cdots + S^x_{h6}\right)^2 + \left(S^y_{h1} + \cdots + S^y_{h6}\right)^2 \right)$$

$$J_\perp << J_z$$

- SU(2) symmetric (=Heisenberg-like) spin models

  Raman-Moessner-Sondhi, Phys. Rev. B **72**, 064413 (2005)
  Ring exchange on the triangular lattice ? (GM *et al.*, PRB 1999)

- Experiments. Candidates to be "spin liquids", apparently all gapless…why ?
  - Cs_2CuCl_4 [Anisotropic S=1/2 triangular lattice, Coldea *et al.* 2003]
  - κ-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ [Shimizu *et al.* 2003]
  - $ZnCu_3(OH)_6Cl_2$ [Helton *et al.* 2007, Mendels *et al.* 2007, Ofer *et al.* 2007, Imai *et al.* 2007]
  - $Na_4Ir_3O_8$ [3D lattice of corner sharing triangles, "hyper kagome", Okamoto *et al.* 2007]
  - He^3 films [Nuclear magnetism on a triangular lattice, Masutomi *et al.* 2004]

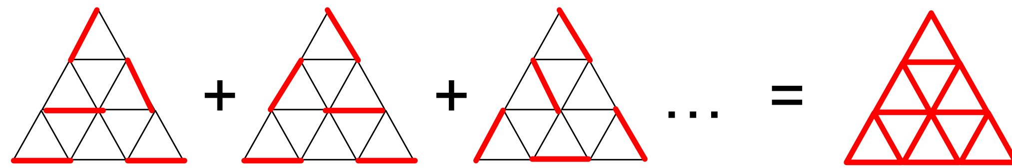
Short-range RVB picture

- P. W. Anderson's idea (1973) : (short-ranged) resonating valence-bond (RVB)

  Linear superposition of many (exponential) low-energy short-range valence-bond configurations

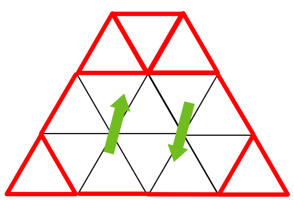
  $$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

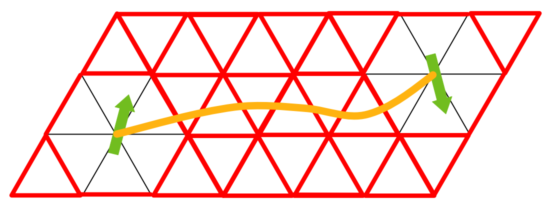
  S=0 spin singlet

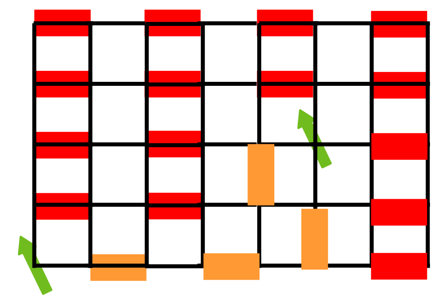


Spatially *uniform* state
(P.W. Anderson, 1973)

- Spin-½ excitations ?
  Crystal of singlets → linear potential between spinons
  Liquid of singlets → we *may* expect deconfined spinons

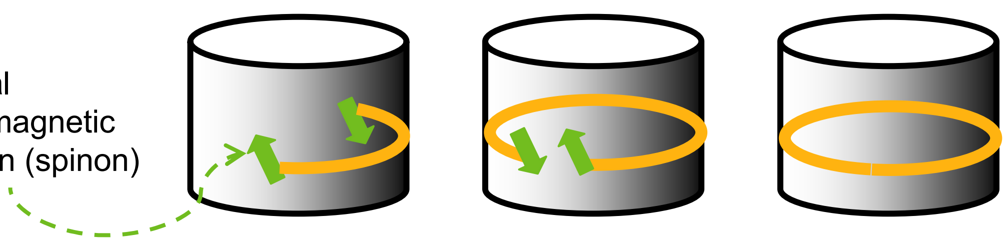






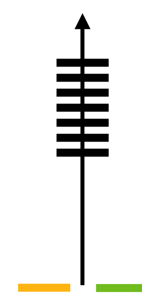
- Topological degeneracy & spinon fractionalization

fractional spin-½ magnetic excitation (spinon)



adiabatic process → New ground-state

Topological degeneracy (X.-G. Wen 1991) ↔ fractionalization
See also Oshikawa & Senthil PRL 96, 060601 (2006)



2-fold degeneracy
⇒ Satisfies LSMH

Schwinger bosons

Schwinger boson representation of the SU(2) spin algebra

$$S^z = \tfrac{1}{2}\left(b_\uparrow^+ b_\uparrow - b_\downarrow^+ b_\downarrow\right)$$

$$S^+ = b_\uparrow^+ b_\downarrow \qquad S^- = b_\downarrow^+ b_\uparrow$$

$b_\uparrow^+$ (or $b_\downarrow^+$) creates a spinon

constraint $b_\uparrow^+ b_\uparrow + b_\downarrow^+ b_\downarrow = 2S$ at each site

$$\vec{S}_r \cdot \vec{S}_{r'} = S^2 - \frac{1}{2} A_{rr'}^+ A_{rr'}$$ Heisenberg interaction

$$A_{rr'}^+ = \left(b_{\uparrow r}{}^+ b_{\downarrow r'}{}^+ - b_{\downarrow r}{}^+ b_{\uparrow r'}{}^+\right)$$ Bond operator - spin singlet

Path integral formulation

$$Z = \mathrm{Tr}\left[\exp(-\beta H)\right]$$

$$= \int D\left[\, b_{r\uparrow}(\tau), b_{r\downarrow}(\tau), \lambda_r(\tau)\right] \exp(-S_{eff})$$

$$S_{\rm eff}' = \int d\tau \sum_r \left(b^+_{r\uparrow}\partial_\tau b_{r\uparrow} + b^+_{r\downarrow}\partial_\tau b_{r\downarrow}\right) + i\int d\tau \sum_r \lambda_r \left(b^+_{r\uparrow} b_{r\uparrow} + b^+_{r\downarrow} b_{r\downarrow} - 2S\right)$$

$$-\int d\tau \sum_{rr'} \left\{ \frac{J_{\rm rr'}}{2} \underbrace{\left(b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'}\right)^+}_{A_{rr'}} \left(b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'}\right) \right\}$$

Read & Sachdev PRL 1989; PRL 1991;…

Hubbard-Stratonovich → spinons interacting with bond fields $Q_{rr'}$

$$Z = \int D\left[\, b_{r\uparrow}, b_{r\downarrow}, Q_{rr'}, \lambda_r \right] \exp(-S_{eff}')$$

$$S_{\rm eff}' = \int d\tau \sum_{rr'} \left\{ 2\frac{\left|Q_{\rm rr'}\right|^2}{J_{\rm rr'}} - Q^+_{rr'} \underbrace{\left(b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'}\right)}_{A_{rr'}} - Q_{rr'}\left(b^+_{\uparrow r} b^+_{\downarrow r'} - b^+_{\downarrow r} b^+_{\uparrow r'}\right) \right\}$$

$$+\cdots$$

(exact so far)

Saddle point approximation

- Saddle point approximation - equivalent to a self-consistent mean field approx.

$$\vec{S}_r \cdot \vec{S}_{r'} = S^2 - \frac{1}{2} A^+_{rr'} A_{rr'}$$

$$\langle A^+_{rr'} A_{rr'} \rangle \approx A^+_{rr'} \langle A_{rr'} \rangle + \langle A^+_{rr'} \rangle A_{rr'} - \langle A^+_{rr'} \rangle \langle A_{rr'} \rangle$$

$$2Q^0_{rr'} = J_{rr'} \langle A_{rr'} \rangle$$

$$\langle b^+_{r\uparrow} b_{r\uparrow} + b^+_{r\downarrow} b_{r\downarrow} \rangle = 2S$$

Note:
Can be motivated as a large-N limit
With *N*: number of boson flavors
(Read & Sachdev)

Arovas & Auerbach, Phys Rev B 38, 316 (1988)

- Mean-field Hamiltonian : $H_{MF} = -\sum_{\langle rr' \rangle} Q^{0+}_{rr'} \underbrace{\left(b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'} \right)}_{A_{rr'}} + \text{H.c}$

$$- \sum_r \lambda^0_r \left(b^+_{\uparrow r} b_{\uparrow r} + b^+_{\downarrow r} b_{\downarrow r} - 2S \right)$$

- Bogoliubov transformation→ mean-field RVB spin liquid with free (gapped) spinons,
  (we assume no Bose condensation (i.e. magnetic/Néel LRO) ok for small enough S)

- Is such an approximation reliable ? Effect of the (neglected) fluctuations ?

Fluctuations about the saddle point: Z_2 gauge field

- Most important fluctuation modes for the long-distance properties of the system ?
  $\Rightarrow$ Gauge modes
- Parametrize the *sign* fluctuations the bond field Q

Mean field value

$$Q_{ij}(\tau) = Q^0_{ij}\,\sigma^z_{ij}(\tau)$$

$$\sigma^z_{ij}(\tau) = \pm 1$$

Read & Sachdev
+ X. G. Wen's work on PSG & IGG

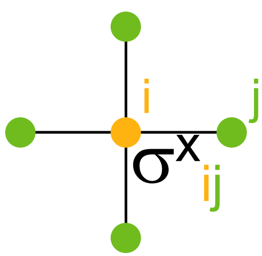
- Z_2 gauge invariance $\eta_i = \pm 1$

$$b_{\uparrow i} \to \eta_i b_{\uparrow i}$$

$$b_{\downarrow i} \to \eta_i b_{\downarrow i}$$

$$\sigma^z_{ij} \to \eta_i \eta_j \sigma^z_{ij}$$

- Quantization → Gauss Law

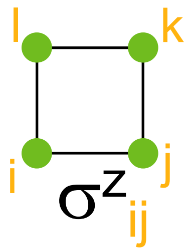


$$G_i = \exp\left[i\pi\left(b^+_{i\uparrow} b_{i\uparrow} + b^+_{i\downarrow} b_{i\downarrow}\right)\right]\prod_j \sigma^x_{ij} \qquad \sigma^x\sigma^z = -\sigma^z\sigma^x$$

$$\forall i \quad G_j\left|\text{phys}\right\rangle = G_j\left|\text{phys}\right\rangle$$

= local constraint of physical states

A toy model describing the fluctuations



$$H = -t\sum_{\langle ij\rangle}\left(b^+_{i\uparrow}b_{j\uparrow}\sigma^z_{ij} + b^+_{i\downarrow}b_{j\downarrow}\sigma^z_{ij} + H.c.\right) + \Delta\sum_i\left(b^+_{i\uparrow}b_{i\uparrow} + b^+_{i\downarrow}b_{i\downarrow}\right)$$

$$-K\sum_{ijkl}\sigma^z_{ij}\sigma^z_{jk}\sigma^z_{kl}\sigma^z_{li} - \Gamma\sum_i \sigma^x_i$$

$$+V\sum_i\left[\left(b^+_{i\uparrow}b_{i\uparrow} + b^+_{i\downarrow}b_{i\downarrow} - \tfrac{1}{2}\right) - \tfrac{1}{4}\right]^2$$

Spinon hopping

Spinon gap

Penalize/favour non-uniform signs

Sign flip

spinon-spinon repulsion

& constraint $(-1)^{b^+_{i\uparrow}b_{i\uparrow}+b^+_{i\downarrow}b_{i\downarrow}}\prod_j \sigma^x_{ij} = 1$

"Toric code" limit $\begin{cases} t = 0 \\ \Gamma = 0 \\ V = \infty \end{cases}$

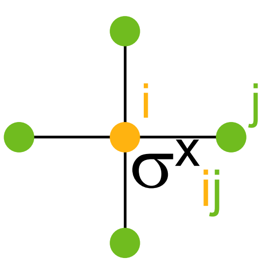
A toy model describing the fluctuations: toric code limit

"Toric code" limit

$$\left.\begin{aligned} t &= 0 \\ \Gamma &= 0 \\ V &= \infty \end{aligned}\right\} \rightarrow H = \Delta \sum_i \left(b^+_{i\uparrow} b_{i\uparrow} + b^+_{i\downarrow} b_{i\downarrow} \right) - K \sum_{ijkl} \sigma^z_{ij} \sigma^z_{jk} \sigma^z_{kl} \sigma^z_{li}$$

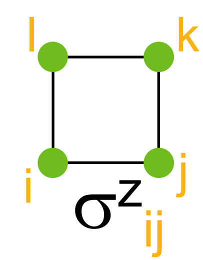
$$\left.\begin{aligned} & b^+_{i\uparrow} b_{i\uparrow} + b^+_{i\downarrow} b_{i\downarrow} = 0 \text{ or } 1 \\ & \underbrace{(-1)^{b^+_{i\uparrow} b_{i\uparrow} + b^+_{i\downarrow} b_{i\downarrow}} \prod_j \sigma^x_{ij} = 1}_{\text{Gauss law}} \end{aligned}\right\} \rightarrow 2\left(b^+_{i\uparrow} b_{i\uparrow} + b^+_{i\downarrow} b_{i\downarrow} \right) = 1 - \prod_j \sigma^x_{ij}$$

$$H = -\frac{\Delta}{2} \sum_i \left(\prod_j \sigma^x_{ij} \right) - K \sum_{ijkl} \sigma^z_{ij} \sigma^z_{jk} \sigma^z_{kl} \sigma^z_{li}$$



= Kitaev's toric code

(=solvable toy model for a topologically ordered ground state



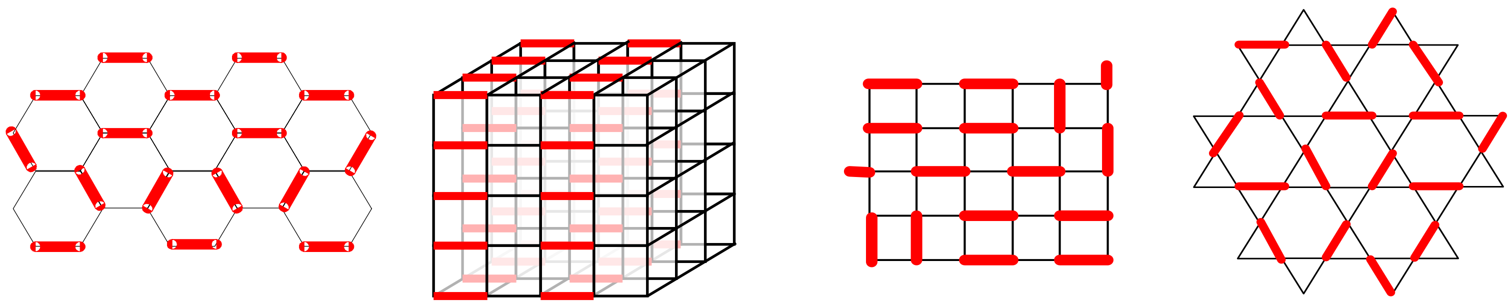
Part III

Quantum dimer models

Quantum dimer models

Rokhsar & Kivelson, Phys. Rev. Lett. (1988)

- Basis states = fully packed dimer coverings of the lattice

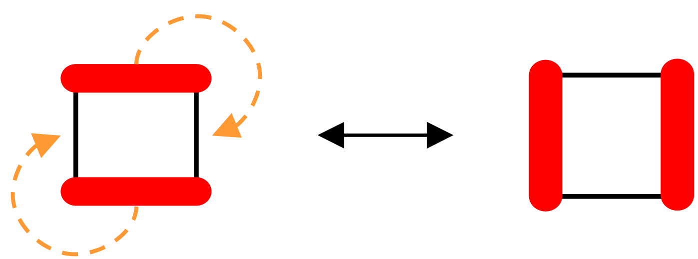


Simplified picture for a S=1/2 antiferromagnet with a “short-range RVB liquid” ground state

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

- Introduce some dynamics

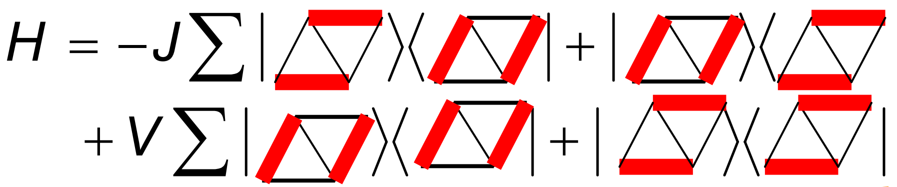
Example:



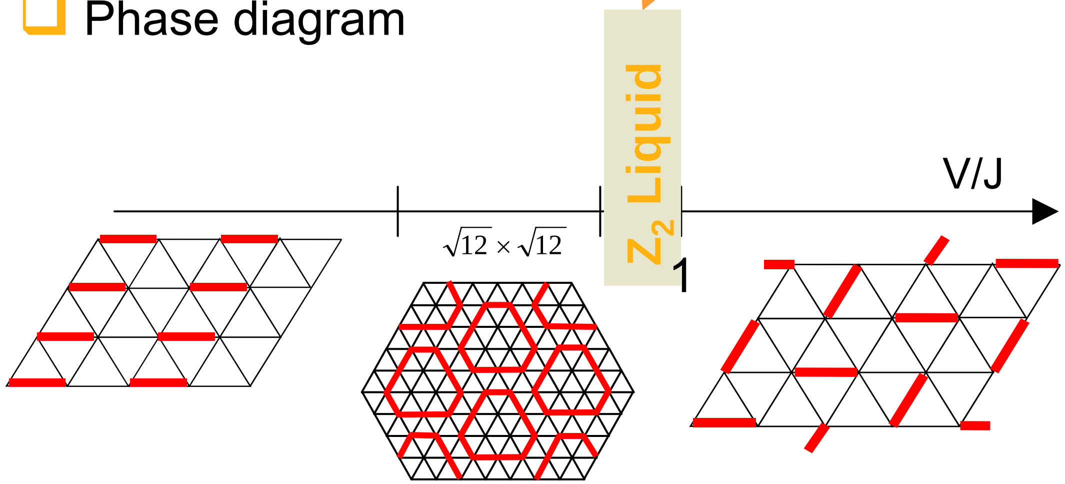
Nature of the ground state ?
Conventional order ?
Topological order ?
Critical phases/points ?

Dimer liquid phase in the triangular QDM

- Hamiltonian [Moessner & Sondhi, 2001]



- Phase diagram



- Liquid phase:
  - Short-range dimer-dimer correlations
  - Gapped excitation spectrum
  - The g. s. degeneracy depends on the topology.
  - The degenerate g. s. cannot be distinguished by any local observable.
  - Effective Z_2 gauge theory

- J=V=1 "Rokhsar-Kivelson point"

  The ground state(s) is(are) known exactly

  =Equal amplitude superposition of all dimer coverings :

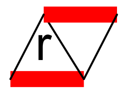
$$\left|g.s.\right\rangle = \sum_{c\in\{\text{coverings}\}} \left|c\right\rangle$$

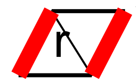
First *simple* model with a short-ranged RVB liquid.
Moessner & Sondhi, Phys. Rev. Lett. (2001)
Ralko *et al.*, Phys. Rev. B 74, 134301 (2006) + refs. Therein
GM and Mila, Phys. Rev. B 77, 134421 (2008)

Rokhsar-Kivelson point - triangular QDM

At J=V=1, the Hamiltonian can be written as

$$H = 2\sum_r |\psi_r\rangle\langle\psi_r|$$ = Sum of projectors

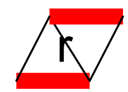
$$|\psi_r\rangle = \frac{1}{\sqrt{2}}\Big(\Big|$$ 

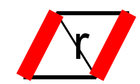
$$\Big\rangle - \Big|$$ 

$$\Big\rangle \Big)$$

$$|\text{g.s.}\rangle = \sum_c |c\rangle$$

$$\langle\Psi_r|c\rangle = \begin{cases} \frac{1}{\sqrt{2}}|c_{\backslash r\backslash}\rangle \text{ if } |c\rangle = |\cdots \quad \cdots\rangle \\ -\frac{1}{\sqrt{2}}|c_{\backslash r\backslash}\rangle \text{ if } |c\rangle = |\cdots \quad \cdots\rangle \\ 0 \text{ otherwise} \end{cases}$$



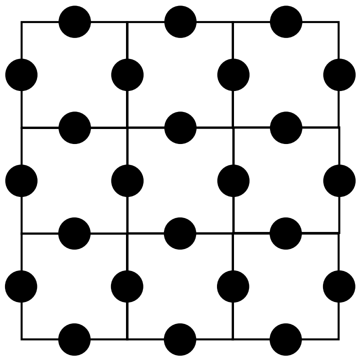


$$\langle\Psi_r|\text{g.s.}\rangle = 0$$

$$\Rightarrow H|\text{g.s.}\rangle = 0 \Rightarrow \text{a ground-state}$$

Excitations of the toric code

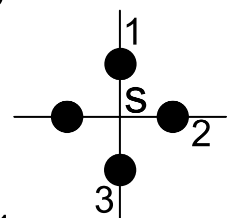
Kitaev 1997

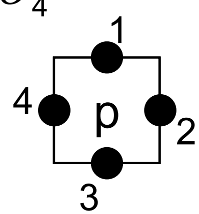


$$H = -\sum_s A_s - \sum_p B_p$$

$$A_s = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$$

$$B_p = \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$$

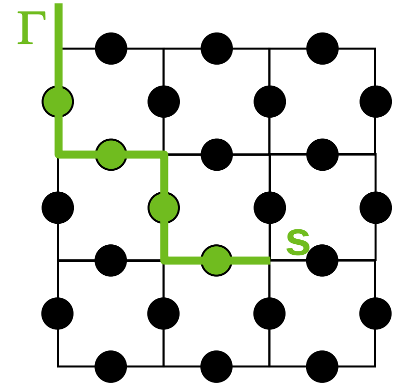




Electric charge

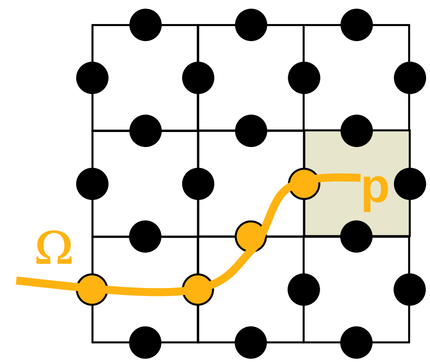
$$A_s = -1$$

$$|s\rangle = \left(\prod_{i \in \Gamma} \sigma_i^z \right) |0\rangle$$



Magnetic charge

$$B_p = -1$$

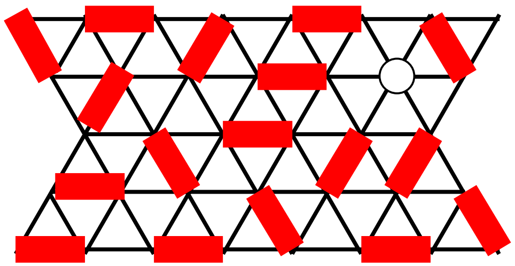


$$|p\rangle = \left(\prod_{i \in \Omega} \sigma_i^x \right) |0\rangle$$

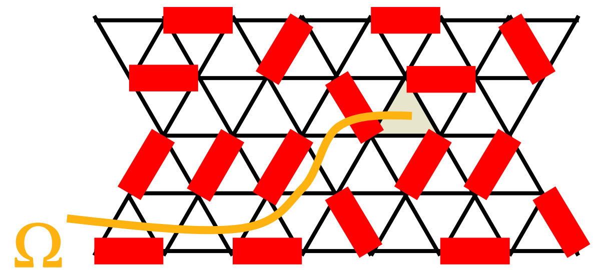
Excitations in the dimer liquid

Magnetic Z_2 vortices (« visons »)
Analogy with the toric code

Electric charge = monomer



Magnetic charge = vison



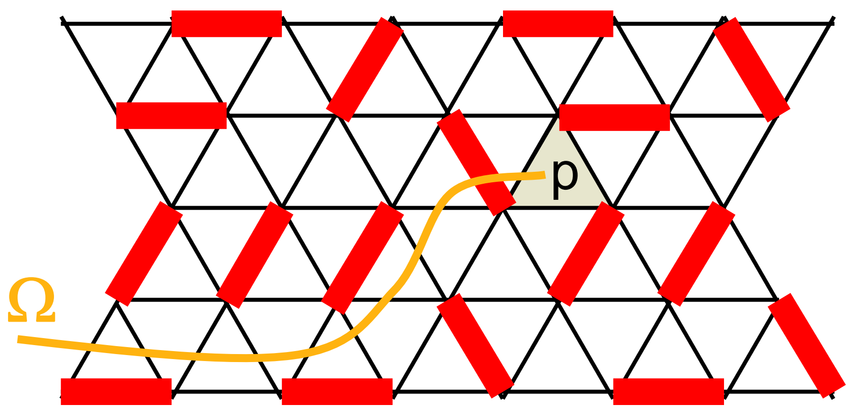
$$|\text{g.s.}\rangle = \sum_{c \in \{\text{coverings}\}} |c\rangle$$

$$|\text{vison}\rangle \approx \sum_{c \in \{\text{coverings}\}} (-1)^{N(\Omega,c)} |c\rangle$$

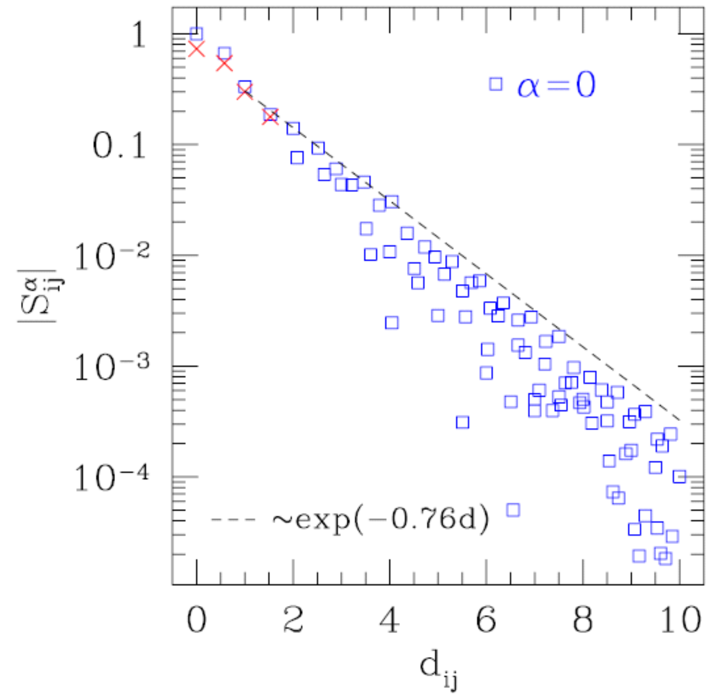
Contrary to the toric code, this trial state is not an exact eigenstate…

See also: GM, Serban & Pasquier PRL (2002)

Variational approximation (I)



$$|p\rangle = \sum_{c\in\{\text{coverings}\}} (-1)^{N(\Omega,c)} |c\rangle$$



GM and Mila, Phys. Rev. B 77, 134421 (2008)

- Not an exact eigenstate…
- Not orthogonal to each other

$$S_{pp'} = \langle p | p' \rangle \neq 0 \quad \text{overlapp}$$

$$\langle p | H | p' \rangle \neq 0 \ \text{vison hopping}$$

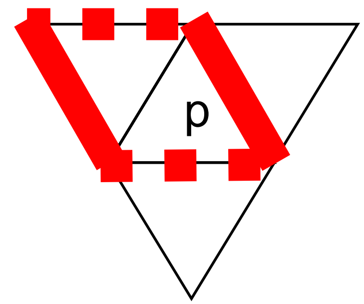
- Overlap and dispersion relation are computed using Pfaffians

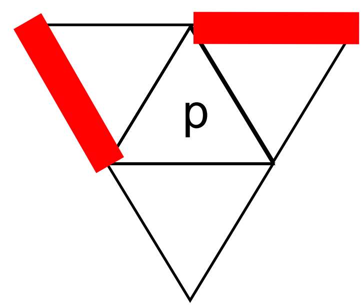
Variational approximation (I)

α: variational parameter

Improved variational wave-function

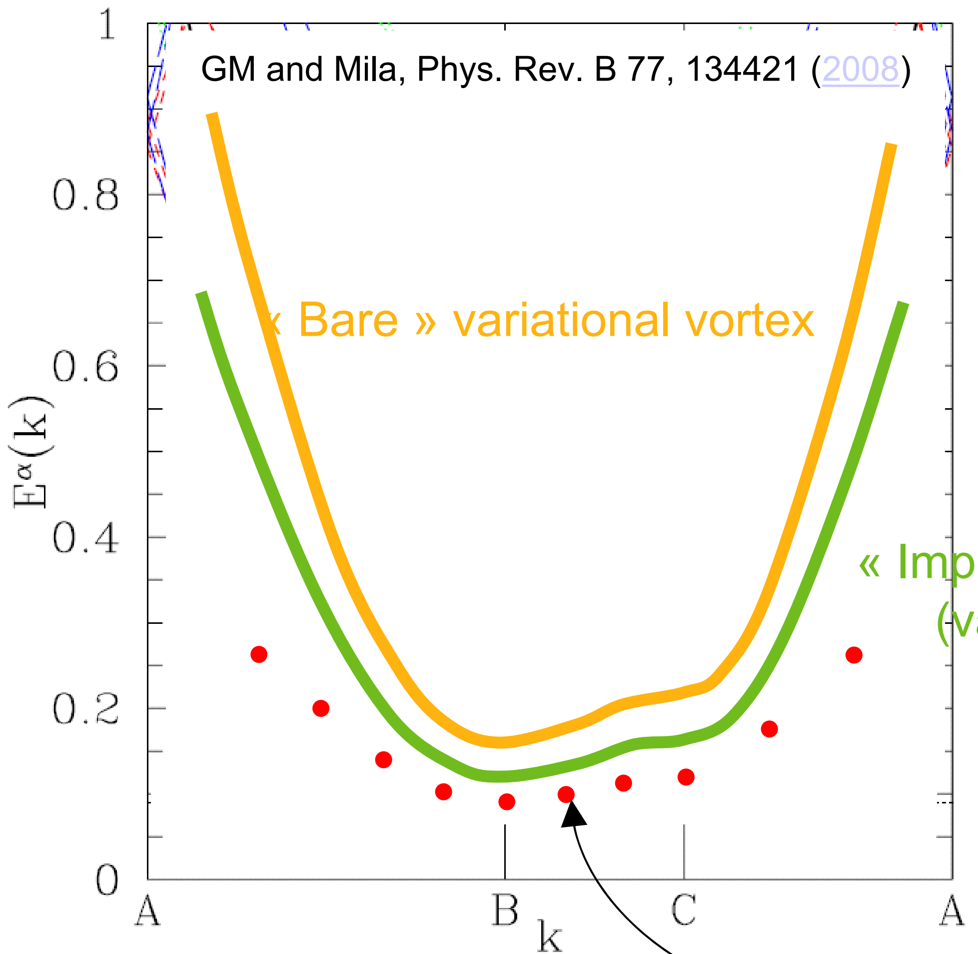
$$|\tilde{p}\rangle = \sum_{c\in\{\text{coverings}\}} (-1)^{N(\Omega,c)} \left[1+\alpha \underbrace{F(c)}_{\substack{\text{1 if c is "flippable" around p} \\ \text{0 otherwise}}} \right] |c\rangle$$

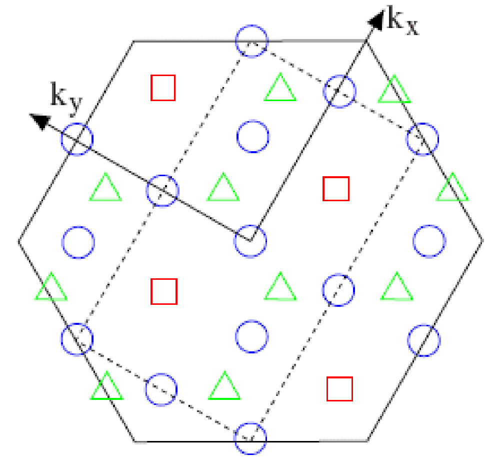
c=  →F(c)=1

c'=  → F(c')=0

Overlap and dispersion relation are computed using Pfaffians (not easy…)

Dispersion relation of the « magnetic » vortices

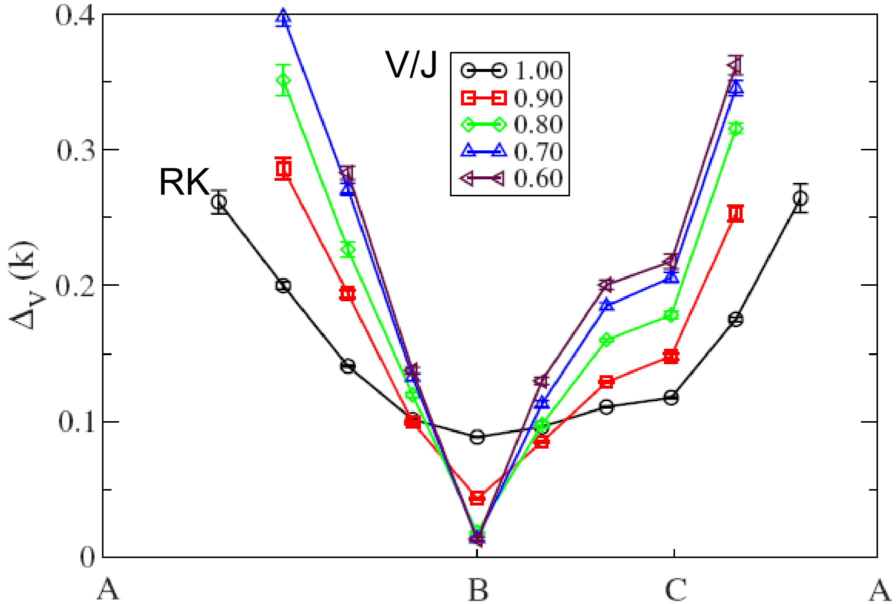




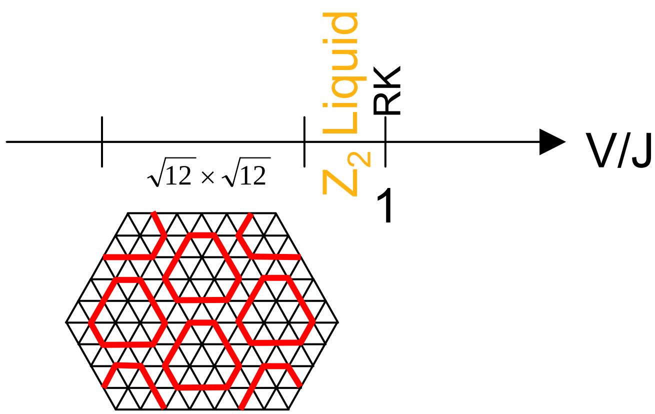
« Improved » variational vortex
(var. parameter α=-0.8)

Points: Green's function Quantum Monte Carlo
Ralko *et al.*, Phys. Rev. B 76, 140404(R) (2007)

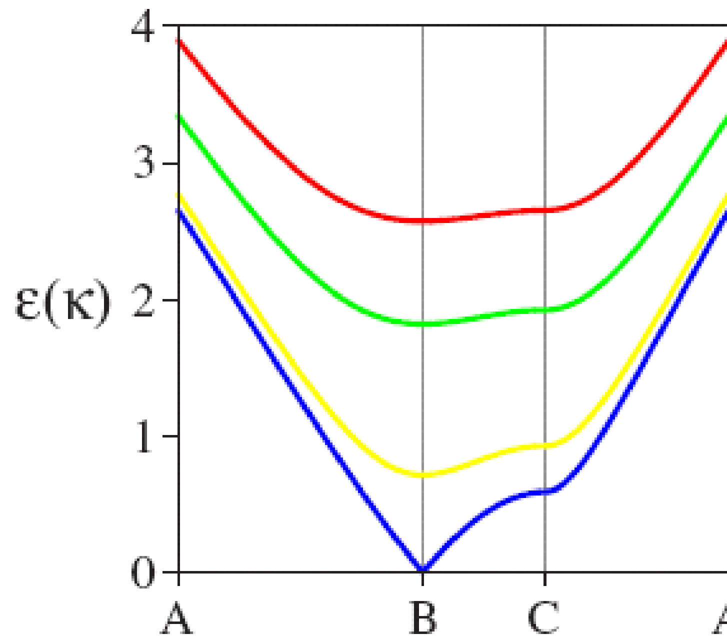
Dimer crystallization – Z_2 vortex condensation



Ralko *et al.*,
Phys. Rev. B **76**, 140404(R) (2007)



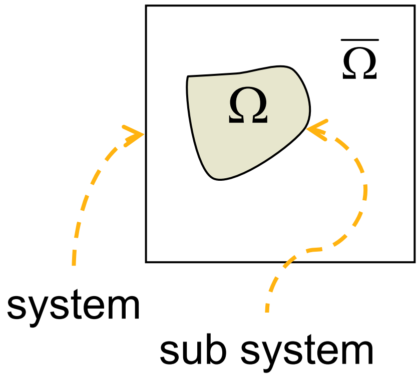
« Soft dimer » approximation



GM and Mila, Phys. Rev. B 77, 134421 (2008)

Reduced density matrices for RK wave functions

Furukawa & GM, Phys. Rev. B 75, 214407 (2007)



$$|\mathrm{g.s}\rangle = \frac{1}{\sqrt{N}} \sum_{c \in \{\text{coverings}\}} |c\rangle$$

$$= \frac{1}{\sqrt{N}} \sum_{b \in \left\{\substack{\text{boundary} \\ \text{conditions}}\right\}} \left(\sum_{\substack{c^{\Omega} \text{ compatible} \\ \text{with b}}} |c^{\Omega}\rangle \right) \otimes \left(\sum_{\substack{c^{\bar{\Omega}} \text{ compatible} \\ \text{with b}}} |c^{\bar{\Omega}}\rangle \right)$$

$$= \sum_{b \in \left\{\substack{\text{boundary} \\ \text{conditions}}\right\}} \sqrt{\frac{N^{\Omega,b} N^{\bar{\Omega},b}}{N}} \left(\frac{1}{\sqrt{N^{\Omega,b}}} \sum_{\substack{c^{\Omega} \text{ compatible} \\ \text{with b}}} |c^{\Omega}\rangle \right) \otimes \left(\frac{1}{\sqrt{N^{\bar{\Omega},b}}} \sum_{\substack{c^{\bar{\Omega}} \text{ compatible} \\ \text{with b}}} |c^{\bar{\Omega}}\rangle \right)$$

$$= \sum_b \lambda_b |b^{\Omega}\rangle |b^{\bar{\Omega}}\rangle =$$ Schmidt decomposition

$$\rho = \sum_b |\lambda_b|^2 |b^{\Omega}\rangle\langle b^{\Omega}|$$

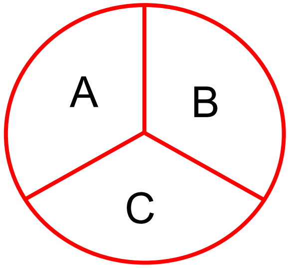
$$S = - \sum_{b \in \left\{\substack{\text{boundary} \\ \text{conditions}}\right\}} p_b \log(p_b)$$

$$p_b = |\lambda_b|^2 = \frac{N^{\Omega,b} N^{\bar{\Omega},b}}{N}$$

Classical “thermal” entropy of the boundary

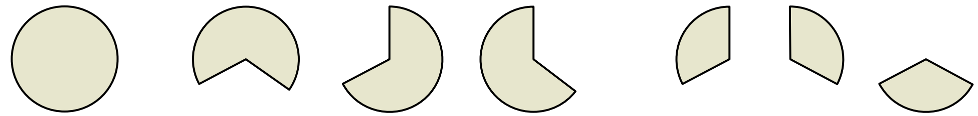
Probing topological order with entanglement entropy

- A. Kitaev and J. Preskill, Phys. Rev. Lett. 96, 110404(2006)

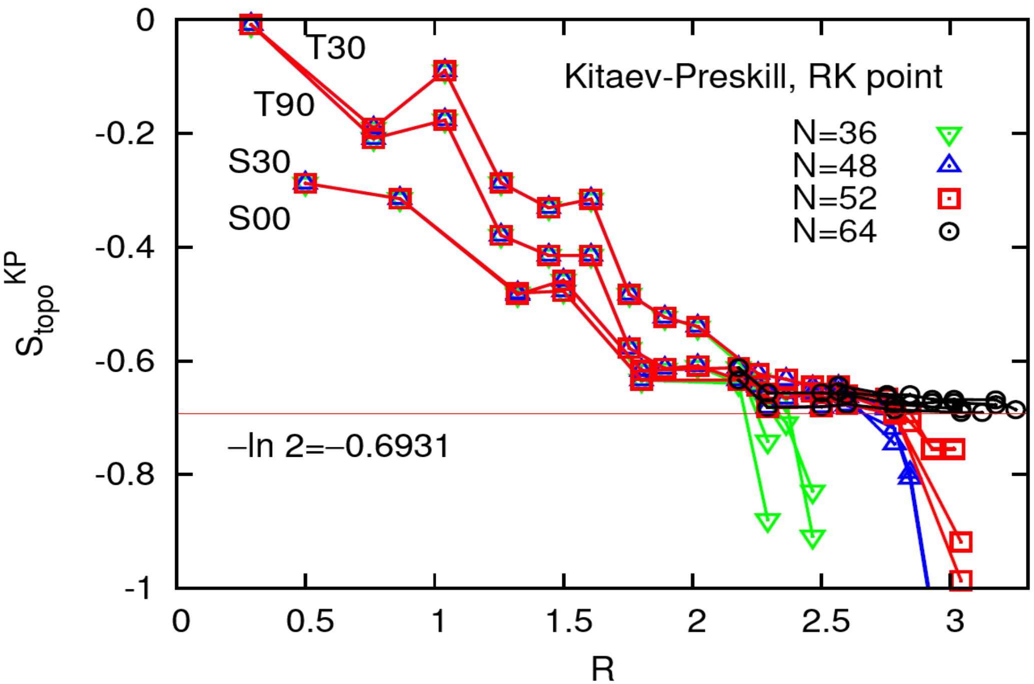


$$S(R) \approx \alpha R + \underbrace{S_{topo}}_{-\log(D_{tot}^{quantum})}$$

$$S_{topo} = S_{ABC} - (S_{AB} + S_{BC} + S_{AC}) + (S_A + S_B + S_C)$$



See also: M. Levin and X.-G. Wen, Phys. Rev. Lett. 96, 110405(2006)



Furukawa & GM, Phys. Rev. B 75, 214407 (2007)

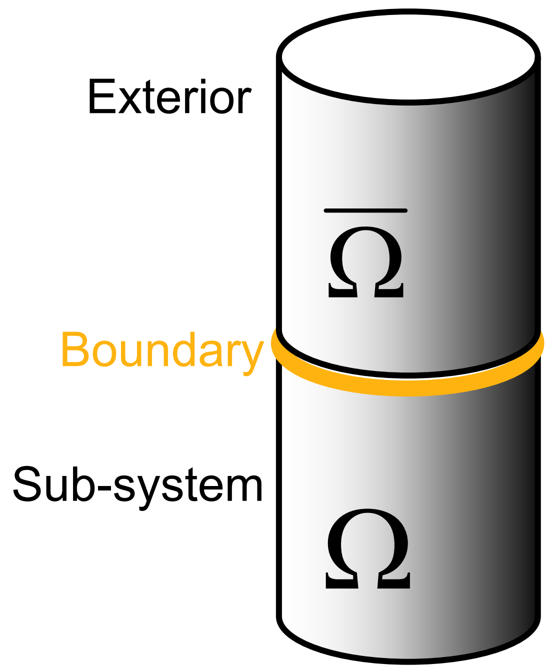
- Z_2 Liquid: D_{tot}=2. Seems ok !
- See also (FQHE) : Haque, *et al.*, Phys. Rev. Lett. 98, 060401 (2007); Zozulya *et al.*, Phys. Rev. B 76, 125310 (2007).

Reduced density matrix for RK wave functions

$$|RK\rangle = \frac{1}{Z}\sum_c e^{-\frac{1}{2}\beta E(c)}|c\rangle$$

GM & V. Pasquier, work in progress…

Cylinder geometry



1) Write the transfer matrix M of the classical problem

2) find the ground state |ψ> of M
(=assume an infinitely long cylinder).
In the dimer case: 1D free fermions.

3) All the probabilities of the boundary states are contained in the ground state of M (the *full* RDM spectrum) :

$$p_b = |\langle b|\psi\rangle|^2$$

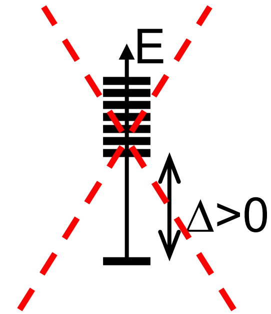
4) Compute the entanglement entropy

$$S = -\sum_b |\langle b|\psi\rangle|^2 \log\left(|\langle b|\psi\rangle|^2\right)$$

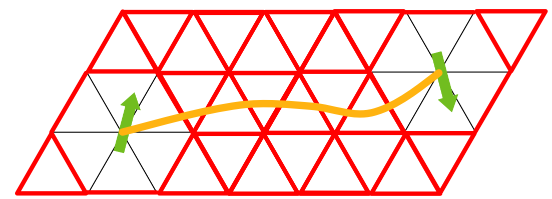
Summary

- Lieb-Shultz-Mattis-Hastings theorem:

  a gapped Mott insulator is either
  - topologically ordered,
  - or "conventionally" ordered.



⇒ Frustrated Heisenberg magnets may provide realizations of gapped topological states
… but no experimental evidence so far.



- Quantum dimer models & RK wave functions

offer tractable models to study entanglement, fractionalization and topological order.