

Excited hadron spectroscopy from lattice QCD

Christopher Thomas, Trinity College Dublin

thomasc@maths.tcd.ie

Irish Quantum Foundations, Castletown House, 3rd May 2013

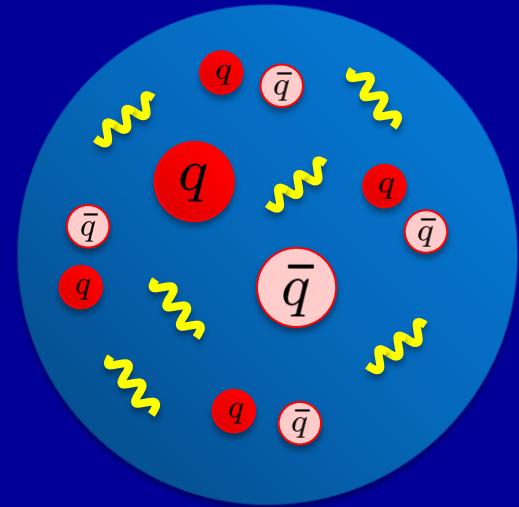
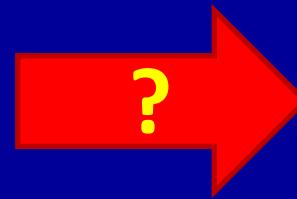
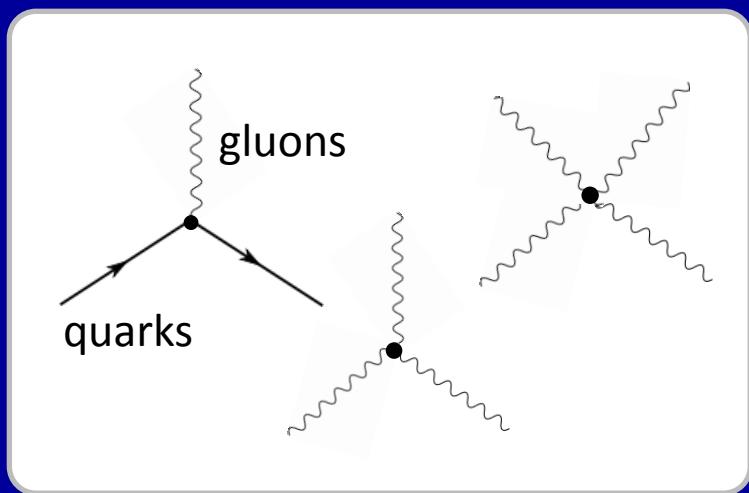
Hadron Spectrum Collaboration



Outline

- Introduction: spectroscopy and lattice QCD
- Some recent work:
 - Excited meson spectra
 - The ρ resonance in isospin-1 $\pi\pi$ scattering
- Summary and outlook

Quantum Chromodynamics

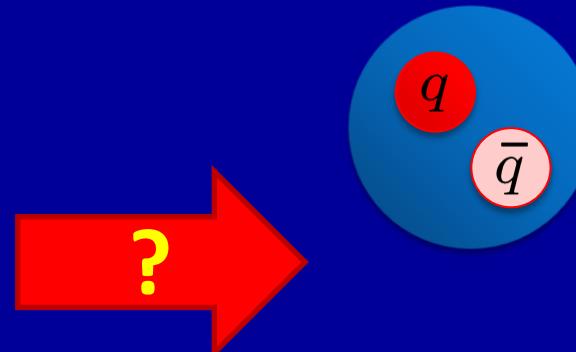
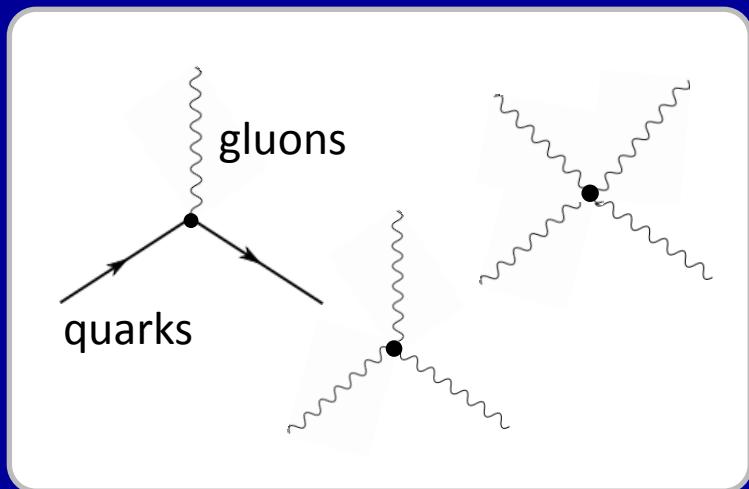


Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

Confinement

Relevant degrees of freedom? Gluons?

Quantum Chromodynamics

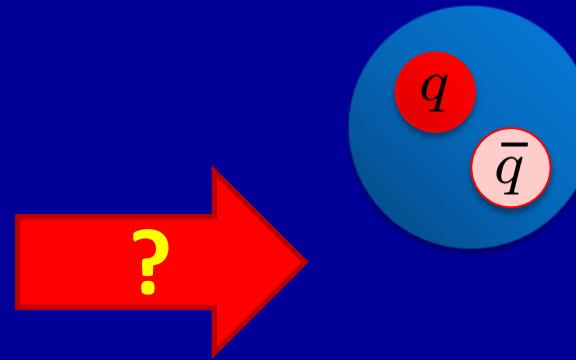
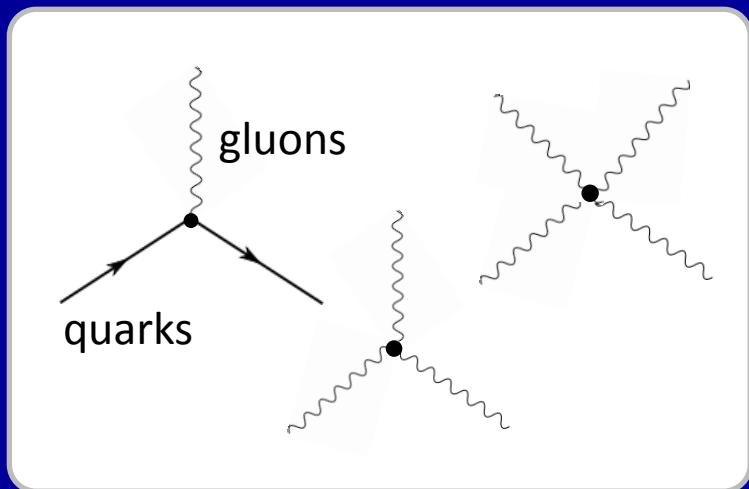


Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

Confinement

Relevant degrees of freedom? Gluons?

Quantum Chromodynamics



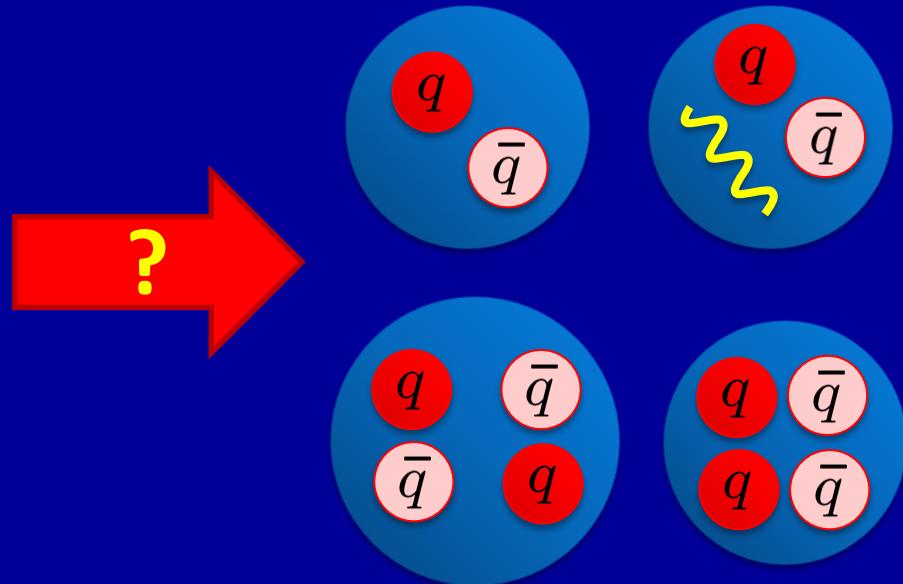
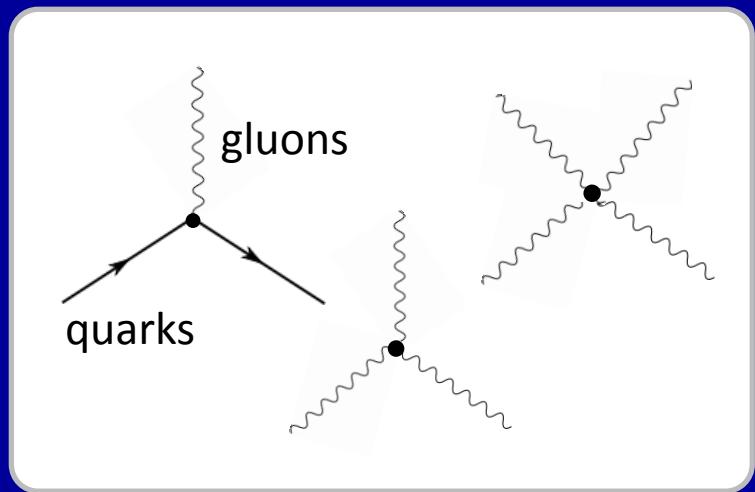
Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

Confinement

Relevant degrees of freedom? Gluons?

Exotic quantum numbers ($J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \dots$)
– can't just be a $q\bar{q}$ pair

Quantum Chromodynamics



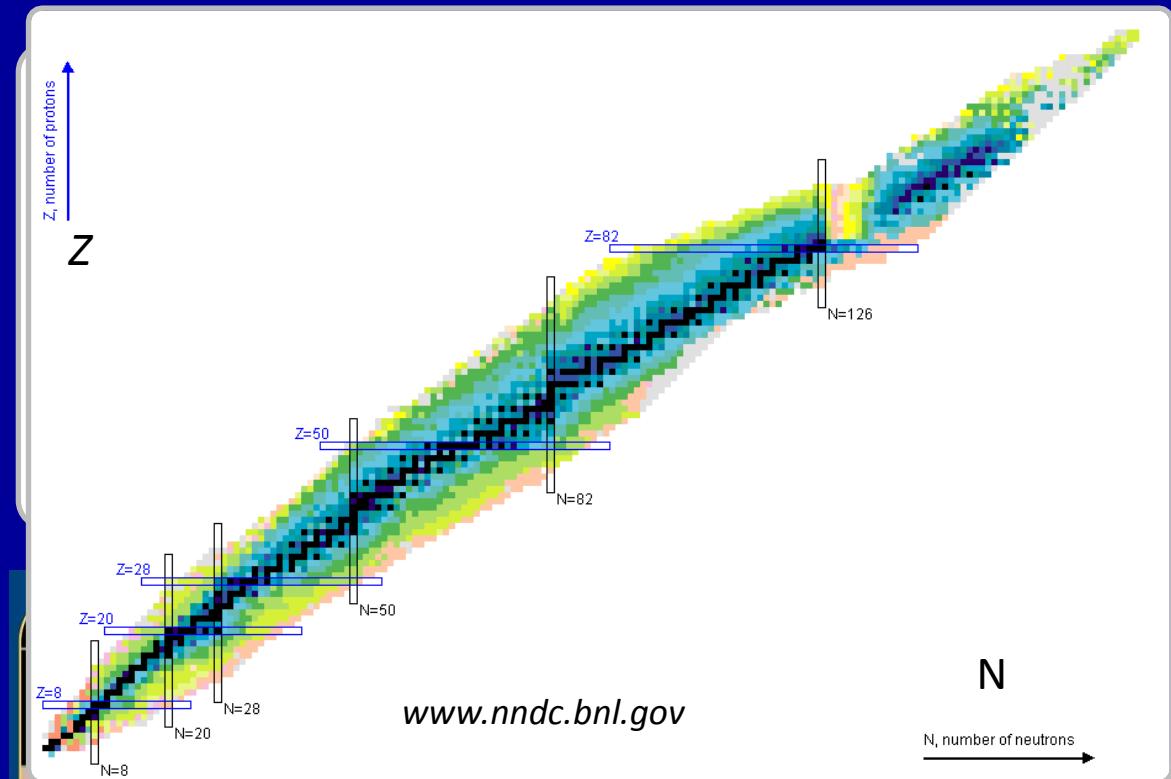
Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.002	2/3
d down	0.005	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	173	2/3
b bottom	4.2	-1/3

Confinement

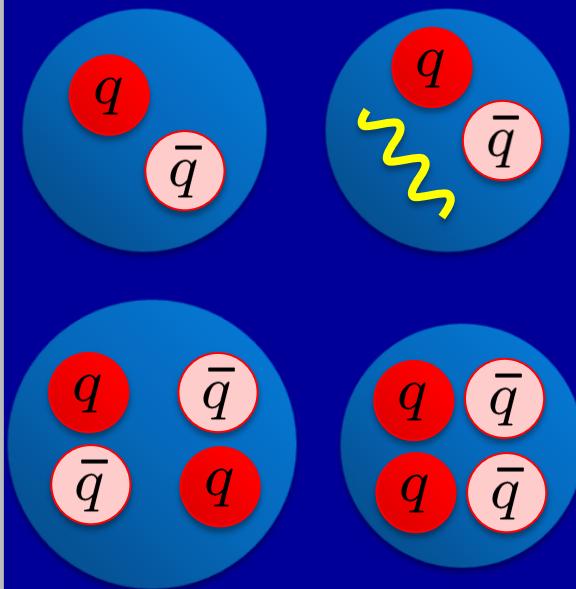
Relevant degrees of freedom? Gluons?

Exotic quantum numbers ($J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \dots$)
– can't just be a $q\bar{q}$ pair

Quantum Chromodynamics



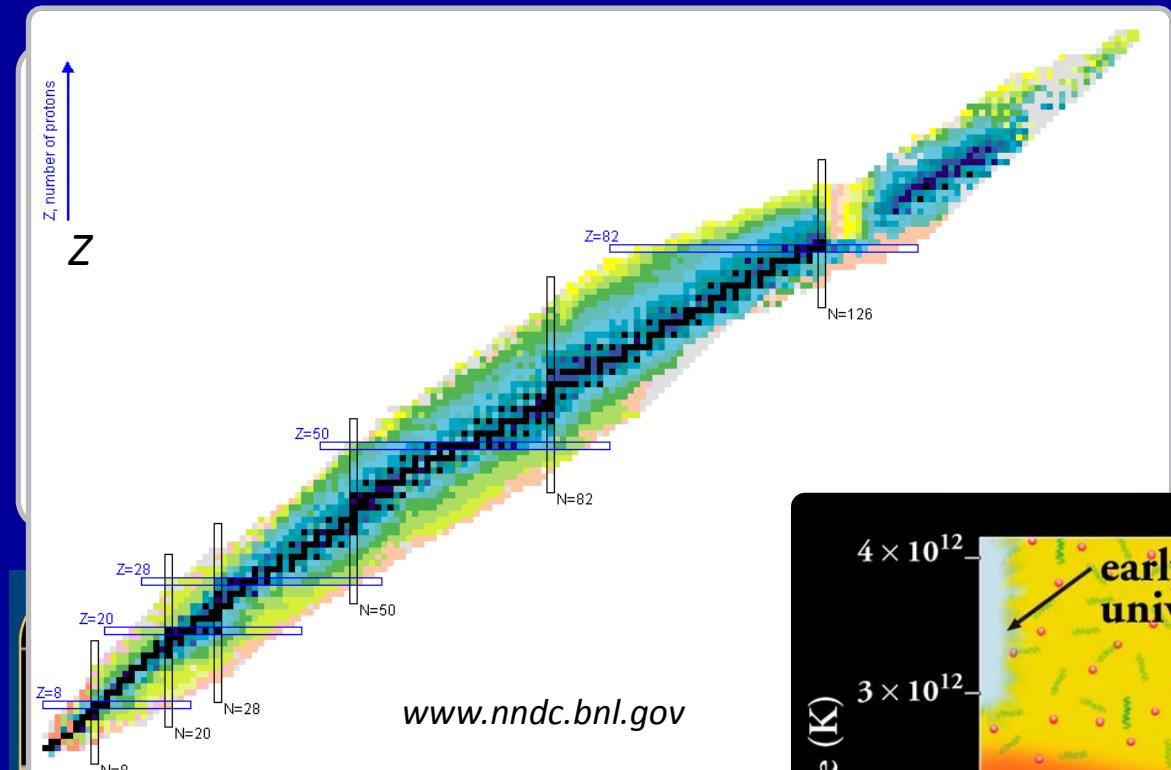
U	up	0.002	2/3
d	down	0.005	-1/3
C	charm	1.3	2/3
S	strange	0.1	-1/3
t	top	173	2/3
b	bottom	4.2	-1/3



Relevant degrees of freedom? Gluons?

Exotic quantum numbers ($J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \dots$)
– can't just be a $q\bar{q}$ pair

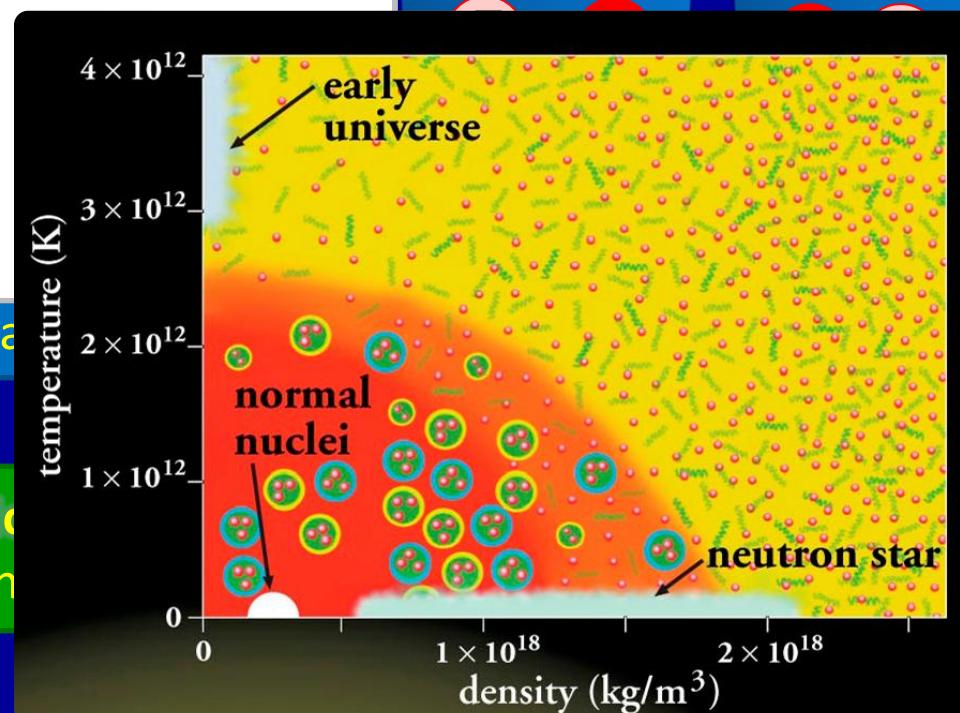
Quantum Chromodynamics



U	up	0.002	2/3
d	down	0.005	-1/3
C	charm	1.3	2/3
S	strange	0.1	-1/3
t	top	173	2/3
b	bottom	4.2	-1/3

Relevan

Exotic
— can



Meson spectroscopy and transitions

Experiments

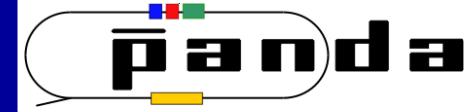
LHC

JLab @ 12 GeV

BESIII

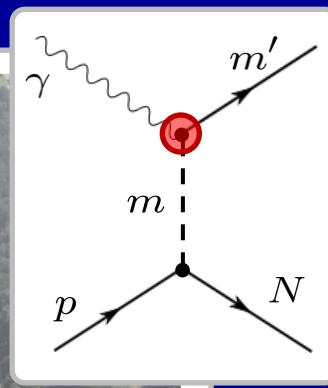
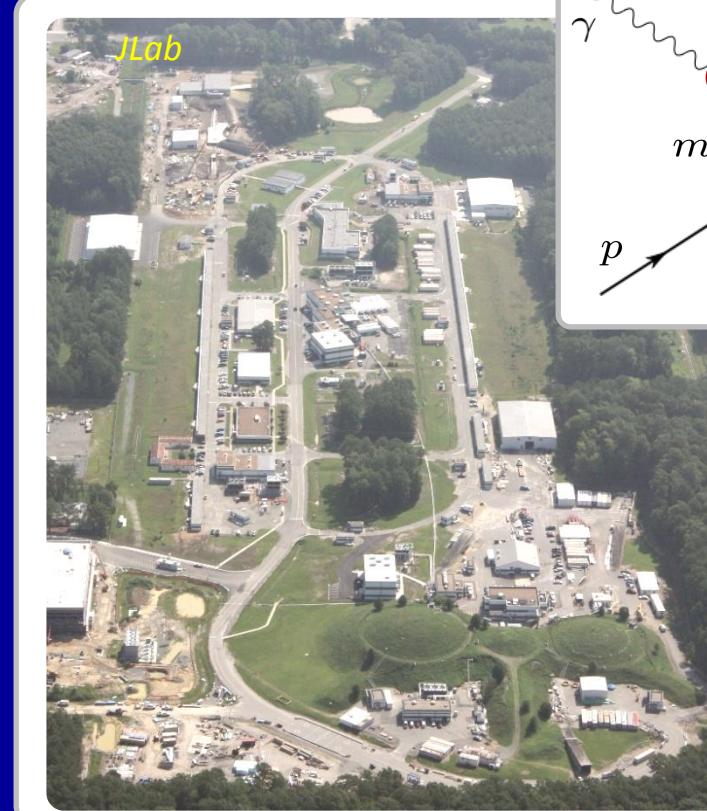
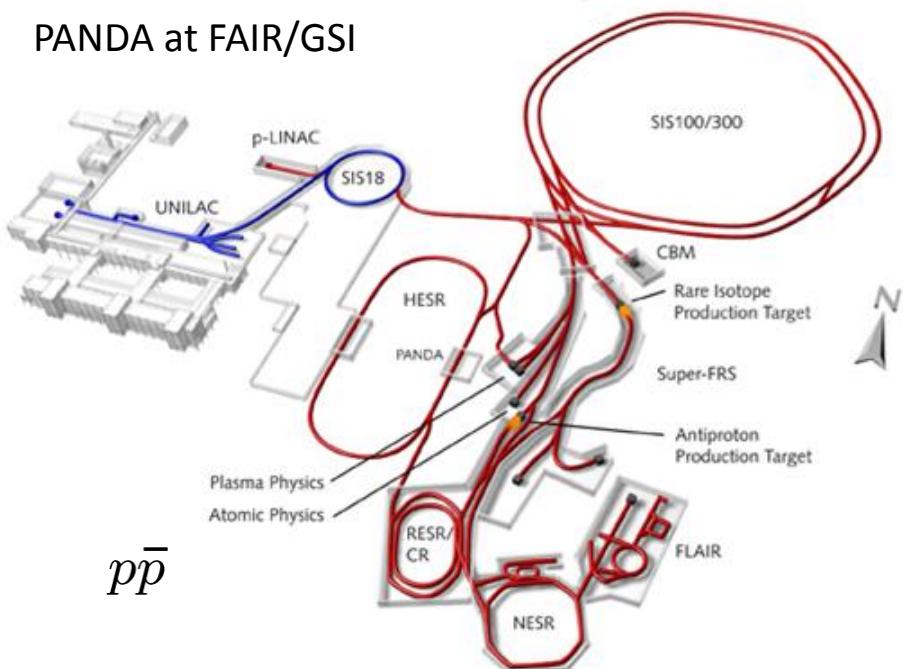
KLOE2

CLAS12

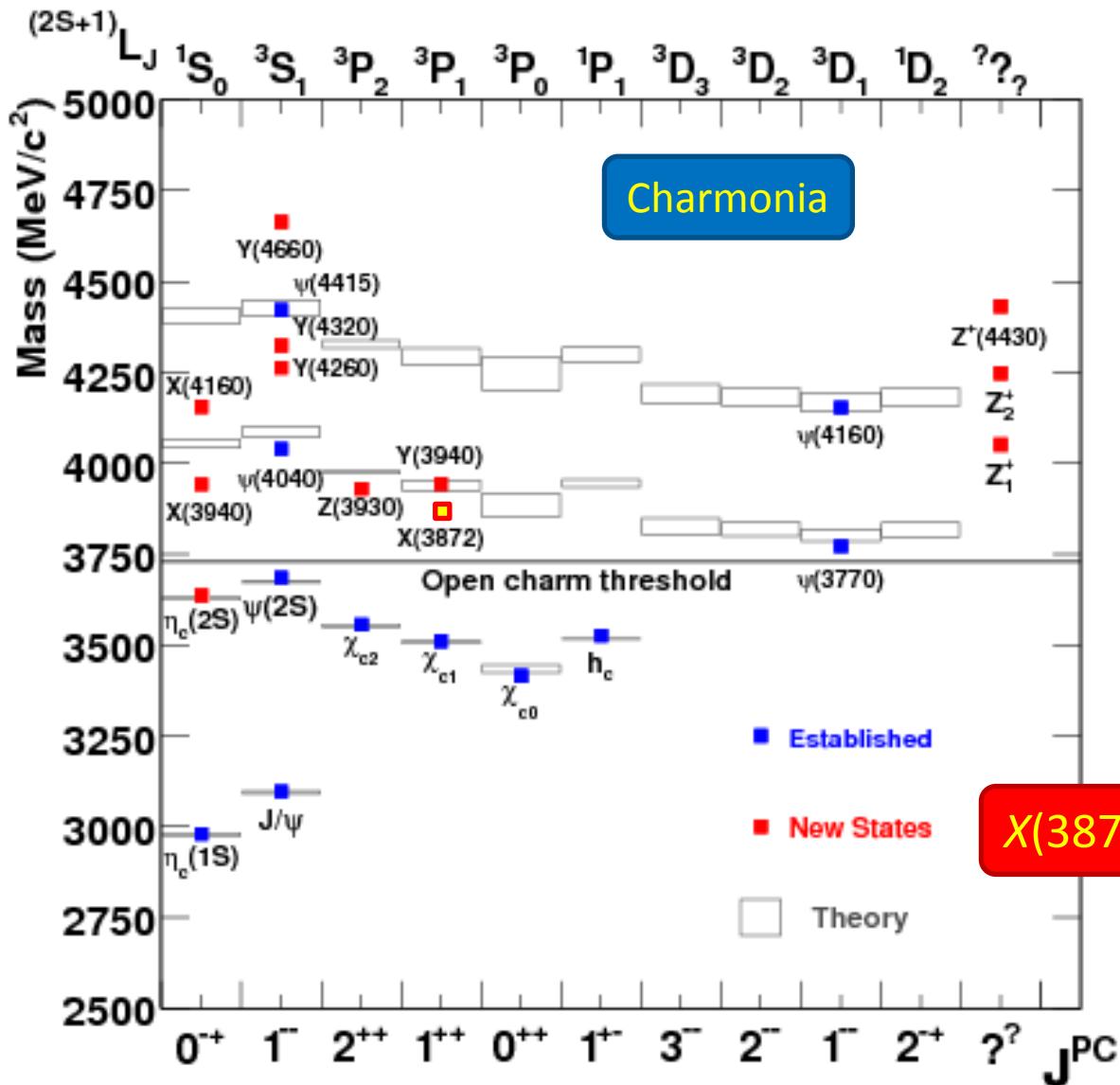


'Unusual' states; exotics?

PANDA at FAIR/GSI



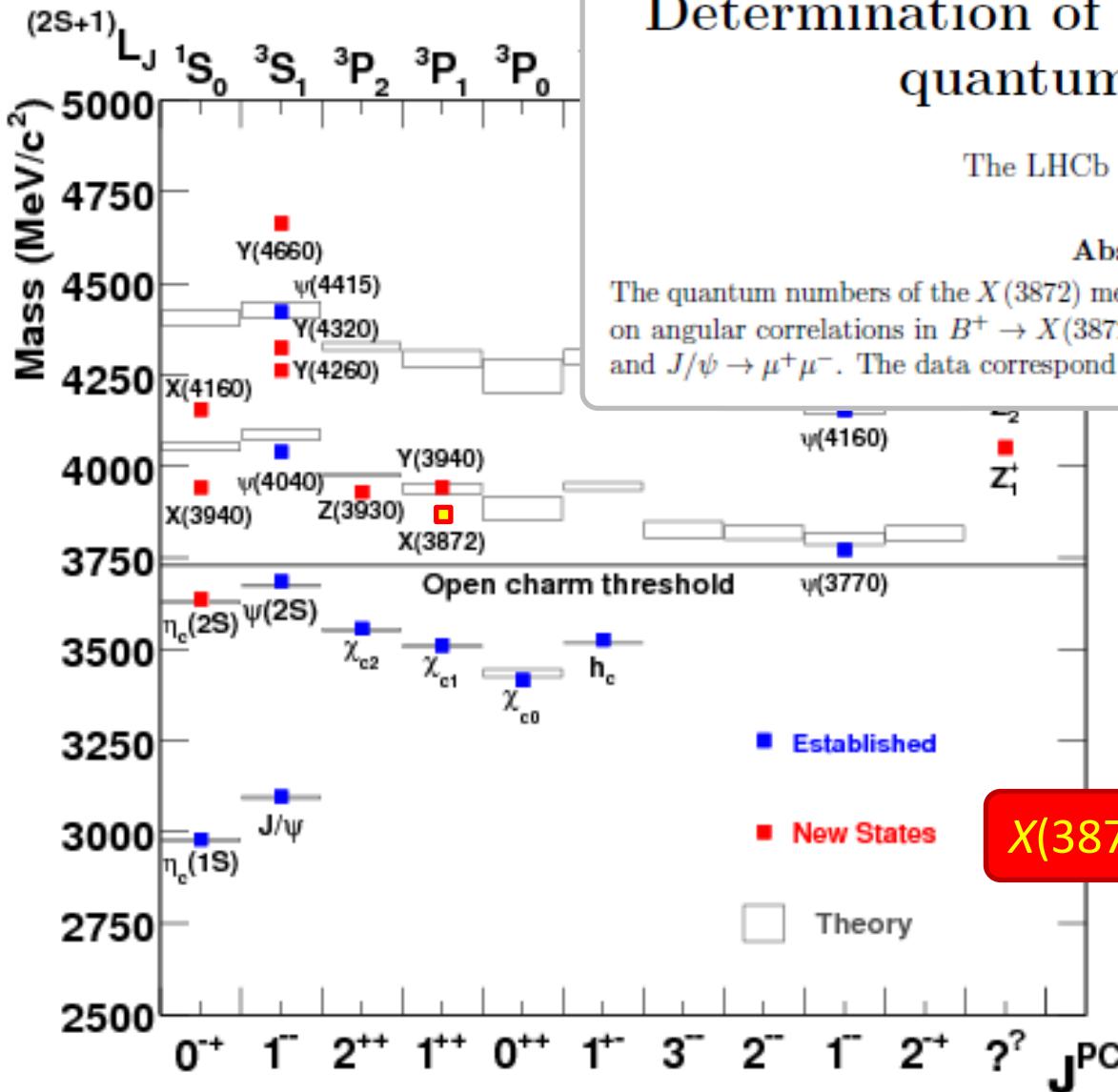
Meson spectroscopy and transitions



SIII

KLOE2





Determination of the $X(3872)$ meson quantum numbers

The LHCb collaboration[†]

Abstract

The quantum numbers of the $X(3872)$ meson are determined to be $J^{PC} = 1^{++}$ based on angular correlations in $B^+ \rightarrow X(3872)K^+$ decays, where $X(3872) \rightarrow \pi^+\pi^-J/\psi$ and $J/\psi \rightarrow \mu^+\mu^-$. The data correspond to 1.0 fb^{-1} of pp collisions collected by the

$\psi(4160)$
 Z_1^+
 $\psi(3770)$
 $X(3872), Y(4260), Z(4430), \dots ?$

$1^{-+} ?$

Meson spectroscopy and transitions

Experiments

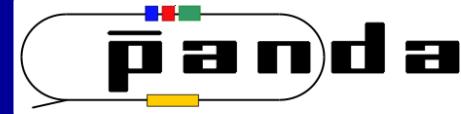
LHC

JLab @ 12 GeV

BESIII

KLOE2

CLAS12



'Unusual' states; exotics?

Need first-principles calculations in QCD

Lattice QCD

Spectroscopy of mesons

Transitions

Meson spectroscopy and transitions

Experiments

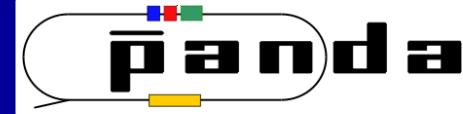
LHC

JLab @ 12 GeV

BESIII

KLOE2

CLAS12



'Unusual' states; exotics?

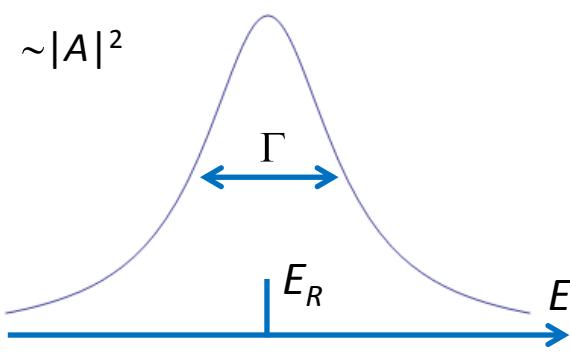
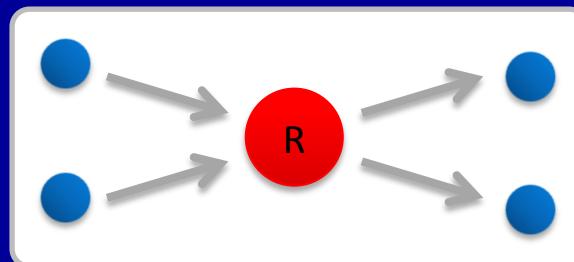
Need first-principles calculations in QCD

Lattice QCD

Spectroscopy of mesons

Transitions

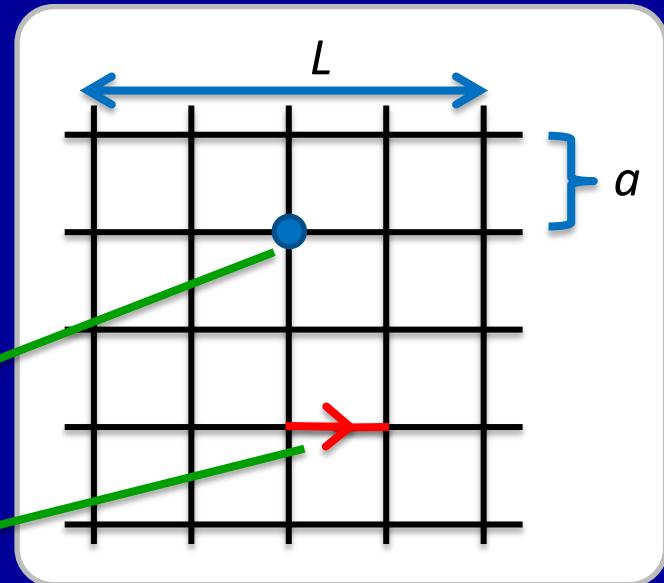
Resonances
(unstable)



QCD on a lattice

Discretise (spacing = a) – regulator
Finite volume \rightarrow finite no. of d.o.f.

Quarks fields
Gauge fields (gluons)



Path integral formulation

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U f(\psi, \bar{\psi}, U) e^{-\tilde{S}[\psi, \bar{\psi}, U]}$$

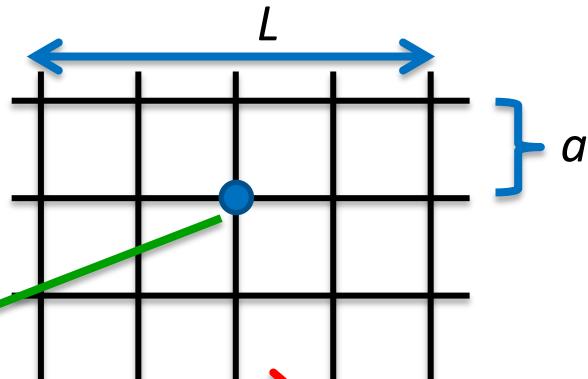
Euclidean time: $t \rightarrow i t$

Numerical methods (Monte Carlo)
 \rightarrow high performance computing

- Finite a and L
(and reduced sym.)
- Unphysical m_π

QCD on a lattice

Discretise (spacing = a) – regulator
Finite volume \rightarrow finite no. of d.o.f.



Quan

Gauge field



JLab

Spectroscopy on the lattice

Energy eigenstates of
Hamiltonian from 2-pt corr. fns.

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Operators (interpolating fields)

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Spectroscopy on the lattice

Energy eigenstates of
Hamiltonian from 2-pt corr. fns.

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Operators (interpolating fields)

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Large basis of ops → matrix of corrs.

Eigenvalue problem

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

Spectroscopy on the lattice

Energy eigenstates of
Hamiltonian from 2-pt corr. fns.

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Operators (interpolating fields)

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Large basis of ops → matrix of corrs.

Eigenvalue problem

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

$$Z_i^{(n)} \equiv \langle 0 | \mathcal{O}_i | n \rangle$$

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

Eigenvectors → $Z^{(n)}$

$(t \gg t_0)$

Excited Charmonium Spectrum

Hadron Spectrum Collaboration

JHEP 07 (2012) 126 – Liuming Liu, Graham Moir, Mike Peardon, Sinéad Ryan, CT, Pol Vilaseca;
Jo Dudek, Robert Edwards, Bálint Joó, David Richards

Excited Charmonium Spectrum

Hadron Spectrum Collaboration

Dynamical (unquenched) u , d and s quarks [$N_f = 2+1$]

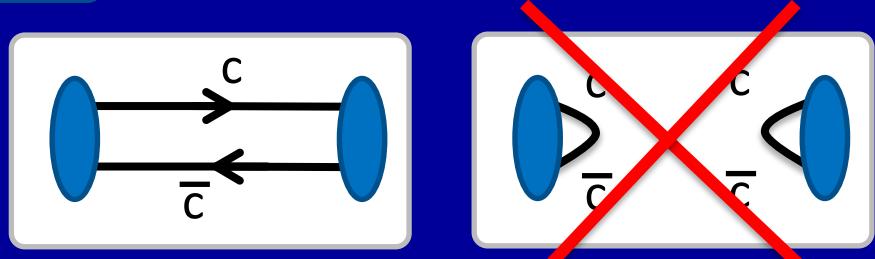
Relativistic charm

Anisotropic – finer in temporal dir ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm

Two volumes: 16^3 , $\mathbf{24}^3$ ($L_s \approx 1.9$, 2.9 fm)

$M_\pi \approx 400$ MeV

Only connected contributions



Excited Charmonium Spectrum

Hadron Spectrum Collaboration

Dynamical (unquenched) u , d and s quarks [$N_f = 2+1$]

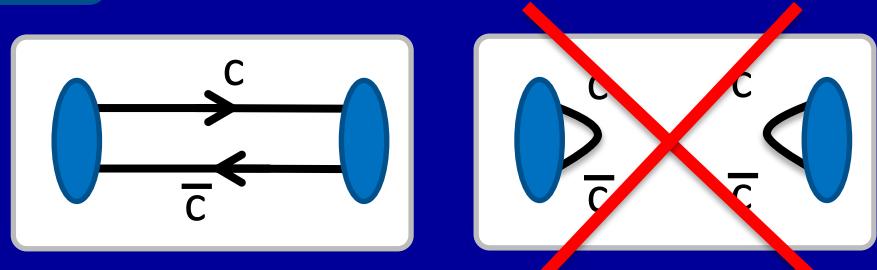
Relativistic charm

Anisotropic – finer in temporal dir ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm

Two volumes: 16^3 , $\mathbf{24}^3$ ($L_s \approx 1.9$, 2.9 fm)

$M_\pi \approx 400$ MeV

Only connected contributions



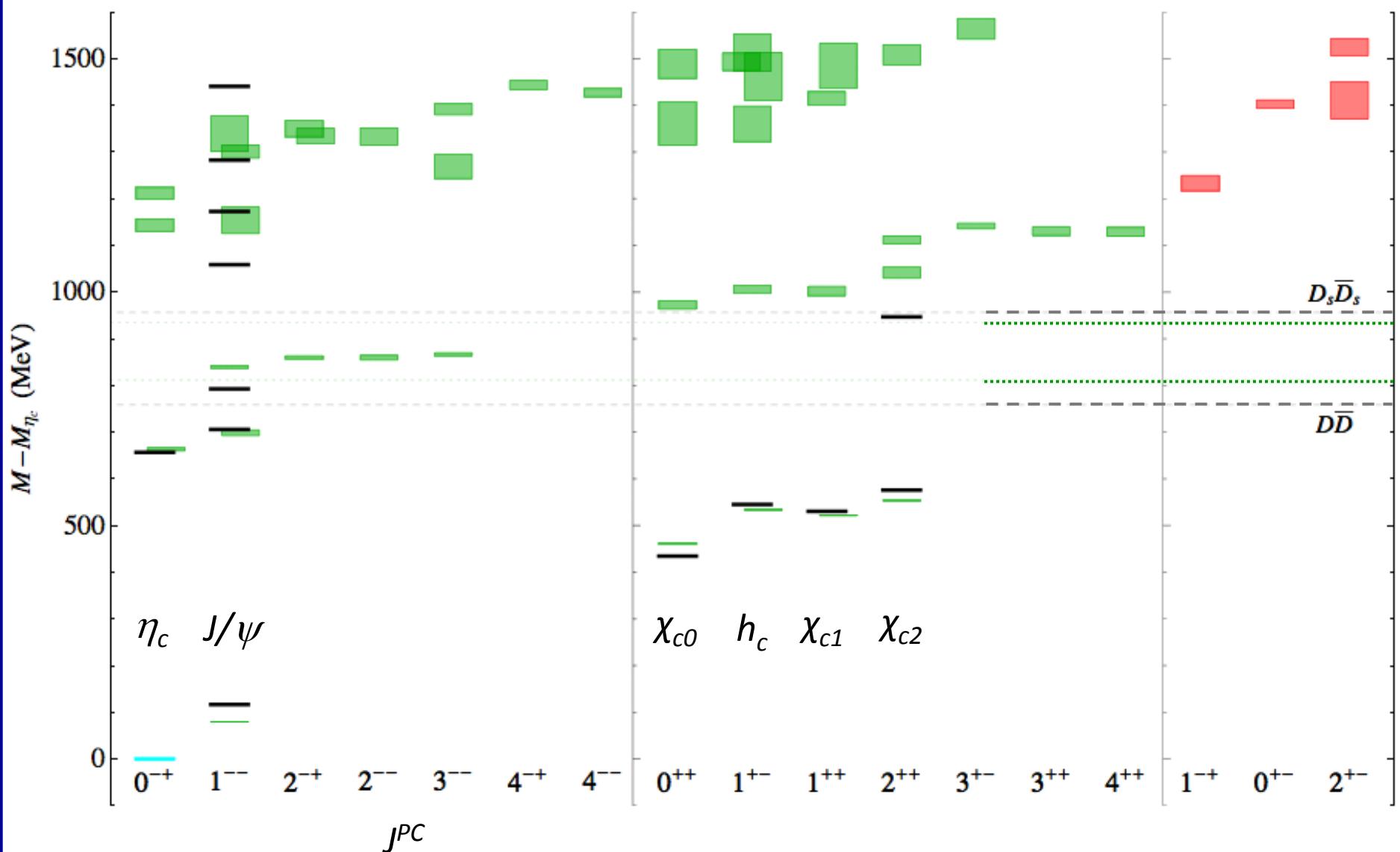
Various techniques

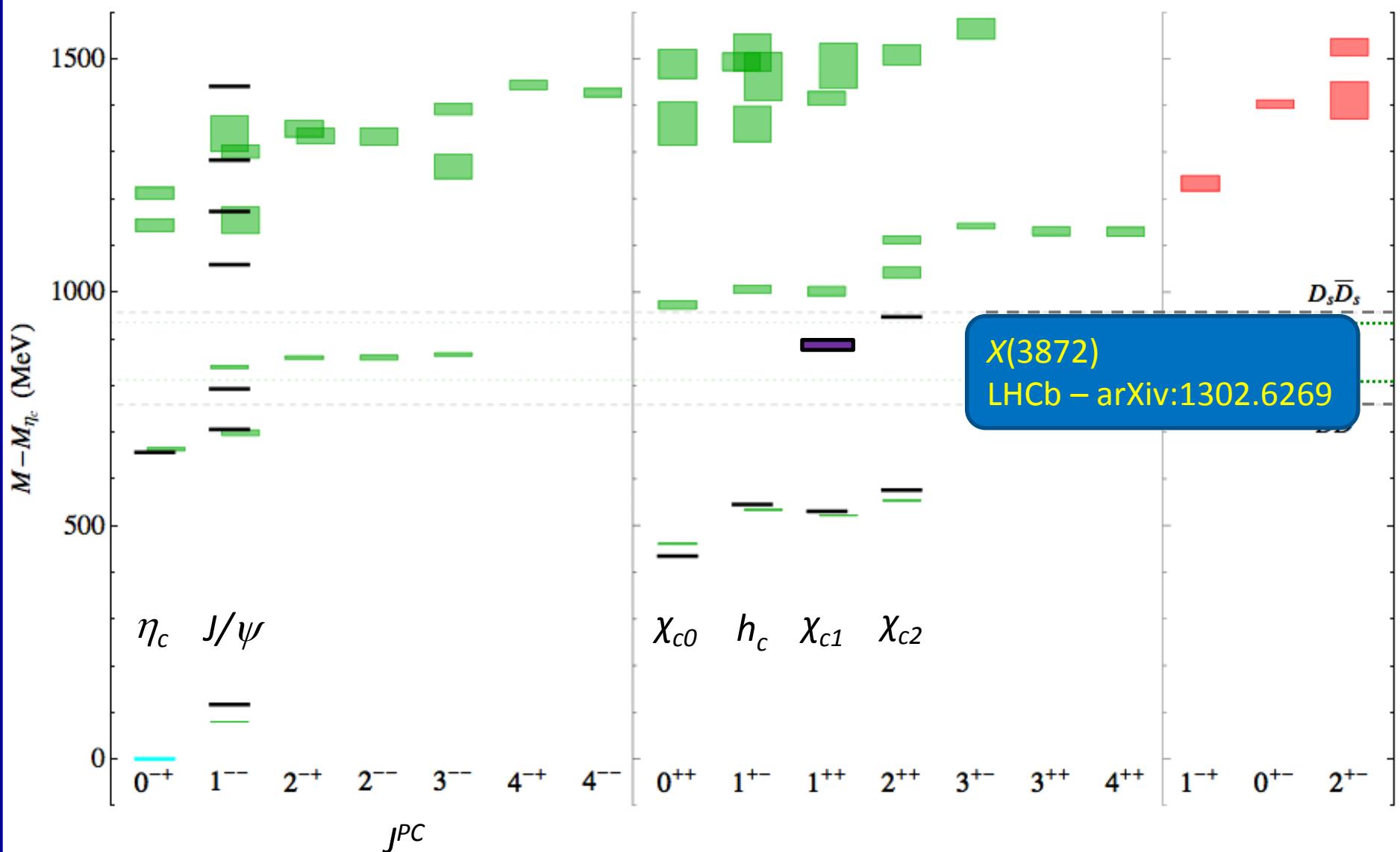
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Large no. of ops. of different structures, including gluonic excitations

→ Large number and variety of states

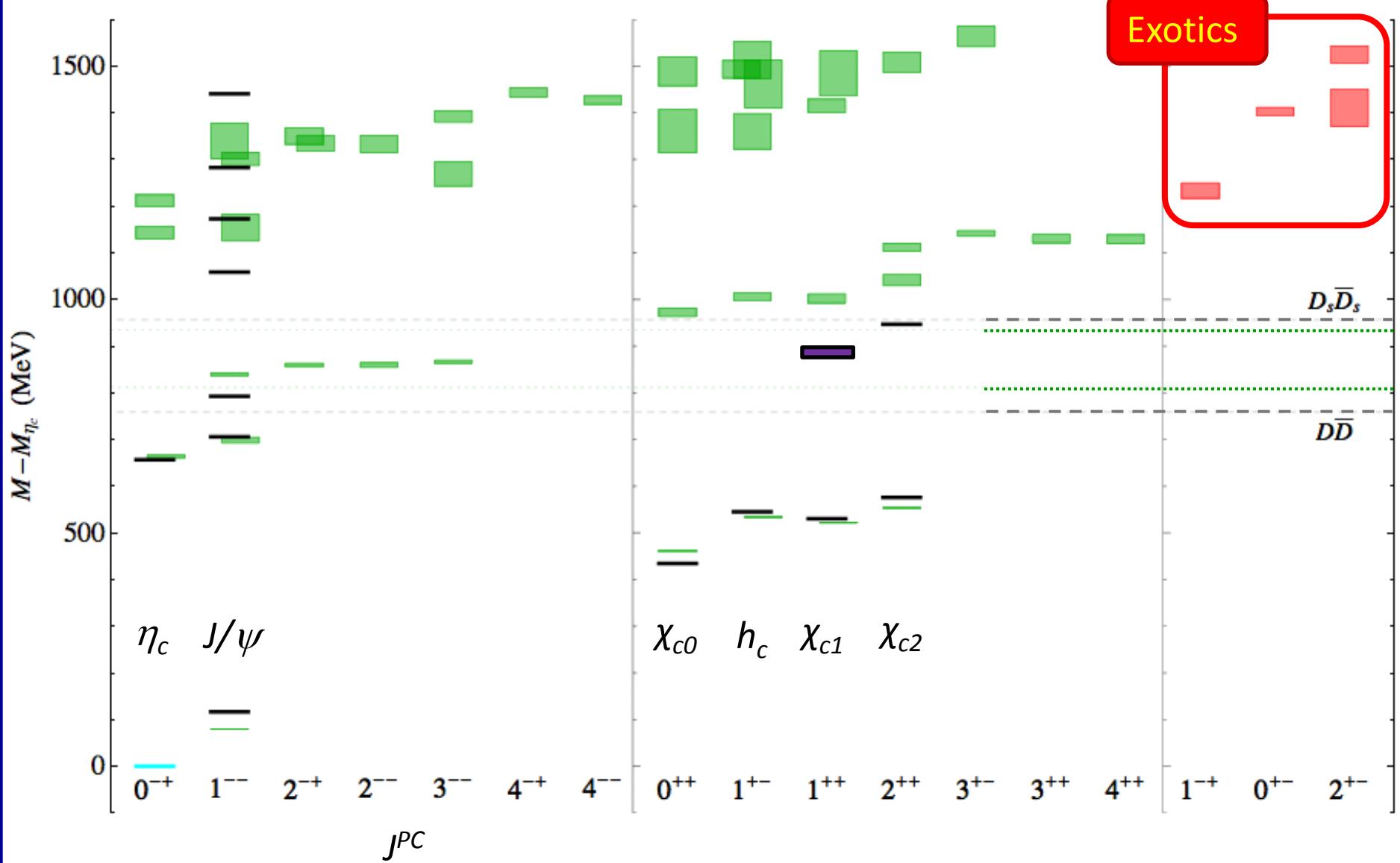
$$\sim \bar{\psi} \Gamma D \dots \psi$$





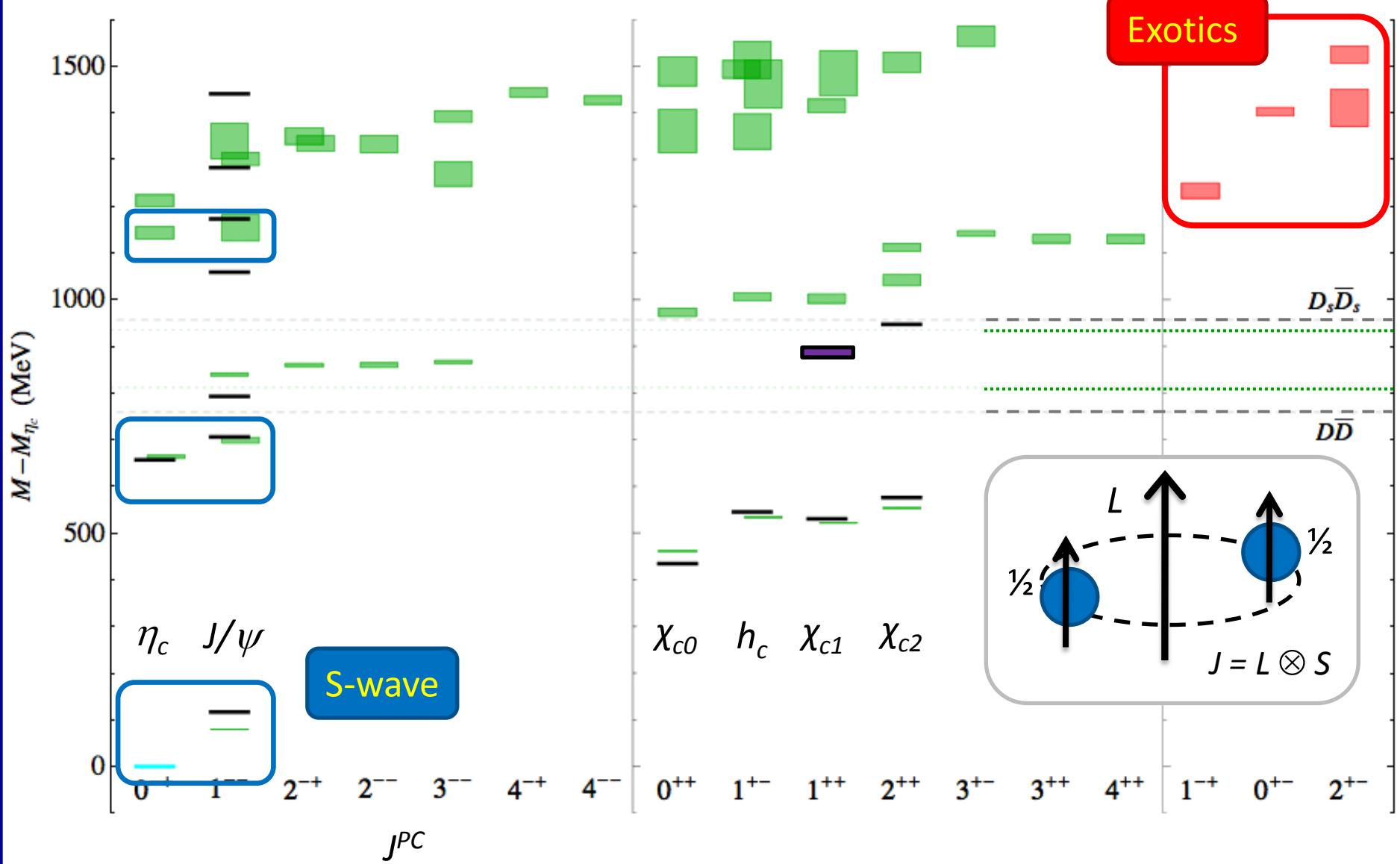
24^3 , $M_\pi \approx 400$ MeV

JHEP 07 (2012) 126



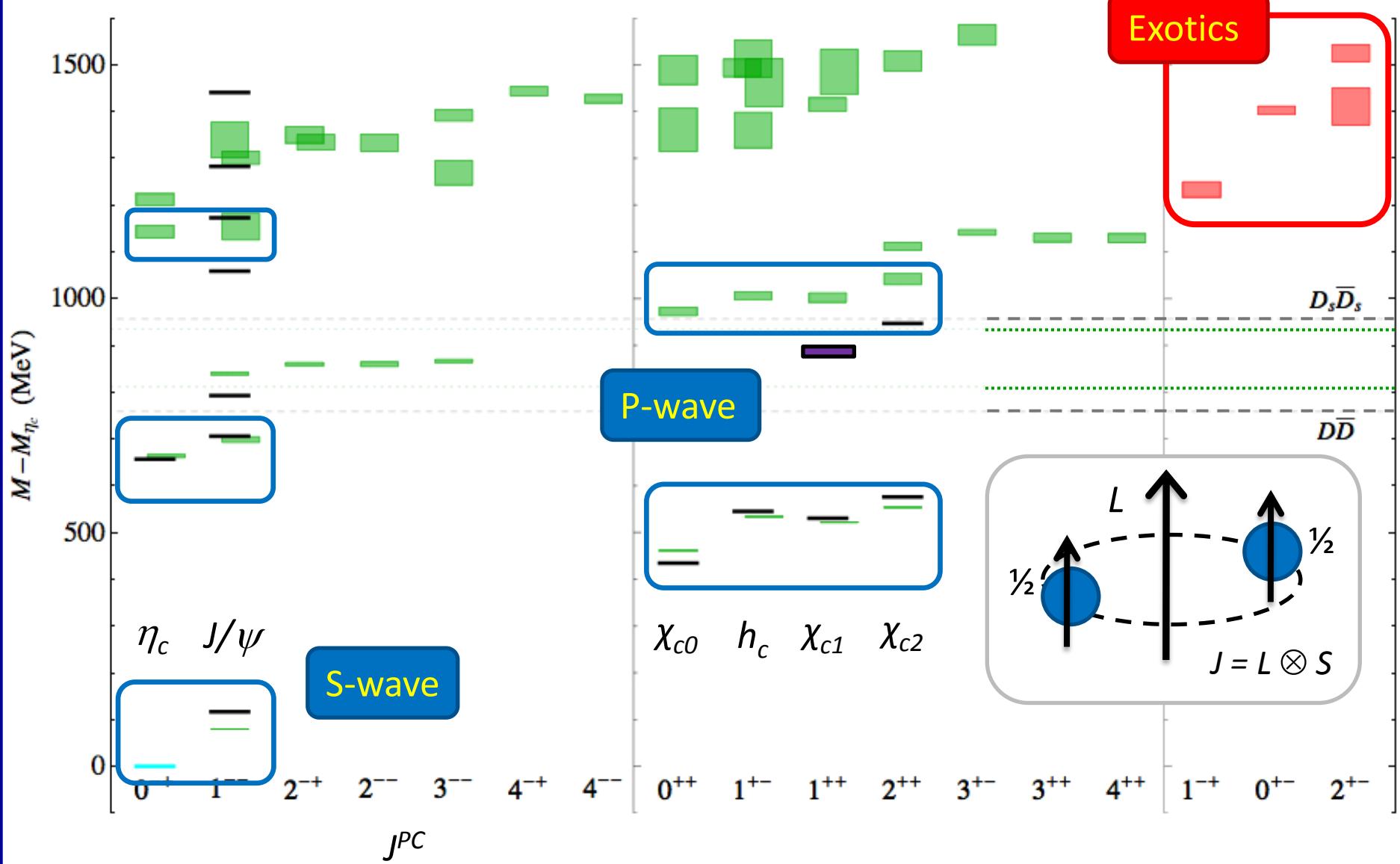
24^3 , $M_\pi \approx 400$ MeV

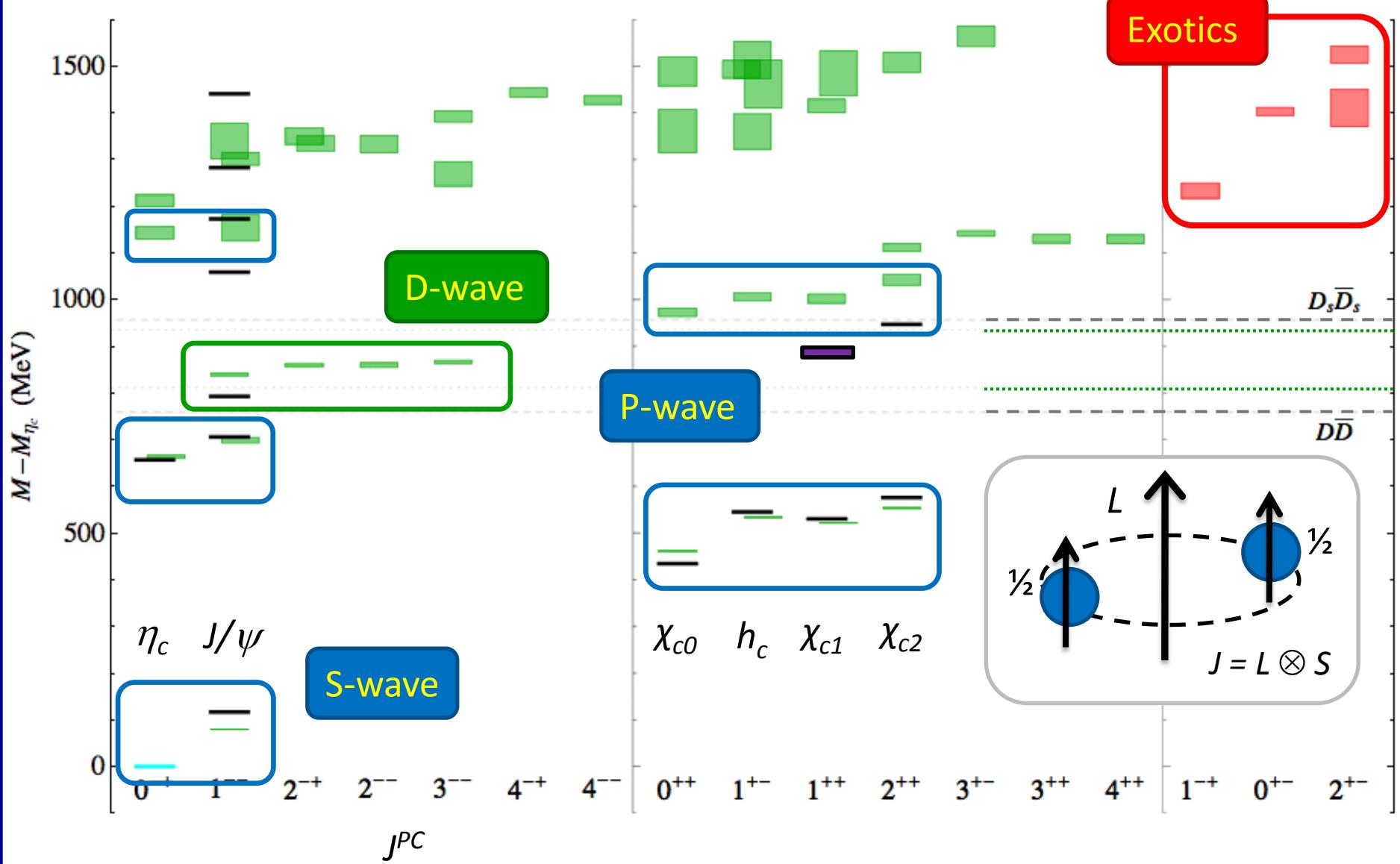
JHEP 07 (2012) 126



$24^3, M_\pi \approx 400$ MeV

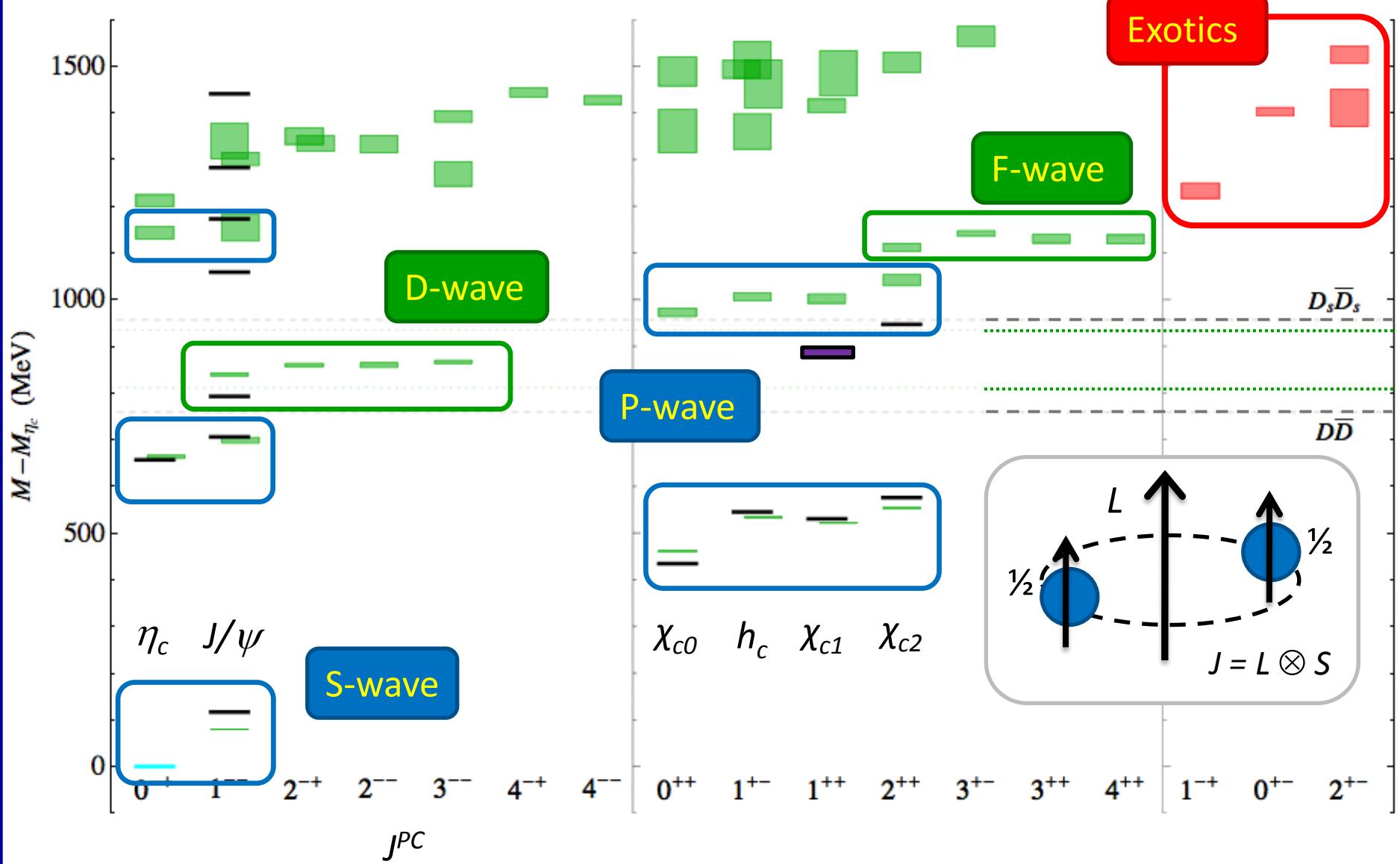
JHEP 07 (2012) 126

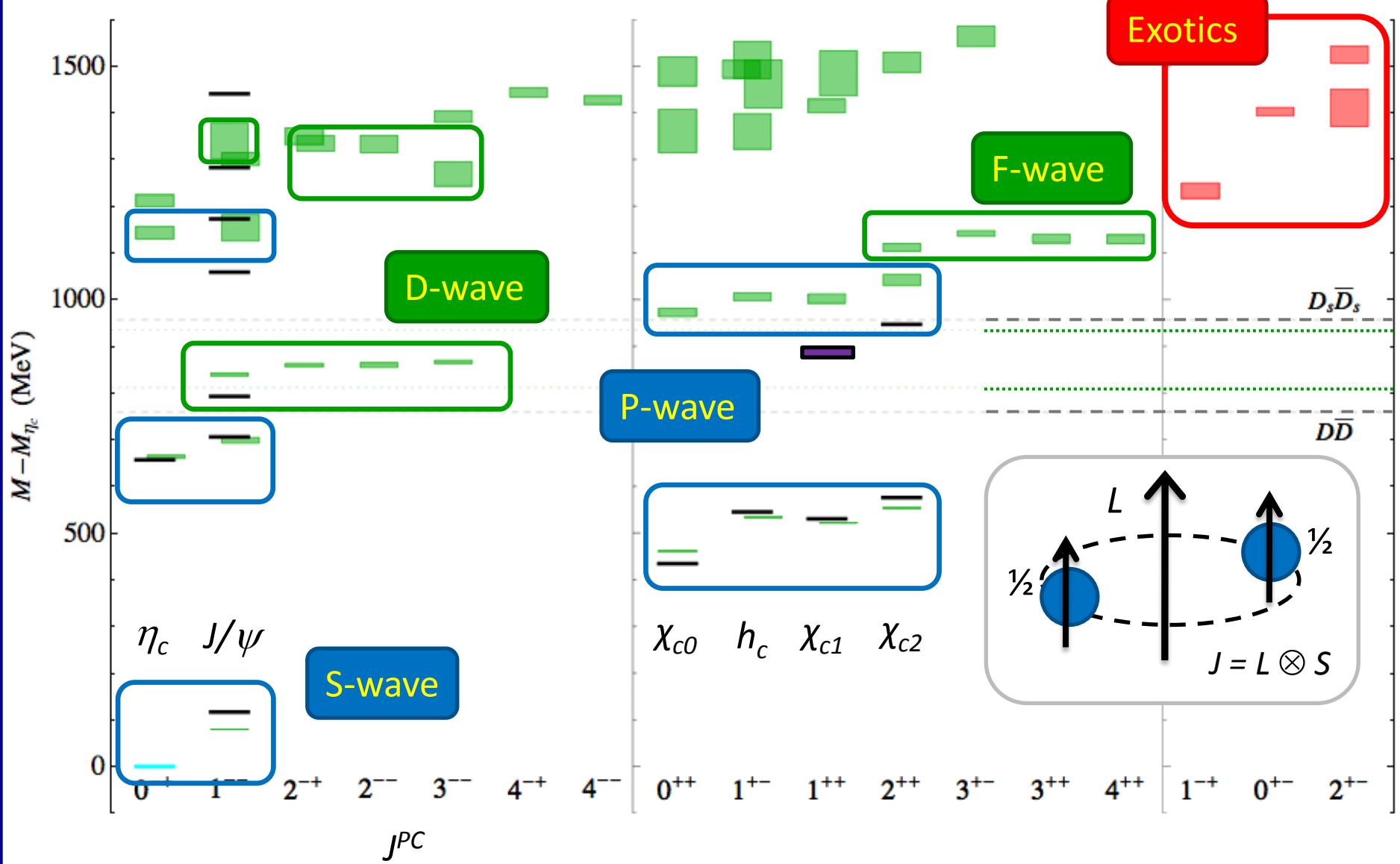




$24^3, M_\pi \approx 400$ MeV

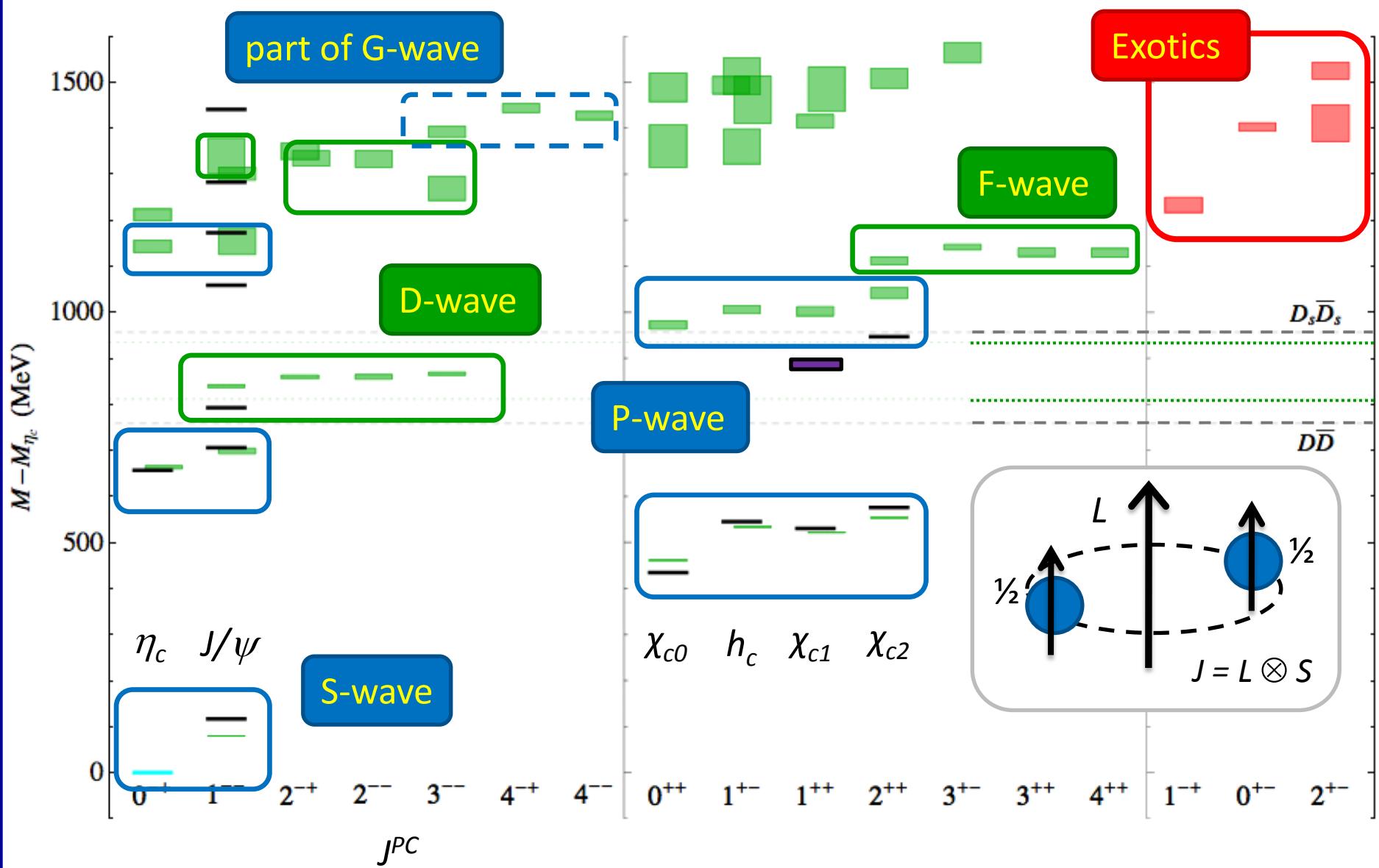
JHEP 07 (2012) 126

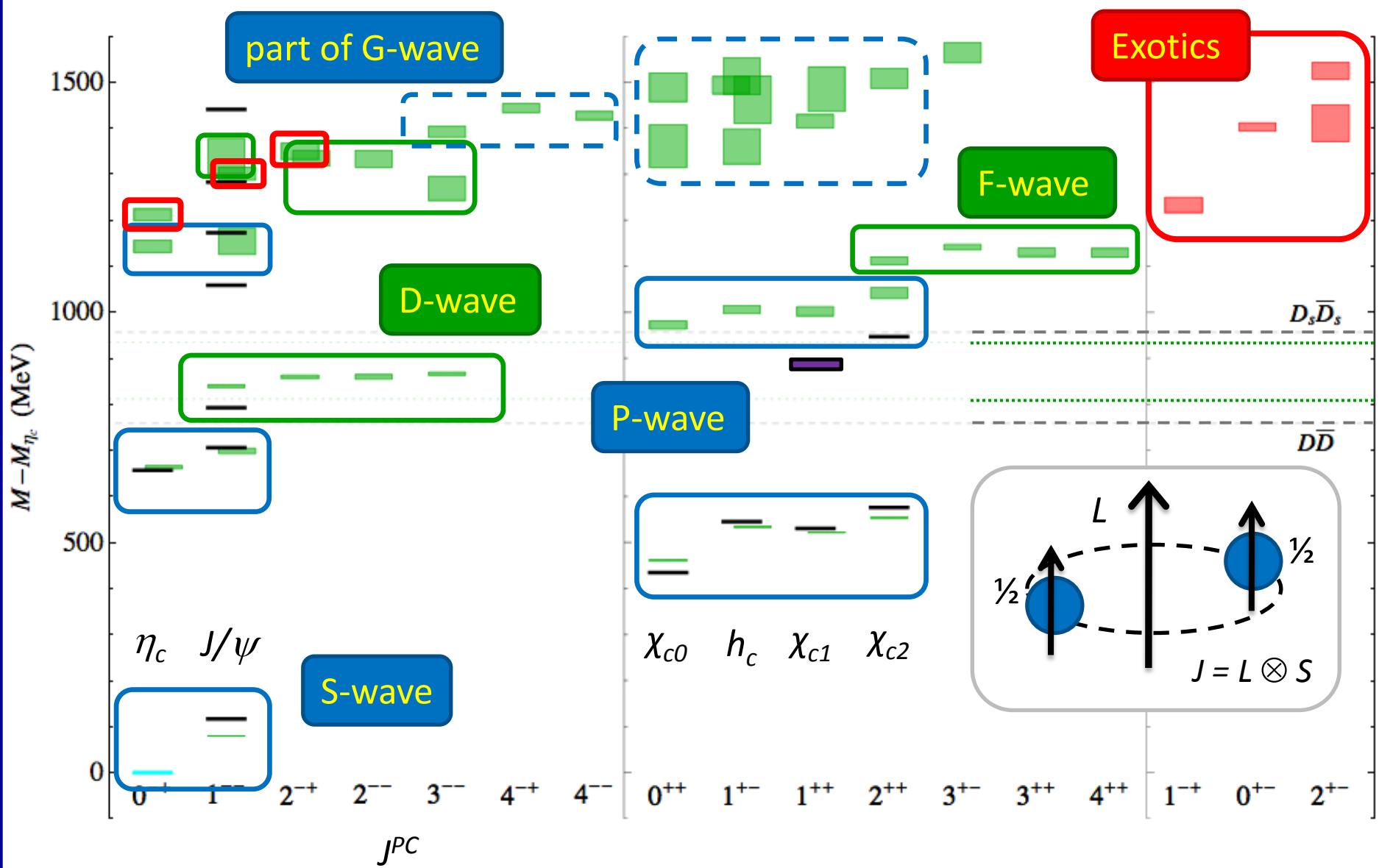




24^3 , $M_\pi \approx 400$ MeV

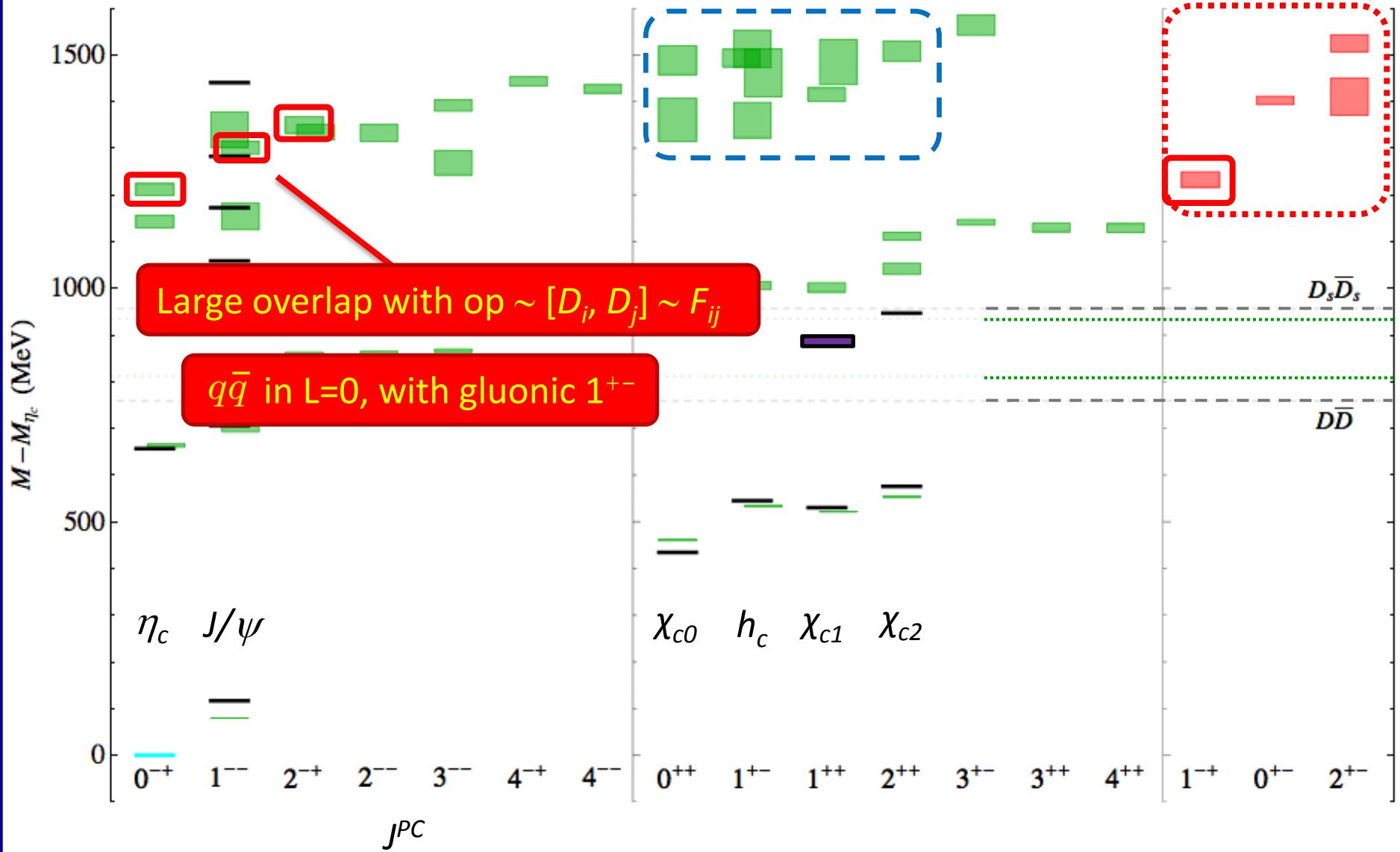
JHEP 07 (2012) 126





24^3 , $M_\pi \approx 400$ MeV

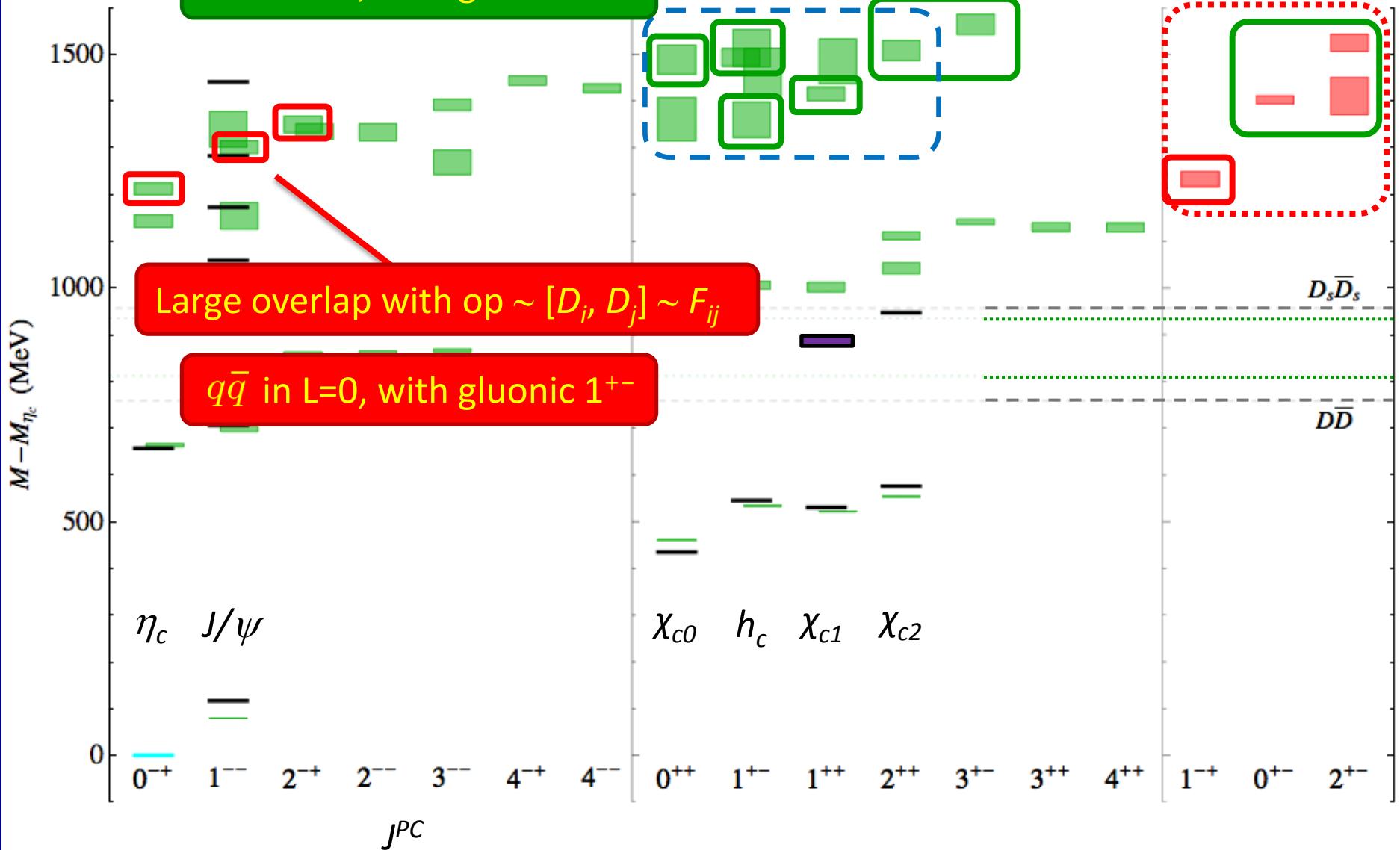
JHEP 07 (2012) 126



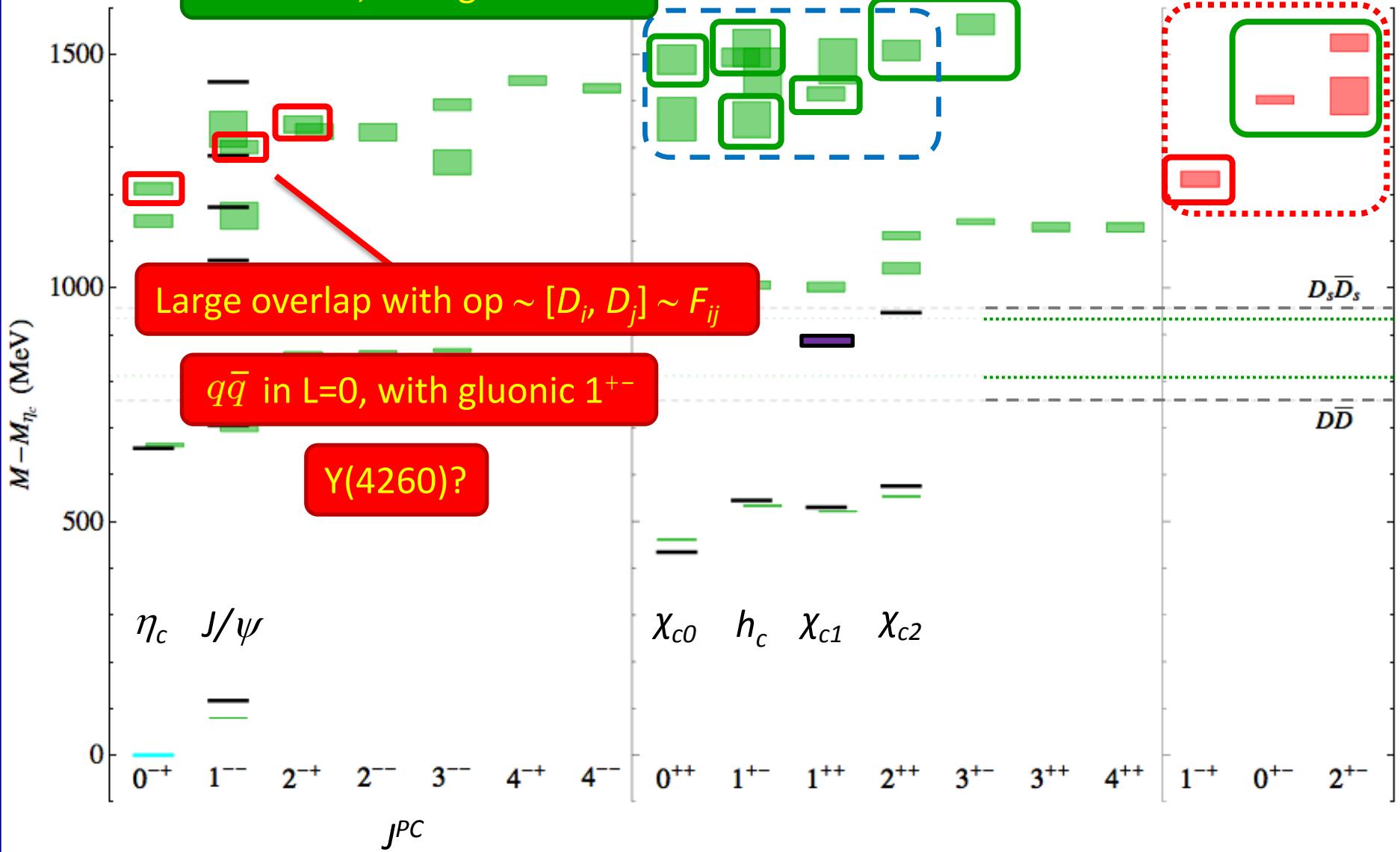
24^3 , $M_\pi \approx 400$ MeV

JHEP 07 (2012) 126

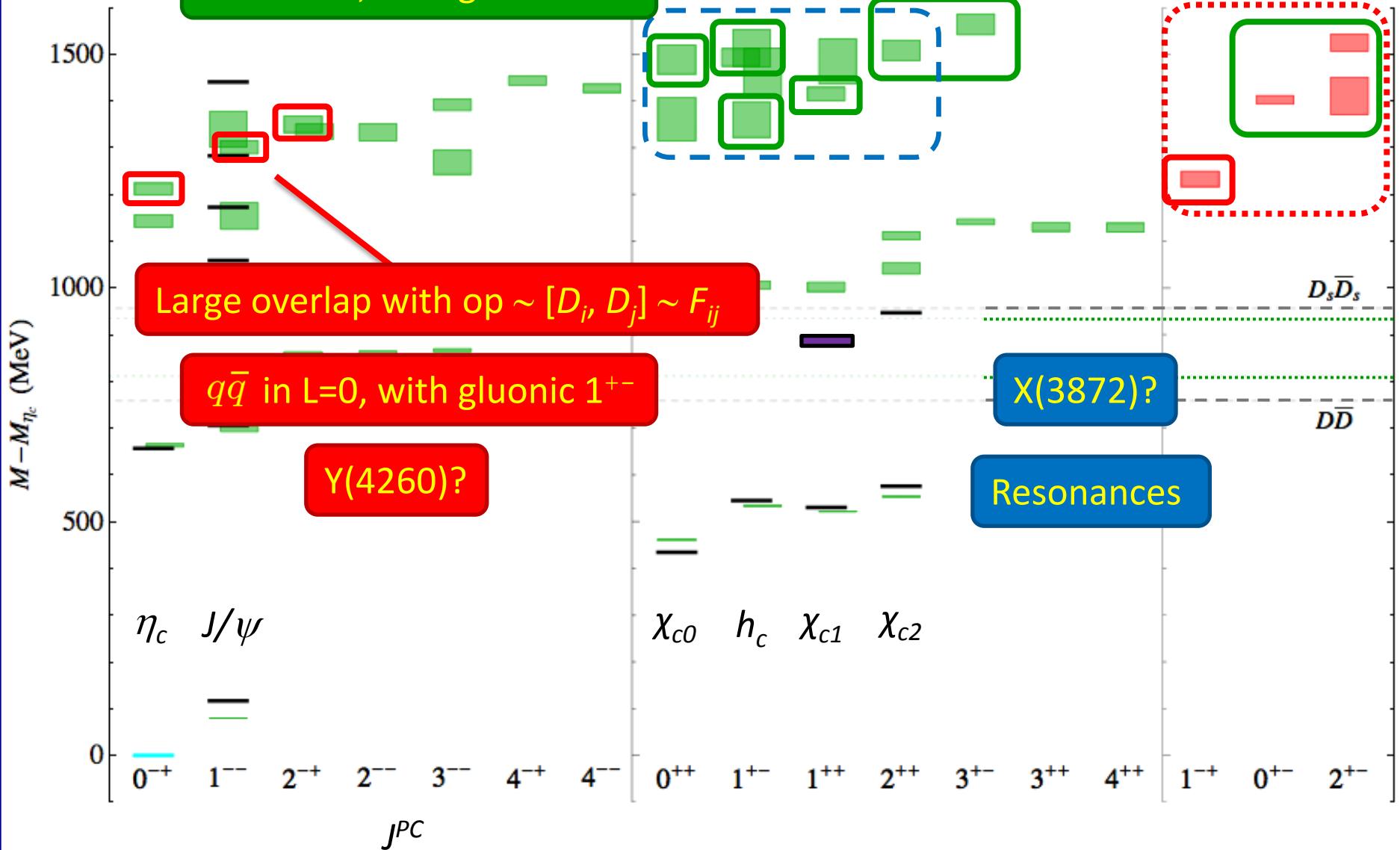
$q\bar{q}$ in L=1, with gluonic 1^{+-}



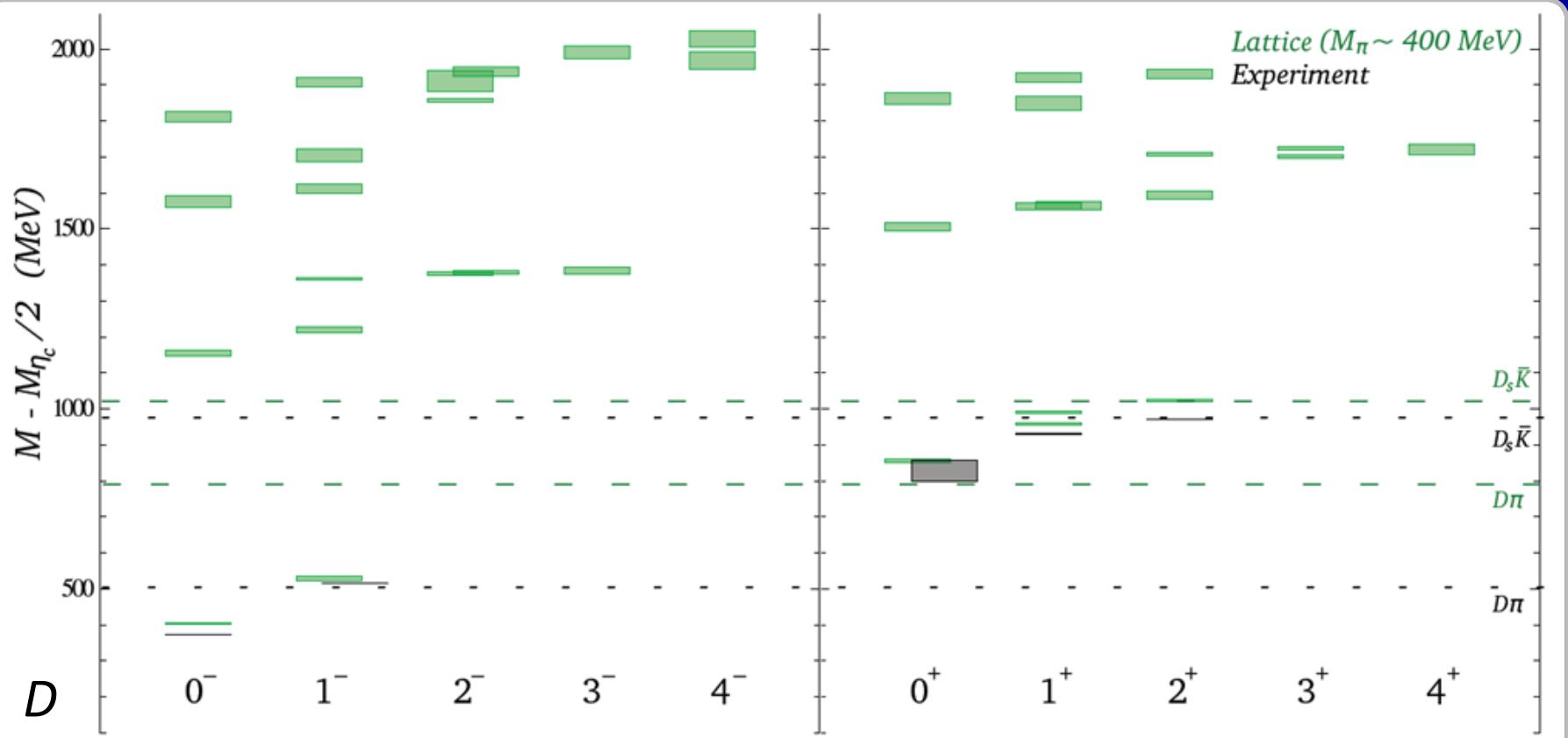
$q\bar{q}$ in L=1, with gluonic 1^{+-}



$q\bar{q}$ in L=1, with gluonic 1^{+-}

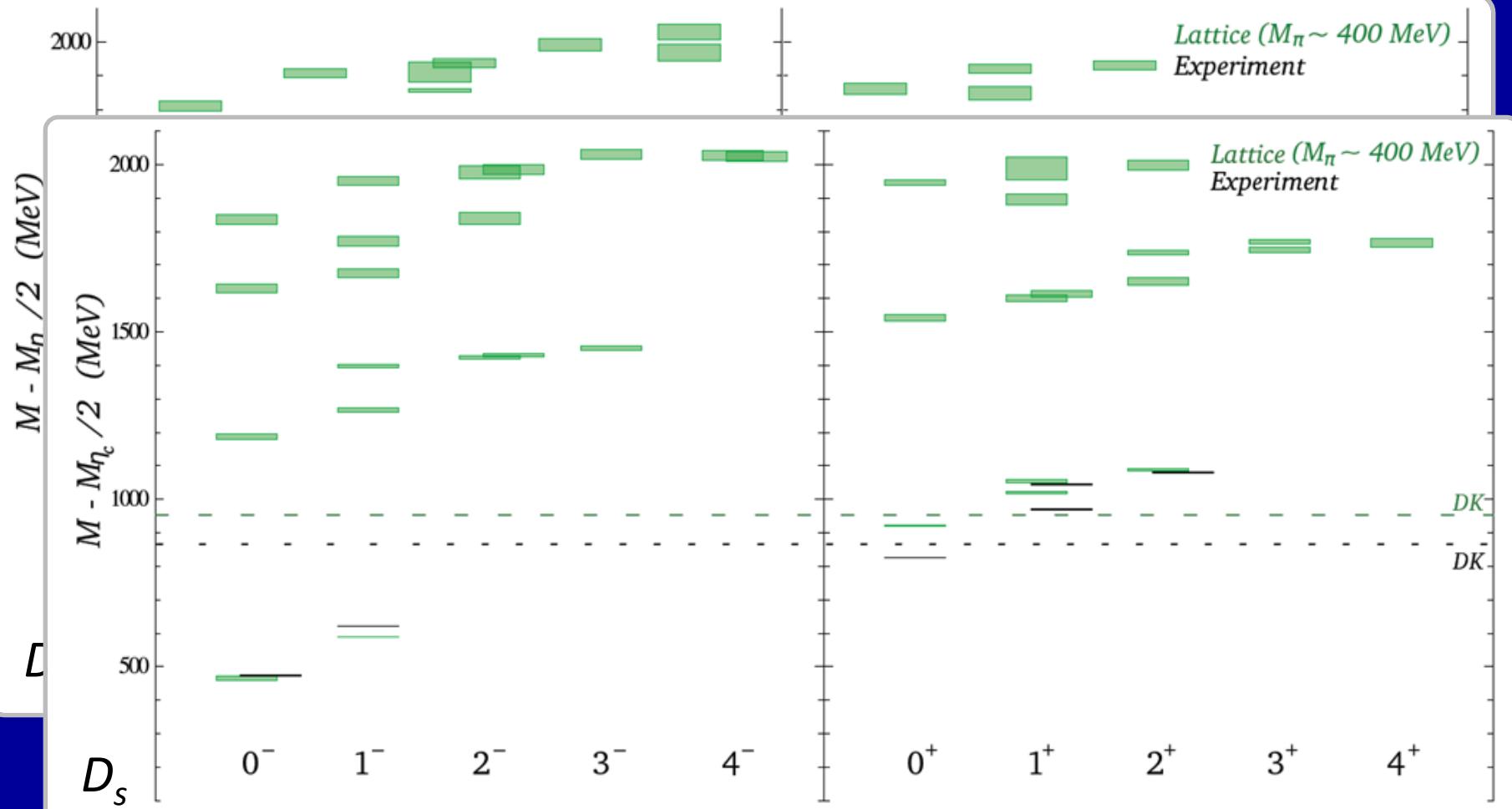


Charmed (D/D_s) mesons



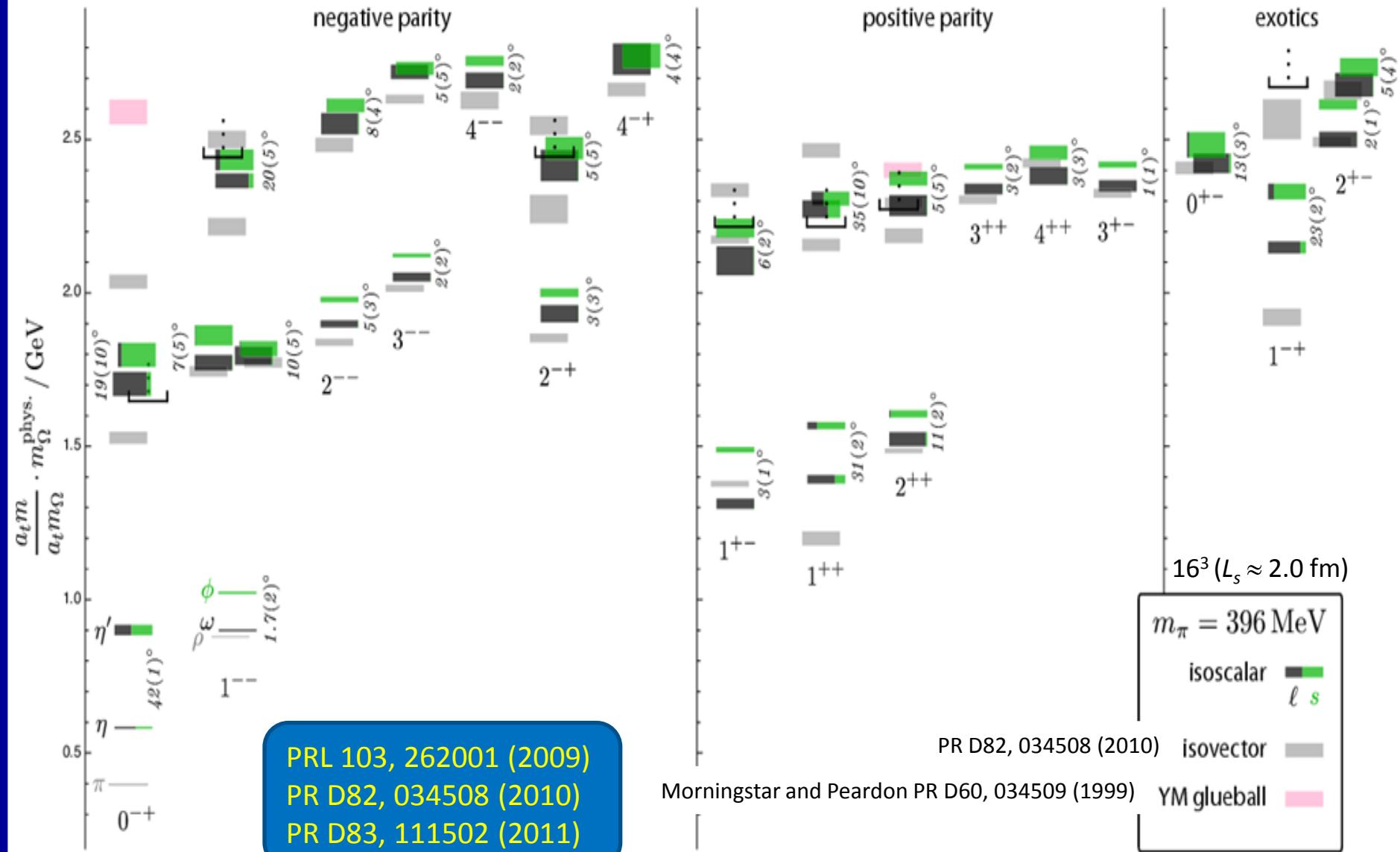
Graham Moir, Mike Peardon, Sinéad Ryan, CT, Liuming Liu, arXiv:1301.7670 [to appear in JHEP]

Charmed (D/D_s) mesons



Graham Moir, Mike Peardon, Sinéad Ryan, CT, Liuming Liu, arXiv:1301.7670 [to appear in JHEP]

Light unflavoured mesons



Resonances on the lattice

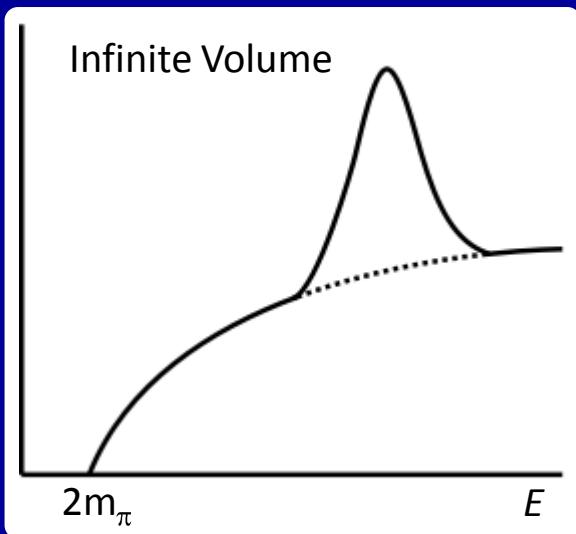
Imaginary time – can't study
dynamics (e.g. scattering) directly

Resonances on the lattice

Imaginary time – can't study dynamics (e.g. scattering) directly

Infinite volume

Continuous spectrum



Resonances on the lattice

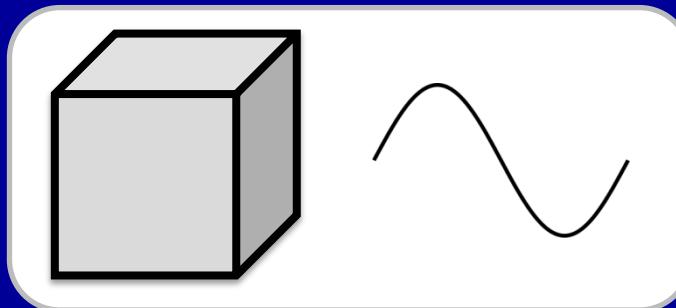
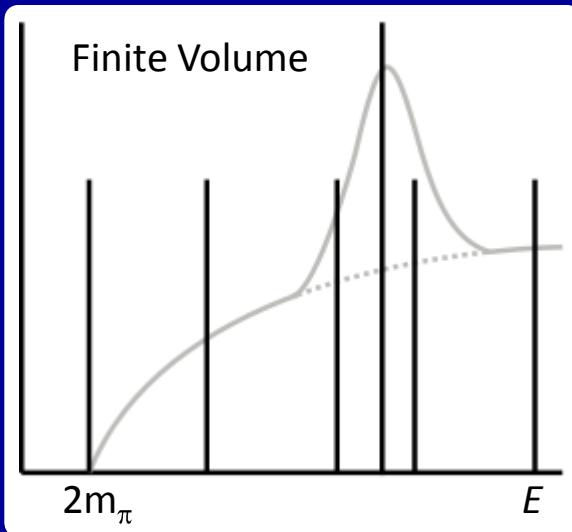
Imaginary time – can't study dynamics (e.g. scattering) directly

Infinite volume

Continuous spectrum

Finite volume

Quantised momenta → discrete spectrum

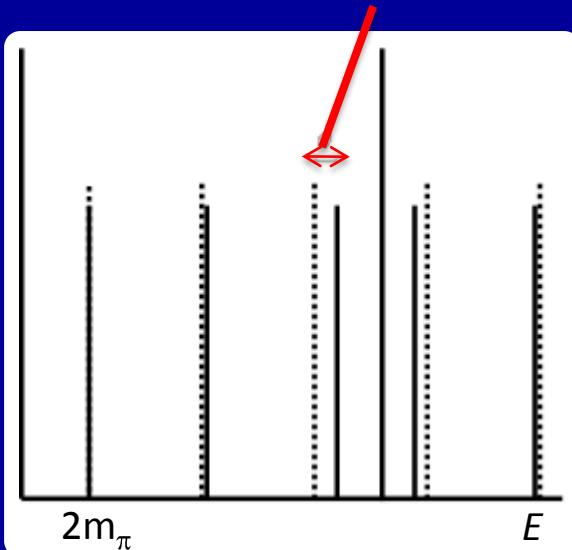


Resonances on the lattice

$$\Delta E(L_s) \rightarrow \delta(E, L_s)$$

Understanding unstable mesons –
need energies of multi-hadron states

Lüscher: (elastic) energy shifts in **finite vol.**
→ **infinite vol.** scattering phase shift



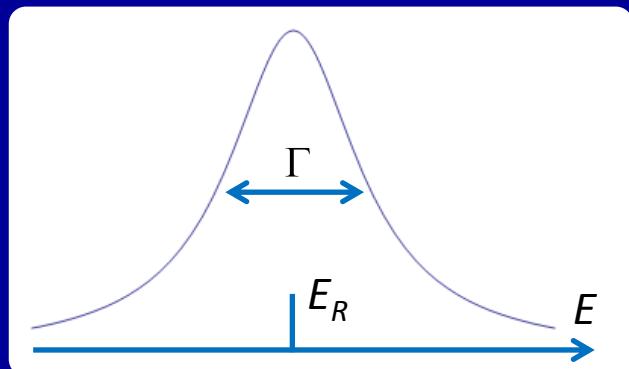
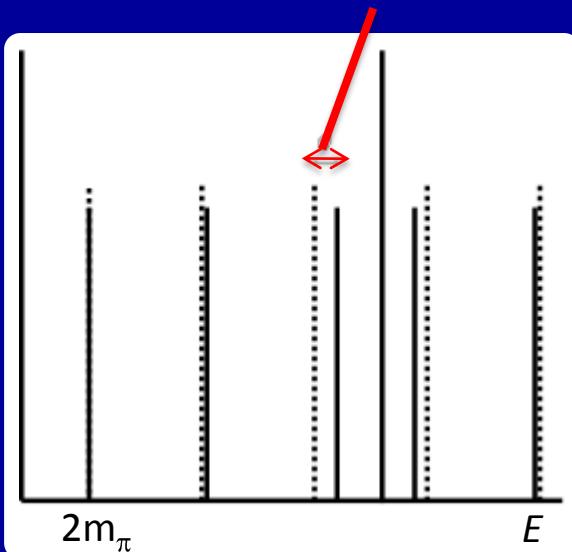
Resonances on the lattice

$$\Delta E(L_s) \rightarrow \delta(E, L_s)$$

Understanding unstable mesons –
need energies of multi-hadron states

Lüscher: (elastic) energy shifts in **finite vol.**
→ **infinite vol.** scattering phase shift

e.g. $\pi\pi \rightarrow \rho \rightarrow \pi\pi$



$$\sigma_l(E) \propto \sin^2 \delta_l(E) = \frac{(\Gamma/2)^2}{(E - E_R)^2 + (\Gamma/2)^2}$$

Extract phase shift at discrete E_{cm}

Map out phase shift → resonance parameters etc

The ρ resonance in $\pi\pi$ scattering

$\pi\pi \rightarrow \rho \rightarrow \pi\pi$

Dudek, Edwards, Thomas, PR D87, 034505 (2013)

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Operators:

single-meson

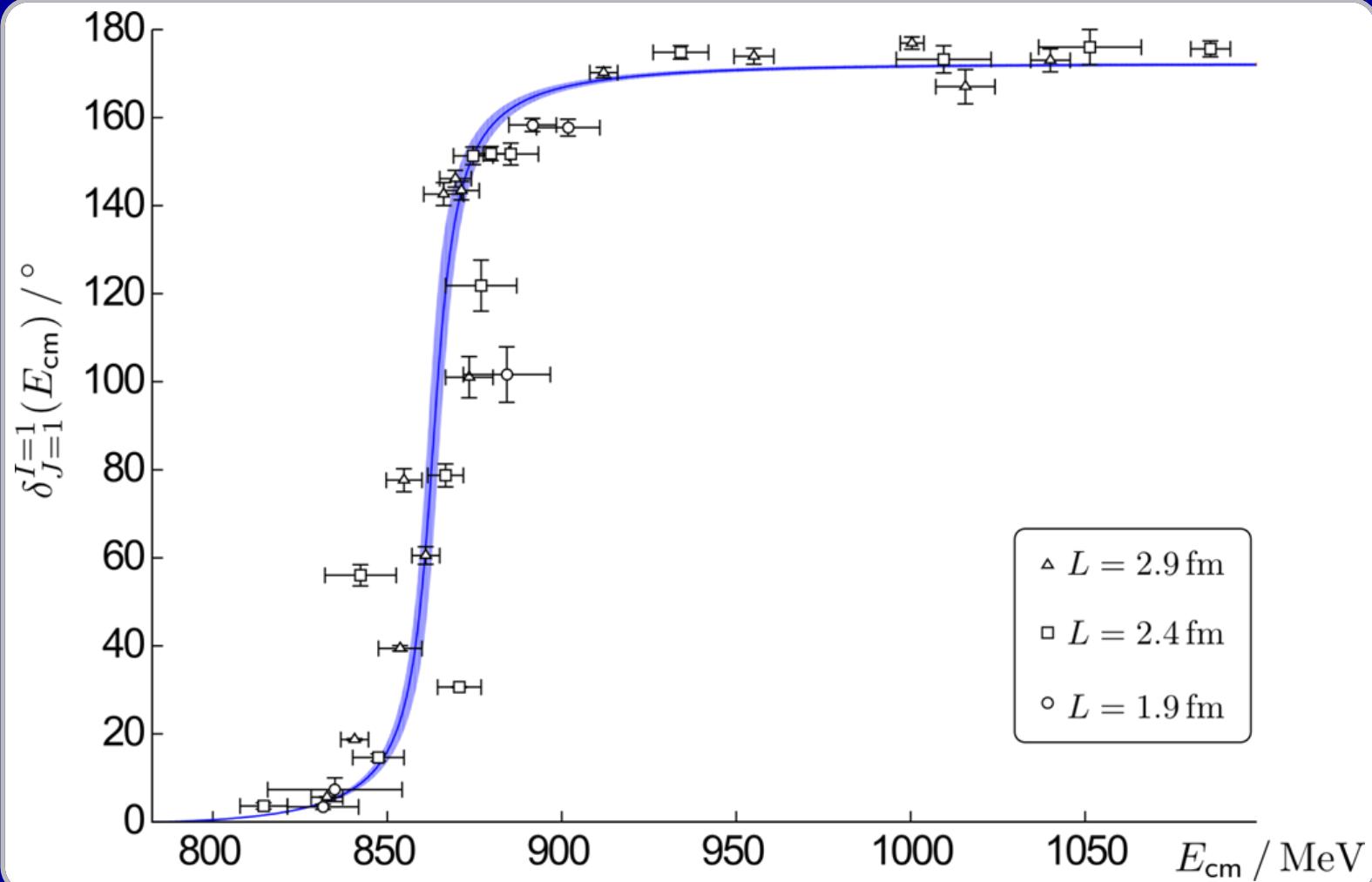
$\sim \bar{\psi} \Gamma D \dots \psi$

and $\pi\pi$

$$\mathcal{O}(\vec{P}) = \sum_{\vec{p}_1, \vec{p}_2} \mathcal{C}_\Lambda(\vec{P}, \vec{p}_1, \vec{p}_2) \mathcal{O}_\pi(\vec{p}_1) \mathcal{O}_\pi(\vec{p}_2)$$

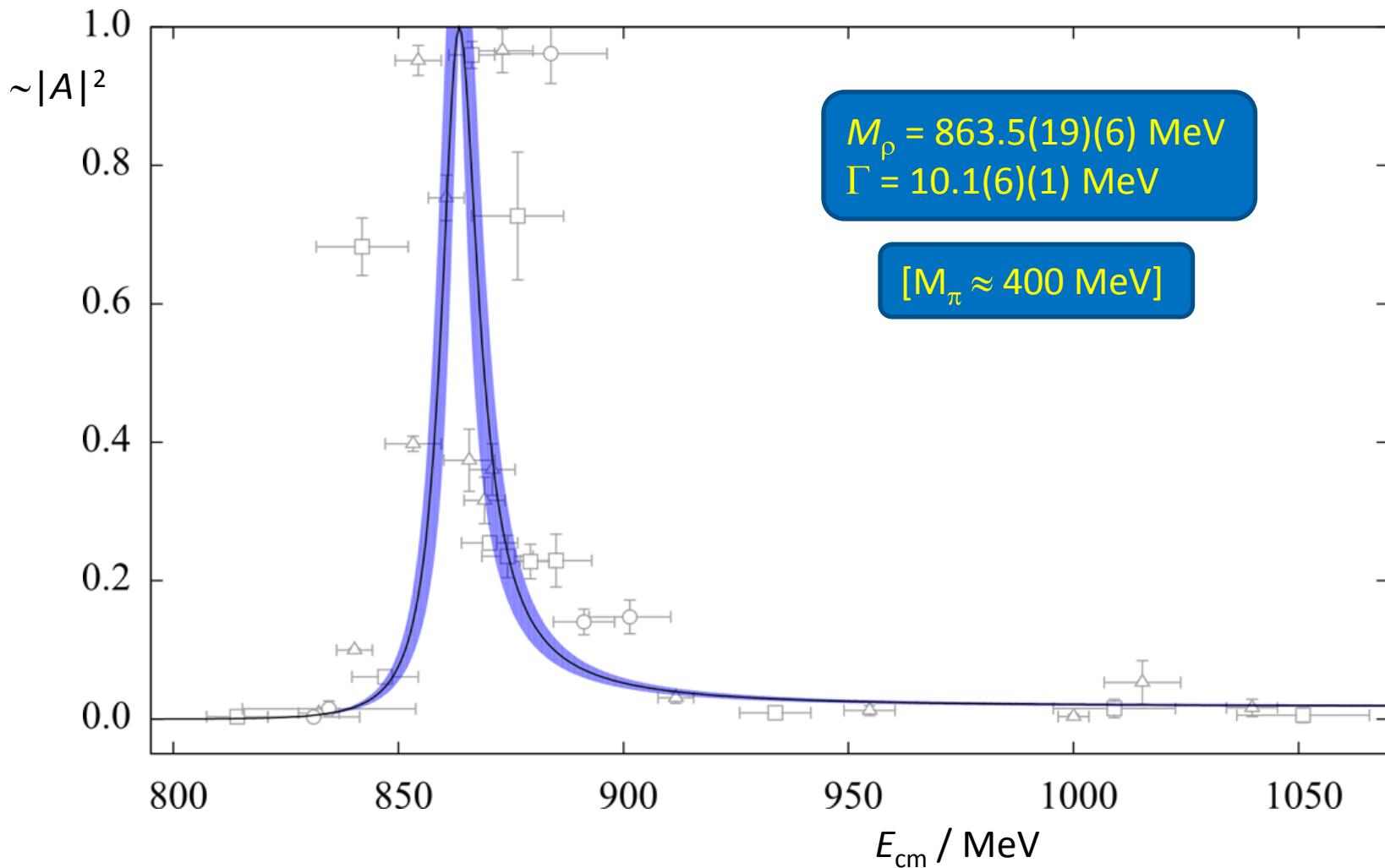
Earlier work on $\pi\pi$ scattering with isospin = 2 (non-resonant):
PR D83, 071504 (2011); PR D86, 034031 (2012)

The ρ resonance in $\pi\pi$ scattering



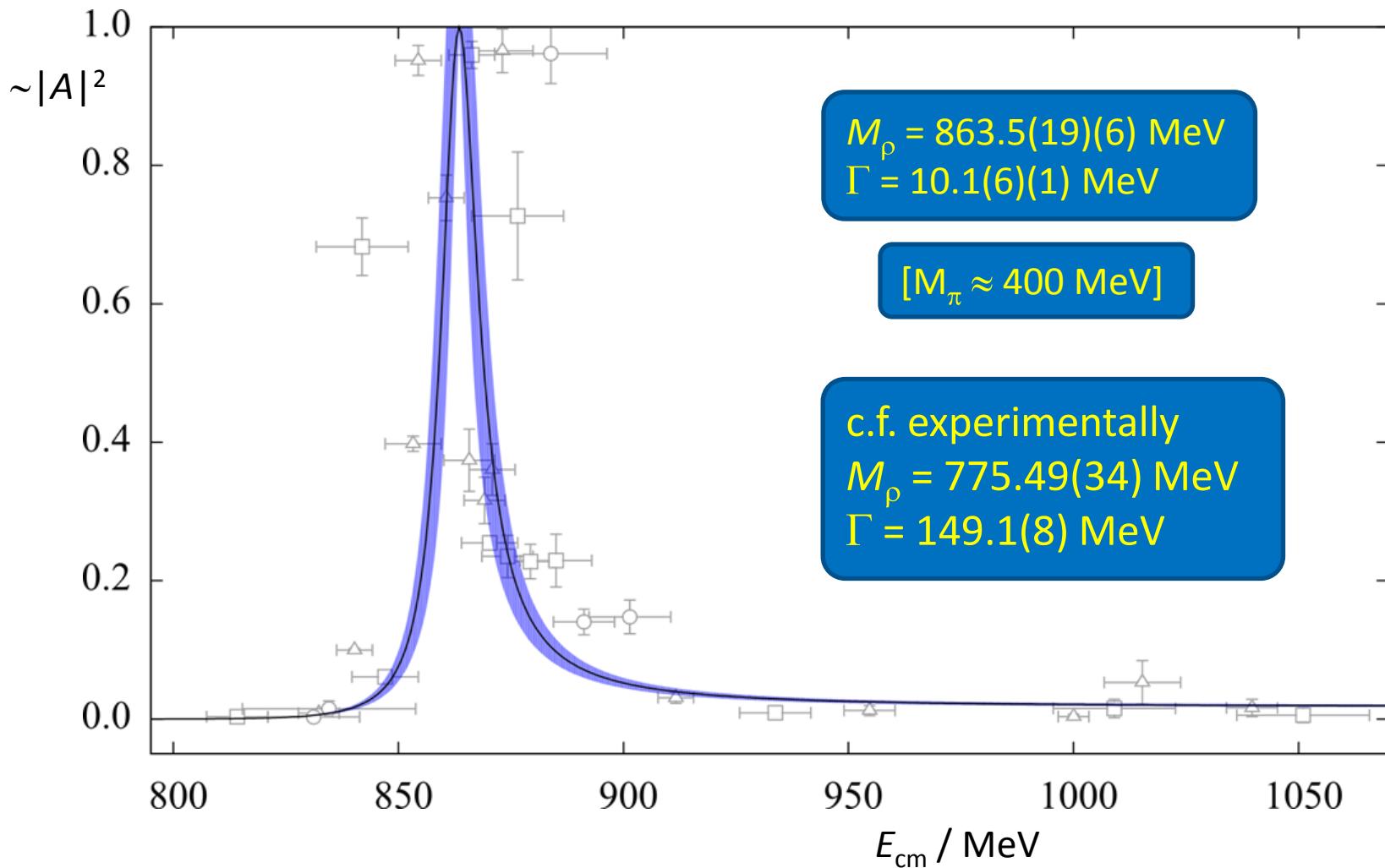
Mapped out in detail

The ρ resonance in $\pi\pi$ scattering



Mapped out in detail

The ρ resonance in $\pi\pi$ scattering



Mapped out in detail

Summary and Outlook

Summary

- Excited charmonia, light mesons, D/D_s mesons, [baryons]
 - Exotics, **hybrid phenomenology**, exp. interest
- **Mapping out** phase shifts → **scattering observables**
 - $\pi\pi$ $|l|=1$ scattering – ρ resonance
 - [$\pi\pi$ $|l|=2$ scattering – S and D -wave ($L=0,2$)]



Summary and Outlook

Summary

- Excited charmonia, light mesons, D/D_s mesons, [baryons]
 - Exotics, **hybrid phenomenology**, exp. interest
- **Mapping out** phase shifts → **scattering observables**
 - $\pi\pi$ $|l|=1$ scattering – ρ resonance
 - [$\pi\pi$ $|l|=2$ scattering – S and D -wave ($L=0,2$)]

Outlook

- **Other scattering channels – resonances, decays, ...**
- Radiative (and other) **transitions**
- Systematics: lighter pion masses, larger volumes, ...
- **Understand strong interaction phenom. within QCD**

