Duality of Cosmic F-String Decay Rates

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Motivation

There has been growing interest in properties of highly excited strings:

- collider physics (for TeV scale strings)[1]
- string/black hole correspondence[2]
- higher-spin gauge theories[3]
- plasmas and fluids[4]
- cosmic strings[5]

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[1] Feng, Lust, Schlotterer, Stieberger (2011); . . .

[2] Sen (1995); Horowitz, Polchinski (1997); Cornalba, Costa, Penedones, Vieira (2006); Amati, Cialafoni, Veneziano (2008); Giddings, Gross, Maharana (2008); . . .

[3] Sagnotti, Taronna (2011); Bianchi, Lopez, Richter (2011); . . .

[4] Dennis et al (2010); Kleckner, Irvine (2013)

[5] Dvali & Vilenkin (2004); Copeland, Myers, Polchinski (2004); Banks & Seiberg (2011); . . .

Example 1

Consider a theory with a local $U(1)$, e.g. the abelian-Higgs[6] approximation,

$$S = \int d^4x - \frac{1}{4g^2}F^2 + (D_\mu \phi)^\dagger D^\mu \phi - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2,$$

the "mexican hat" potential leads to an S^1 vacuum manifold with

$$\pi_1(S^1) = \mathbb{Z} \qquad \Rightarrow \qquad \textit{strings}$$

The presence of strings is hinted towards in the flux through area C,

$$\int_C F = -\frac{2\pi n}{g}, \qquad n \in \mathbb{Z}$$

which is quantized, implying an integer number of strings can pass through C, each with integer number of flux quanta, $2\pi/g$

[6] Nielsen & Olesen (1973)

Example 2

Same principle also applies to the cosmic strings of string theory

Consider D-brane inflation: a BPS D3-brane rolls down a CY throat and meets a BPS $\bar{\mathrm{D}}3$-brane

F-string stretch from one stack to the other
There is a $U(1)$ symmetry associated to a either of these stacks

When the length of open strings approaches the string scale there appears a tachyon in the open string spectrum, leading to an instability, and the branes annihilate (tachyon condensation)[7]

$$\text{D3-}\bar{\text{D}}\text{3-brane pairs} \quad \rightarrow \quad \text{BPS D-strings}$$

These strings would appear as cosmic strings

[7] Sen (1998)

Cosmic String Zoo

Compactifications of string theory lead to many potential cosmic string candidates:[8]

- elementary strings
- D-strings
- wrapped D-branes
- solitonic strings
- electric and magnetic flux tubes

$\vdots$

[8] Dvali, Vilenkin (2004); Copeland, Myers, Polchinski (2004); Polchinski (2006); Banks, Seiberg (2011)

Astrophysical Signatures I

Cosmic strings produce a variety of signatures:[9]

- gravitational-wave bursts [Damour & Vilenkin (2000); O Callaghan et al (2010);. . .]
- gravitational-wave stochastic background [Olmez, Mandic, Siemens (2010);. . .]
- gamma ray bursts [Bhattacharjee & Sigl (1998); . . .]
- ultra-high energy cosmic rays [Berezinsky, Hnatyk, Vilenkin (2001);. . .]
- magnetogenesis [Battefeld, Battefeld, Wesley, Wyman (2008);. . .]
- lensing [Chernoff, Tye (2007); Kuijken, et al (2007); Ringeval & Bouchet (2012);. . .]
- effects on 21cm power spectrum [Khatri & Wandelt (2008);. . .]
- effects on CMB at small scales [Fraisse et al (2007); Dvorkin et al. (2011). . .]
- effects on CMB polarisation [Baumann et al (2008);. . .]

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[9] for an overview see Hindmarsh (2011)

Astrophysical Signatures II

The most stringent constraints[10] come from gravitational waves created by decaying loops

. . . but this is also where the *largest theoretical uncertainty* is:

1. network loop production scale
2. nature of string radiation from cusps

Current constraints (a) are inapplicable when decay channels other than GW available[11]; and (b) neglect radiative backreaction

[10] van Haasteren et al. (2011); Sanidas et al. (2012)

[11] Iengo & Russo (2006); Chialva, Iengo, Russo (2005); Skliros, Copeland, Saffin (to appear)

String Decay

String Decay

Highly excited strings (of mass M and string tension μ) are generically unstable, typical lifetimes (if massive radiation negligible):[12]

$$\tau \sim (G_D \mu)^{-1} (M/\mu)^{D-3}, \qquad \mu = \frac{1}{2\pi\alpha'}$$

by emitting strong bursts of anisotropic radiation.[13]

Example: $D = 4$ low GUT strings the size of a galaxy, the solar system or an atom would survive respectively:

$$\tau_{\text{galaxy}} \sim 10^{14}\text{y}, \qquad \tau_{\text{solar syst.}} \sim 10^{4}\text{y}, \qquad \tau_{\text{atom}} \sim 10\text{ps}$$

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[12] Iengo & Russo (2006); Chialva, Iengo, Russo (2005); Skliros, Copeland, Saffin (2013)

[13] Damour & Vilenkin (2000-2001); Skliros, Copeland, Saffin (2013)

History

A handful of references on massive string decay:

- Wilkinson, Turok, Mitchell (1990): leading Regge (bosonic) states, $\mathbb{R}^{25,1}$, numerical, $\Gamma_{d=4} \propto L$ and $\Gamma_{d=26} \propto L^{-1}$
- Dabholkar, Mandal, Ramadevi (1998): higher genus bound on leading Regge Heterotic states, $\mathbb{R}^{3,1} \times T^6$, $\Gamma \lesssim M^{-1}$
- Iengo, Russo (2002-6); Chialva, Iengo, Russo (2004-5): leading Regge superstring states, $\mathbb{R}^{D-1,1} \times T^{10-D}$, (numerical),

$$\Gamma \sim G_D \mu^2 (M/\mu)^{5-D}$$

- Gutplerle & Krym (2006); leading Regge Heterotic states, $\mathbb{R}^{8,1} \times S^1$, (numerical)

$\vdots$

Open Questions

Dependence on energy of emitted radiation & UV/IR interplay

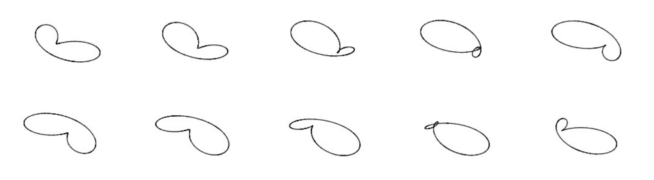
What about the anisotropy of the decay?

Beyond leading Regge?

Radiative backreaction important?

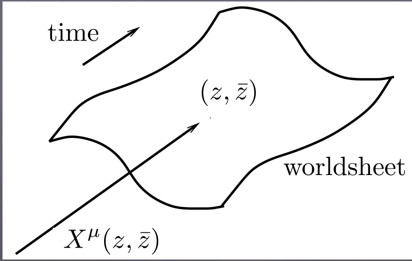
How does classical limit arise?

⋮



Notation

Classical equations of motion of a string loop, $\partial\bar{\partial}X^{\mu} = 0$, with constraints $(\partial X)^2 = (\bar{\partial}X)^2 = 0$,



Vertex Operators

Given any classical solution, there is a one-to-one map to the corresponding quantum vertex operators[14]

For example,

$$X(z,\bar{z}) = \frac{i}{n}\left(\lambda_n\, z^{-n} - \lambda_n^*\, z^n\right) + \frac{i}{m}\left(\bar{\lambda}_m\, \bar{z}^{-m} - \bar{\lambda}_m^*\, \bar{z}^m\right),$$

corresponding to vertex operators,[15]

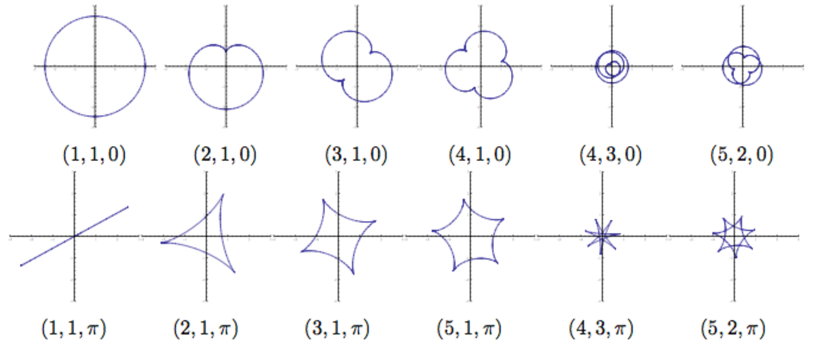
$$\begin{aligned}\mathcal{O}(z,\bar{z}) = \,:C \int_0^{2\pi} \text{đ}s \exp\left(\frac{i}{n}\, e^{ins}\lambda_n \cdot D_z^n X\, e^{-inq\cdot X(z)}\right) \\ \times \exp\left(\frac{i}{m}\, e^{-ims}\bar{\lambda}_m \cdot \bar{D}_{\bar{z}}^m X\, e^{-imq\cdot X(\bar{z})}\right) e^{ip\cdot X(z,\bar{z})}:\end{aligned}$$

[14] Skliros & Hindmarsh PRL (2011)

[15] Skliros & Hindmarsh (2011)

Example

Some explicit string trajectories,[16]



[16] $X = (n, m, \psi)$; the n, m label harmonics and ψ a polarisation tensor parameter

Dual Vertex Operators

Any classical string trajectory $X = X_L(z) + X_R(\bar{z})$, with $\partial\bar{\partial}X = 0$, and has a dual, defined by:[17]

$$\big(X_L(z), X_R(\bar{z})\big) \rightarrow \big(X_L(z), -X_R(\bar{z}^{-1})\big)$$

In the quantum theory, the X are mapped to coherent vertex operators[18] $\mathcal{O}(z, \bar{z})$. Their *duals* are generated by:

$$\{n, m; \lambda_n, \bar{\lambda}_m\} \rightarrow \{n, m; \lambda_n, \bar{\lambda}^*_m\}$$

with $\lambda_n, \bar{\lambda}_m$ polarisation tensors of $\mathcal{O}(z, \bar{z})$.

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[17] Contrast with usual T-duality, $(X_L(z), X_R(\bar{z})) \rightarrow (X_L(z), -X_R(\bar{z}))$. Here dual directions non-compact, see e.g. Berkovits, Maldacena (2008)

[18] Skliros,Hindmarsh (2011)

String Decay Rates I

To compute quantum decay rates:

1. specify X,

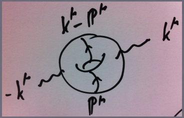
$$X = \frac{i}{n}\left(\lambda_n z^{-n} - \lambda_n^* z^n\right) + \frac{i}{m}\left(\bar{\lambda}_m \bar{z}^{-m} - \bar{\lambda}_m^* \bar{z}^m\right),$$

2. map[19] X to coherent vertex operators $\mathcal{O}$,
3. use a standard unitarity argument familiar from field theory to extract decay rates from:

$$\begin{aligned}\Gamma &= \frac{1}{M}\,\mathrm{Im}\,\langle \mathcal{O}^\dagger \mathcal{O}\rangle \\ &= \frac{1}{M}\int d^D\mathbb{P}\,\mathrm{Im}\left\langle \mathcal{O}^\dagger \mathcal{O}\,\delta^D\Big(\mathbb{P} - \oint_A (\partial X - \bar{\partial} X)\Big)\right\rangle\end{aligned}$$

[19] using one-to-one map of Skliros and Hindmarsh (2011)

String Decay Rates II



$$\begin{aligned}\Gamma &= \frac{1}{M}\int d^D\mathbb{P}\;\mathrm{Im}\left\langle \mathcal{O}^\dagger\mathcal{O}\,\delta^D\Big(\mathbb{P} - \oint_A(\partial X - \bar{\partial}X)\Big)\right\rangle\\ &= \frac{1}{M}\int d^D\mathbb{P}\sum_{\{m_j,\,k^\mu\}}|\ldots|^2\,\delta(\mathbb{P}^2+m_1^2)\delta\big((k-\mathbb{P})^2+m_2^2\big),\end{aligned}$$

...fixed loop momenta[20] makes it manifest that:[21]

1. decay rates non-zero iff strings in loop go onshell
2. $\mathbb{P}^\mu$ labels momentum of decay products
3. Tachyon contribution additive, so easy to drop!

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[20] Dijkgraaf, Verlinde, Verlinde (1988); D'Hoker & Phong (1989, 2002-2008)
[21] Skliros, Copeland, Saffin (to appear)

String Decay Rates III

In the IR and taking $n, m = 1$, the result ressums, leading to:[22]

$$\frac{d\Gamma_N}{d\Omega_{S^{D-2}}}\Big|_{m^2=0} = \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}}\,\omega^{D-4-\delta} N^2 \left[J'^2_N(A) + \Big((N/A)^2 - 1\Big) J^2_N(A)\right] \left[J'^2_N(\bar{A}) + \Big((N/\bar{A})^2 - 1\Big) J^2_N(\bar{A})\right]$$

where the frequency of emitted radiation,[23]

$$\omega = \frac{4\pi N}{L}, \qquad \text{with} \qquad N = 1, 2, \ldots$$

Taking $\delta = 1$ yields a decay rate, $\delta = 0$ yields a power.

[22] Skliros, Copeland and Saffin (2013)

[23] Here $A = N\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_{\mathbf{1}}|$, $\bar{A} = N\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_{\mathbf{1}}|$ and the $J_{\boldsymbol{n}}(z)$ are Bessel functions

Effective Description

Remarkably, the above was shown[24] to agree precisely with the effective theory,

$$
\begin{aligned}
S_{\mathrm{eff}} = \frac{1}{16\pi G_D}\int d^D x\sqrt{-G}\, e^{-2\Phi}\Big(R_{(D)} + 4(\nabla\Phi)^2 - \frac{1}{12}H_{(3)}^2 + \dots\Big) \\
- \mu\int_{S^2}\partial X^\mu \wedge \bar{\partial}X^\nu\big(G_{\mu\nu} + B_{\mu\nu}\big) + \dots,
\end{aligned}
$$

where Φ, $G_{\mu\nu}$ and $H_{(3)}$ are the dilaton, spacetime metric and 3-form field strength, $H = dB$, respectively

(We plug classical solutions for X (from $X - \mathcal{O}$ map) and compute perturbations in G, B and Φ)

[24] Skliros, Copeland and Saffin (2013)

Higher Harmonics

...the above correspondence acts as a guiding principle to write down the general result for arbitrary harmonics:[25]

$$\frac{d\Gamma_N}{d\Omega_{S^{D-2}}}\Big|_{m^2=0} = \frac{16\pi G_D \mu^2}{(2\pi)^{D-4}} \omega^{D-4-\delta} (Nuwg)^2$$
$$\left[J'^2_{Nw}(A) + \Big((Nw/A)^2 - 1\Big) J^2_{Nw}(A)\right]$$
$$\left[J'^2_{Nu}(\bar{A}) + \Big((Nu/\bar{A})^2 - 1\Big) J^2_{Nu}(\bar{A})\right]$$

with $n \equiv gu$, $m \equiv gw$, integers and u, w relatively prime

[25] Here $A = Nw\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\lambda}_{\boldsymbol{n}}|$, $\bar{A} = Nu\sqrt{2}|\hat{\mathbb{P}} \cdot \hat{\bar{\lambda}}_{\boldsymbol{m}}|$

Results

- Discussed construction of covariant closed string coherent vertex operators (classical/quantum manifest)
- Discussed computation of amplitude with these insertions
- Two-point amplitudes invariant under the duality[26] $\{n, m; \lambda_n, \bar{\lambda}_m\} \to \{n, m; \lambda_n, \bar{\lambda}^*_m\}$ to leading order in the string coupling
  ($\to$ e.g., decay rates of epicycloids equal those of hypocycloids)
- Analytic results for decay rates and power associated to massless radiation
- Found that the result can be precisely reproduced by an effective theory in the IR, enabling us to conjecture result for higher harmonics

[26] Non-compact version of T-duality???