EVH/CFT Correspondence Near-Extremal Vanishing Horizon AdS_5 Black Holes and Their CFT Duals

Maria Johnstone[1] M.M. Sheikh-Jabbari[2] Joan Simón[3]
Hossein Yavartanoo[4]

[1]University of Edinburgh, UK
[2]Institute for Research in Fundamental Sciences, Iran
[3]University of Edinburgh, UK
[4]Kyung Hee University, Korea

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Introduction

Black Holes

1. solutions to general relativity
2. behave like thermodynamic systems:
    - satisfy thermodynamic laws
    - have a thermodynamic entropy:

$$S_{BH} = \frac{A_d}{4G_d}$$

- **Question**: Why does the entropy scale like the horizon area? $\Rightarrow$ Holography: “the fundamental degrees of freedom describing the system are described by a quantum field theory with one less dimension.”
- **Question**: What are the underlying states responsible for entropy?

Introduction

AdS/CFT Correspondence

- conformal field theory (CFT) in d dimensions can be described by string theory on $\mathrm{AdS}_{d+1} \times \mathrm{M}$
- Seminal example: type IIB string theory on $\mathrm{AdS}_5 \times S^5$ and its dual description of $\mathcal{N} = 4$ Super Yang–Mills theory:
- Any state/physical process in the asymptotically $\mathrm{AdS}_5 \times S^5$ geometry is dual to a (perturbative) deformation of $\mathcal{N} = 4$ 4d SYM
- A class of these deformations are solutions to $\mathcal{N} = 2$ 5d gauged supergravity: black hole solutions.
- We think the degrees of freedom of the dual CFT live on/near the horizon of the black hole.

Introduction

Kerr/CFT (Extremal Black Hole/CFT) Correspondence

- Statement of Extremal Black Hole/CFT Correspondence: **Near horizon quantum states** $\Longleftrightarrow$ **quantum states of a chiral 2d CFT** such that

$$\boxed{S_{Cardy} = \frac{\pi^2}{3} c_L T_L = S_{BH}}$$

- Extremal black hole: $T = 0$, $r_+ = r_-$.
- Not very much known about this 2d CFT except
  1. It is chiral: states in right moving sector are frozen to ground state ($c_L = T_L = 0$.)
  2. Its central charge c_L and Frolov Thorne temperature T_L
  3. There are no dynamics in the chiral sector.

Introduction

Questions

- What are the generators?
- What is the full non-chiral theory?
- When can one have non-trivial dynamics in the CFT_2?
- Can an extremal black hole have a near horizon AdS_3 throat that's dual to the full non-chiral CFT_2
- What is the relation between the 4d dual and the 2d dual?

Introduction

In this Talk...

- Introduce a set of black holes that have non-chiral 2d CFT descriptions: near-Extremal Vanishing Horizon black holes: a limit of black hole solutions that have vanishing temperature and horizon size
- These black holes have a near horizon geometry that is a 3d rotating BTZ black hole. The first law of thermodynamics reduces to the first law for a BTZ black hole in the near-EVH limit.
- Specify the generators and quantum numbers of the state in the CFT using the AdS_3/CFT_2 dictionary.
- Outline a map between the quantum numbers of the 4d CFT and the quantum numbers of the 2d CFT

5d Supergravity Solution

- **Rotating Charged Black Hole Solution** to $U(1)^3$ 5d gauged supergravity:

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu$$
$$\mu = t, r, \theta, \phi, \chi_1$$
$$ds_5^2 \xrightarrow[r \to \infty]{} ds^2_{\mathrm{AdS}_5}$$

- Four Dimensional Parameter Space Spanned by $a_a := a_\phi, a_\chi, a_q, a_m(r_+)$
- $r_\pm$=Black hole inner and outer horizons
- Gauge and Scalar Fields:

$$A^i = A^i_\mu(r; a_a) dx^\mu, \qquad X_i = X_i(r; a_a),$$

Thermodynamic Quantities

Rotation in ϕ, χ_1:

- Angular velocities in AdS_5: $\Omega_\phi, \Omega_{\chi_1}(a_a)$
- Angular momenta in AdS_5: $J_\phi, J_{\chi_1}(a_a)$.

Gauge Fields A_i, $i = 1, ., 3$

- Chemical Potentials: $\Phi_1 = \Phi_2(a_a), \Phi_{\chi_2}(a_a)$
- Electric Charges: $Q_1 = Q_2(a_a), Q_{\chi_2}(a_a)$
- **Note:** Φ_{χ_2}, Q_{χ_2} not independent.

Thermodynamic Quantities

- **First Law of Thermodynamics:**

$$\boxed{T_{\rm H}\, dS_{\rm BH} = dE - \Omega_\phi\, dJ_\phi - \Omega_{\chi_1}\, dJ_{\chi_1} - \Omega_{\chi_2}\, dJ_{\chi_2} - 2\Phi_1\, dQ_1}$$

- $T_{\rm H}$ = **Hawking Temperature**: Given by horizon surface gravity; function of a_a
- $S_{\rm BH}$ = **Beckenstein-Hawking Entropy:** function of a_a.

$$\boxed{S_{\rm BH} = \frac{A_H}{4G_5}}$$

- Integrate: **Black hole mass:**

$$E = E(a_a).$$

10d Embedding

- Solution to 10d IIB supergravity:

$$\boxed{ds_{10}^2 = \sqrt{\widetilde{\Delta}}\, ds_5^2 + \frac{\ell^2}{\sqrt{\widetilde{\Delta}}} d\Sigma_5^2}$$

- ds_5^2: 5d black hole metric
- deformed S^5:

$$d\Sigma_5^2 = \sum_{i=1}^{3} X_i^{-1}(d\mu_i^2 + \mu_i^2(d\psi_i + A^i/\ell)^2).$$

- also: $F_5 = \star F_5$ with flux N
- Newton's constants:

$$\boxed{G_5 = G_{10}\frac{1}{\pi^3\ell^5} = \frac{\pi}{2}\frac{\ell^3}{N^2}}.$$

10d Embedding

Solutions Can be Uplifted to 10d Supergravity Solutions

- 5d electrostatic potential Φ = 10d angular velocity Ω on S^5.
- 5d electric charge Q = 10d angular momentum J on S^5.

Dual 4d Description

- AdS/CFT:

Black Hole in $AdS_5 \times S^5 \leftrightarrow$ mixed state in dual $\mathcal{N} = 4$ SYM.

- N is the rank of the dual SU(N) gauge group
- States carry conserved charges given by gravity conserved charges:

$$\Delta = \ell E\,, \qquad \mathcal{J}_1 = \mathcal{J}_2 = Q_1\,, \qquad \mathcal{S}_\phi = J_\phi, \qquad \mathcal{S}_\chi = J_\chi$$

The Set of EVH Black Holes

EVH Black Holes

- 4-dimensional black hole parameter space: $(a_\phi, a_\chi, a_q, a_m(r_+))$.
- EVH black holes: $A_{BH} = T_H = 0 \Rightarrow$

$$\boxed{r_+ = 0 \quad \text{and} \quad a_\chi = 0\,.}$$

- The EVH surface is a 2d surface in 4d parameter space:

$$a_m = a_m(a_\phi, a_q).$$

Point on the EVH surface $\Leftrightarrow$ EVH black hole

The Near Horizon Limit of EVH Black Holes

- EVH black hole: $S = T = r_+ = a_\chi = 0$
- Near Horizon Limit:

$$\boxed{t \sim \frac{\tau}{\epsilon}, \qquad \chi \sim \frac{\tilde{\chi}}{\epsilon}, \qquad r \sim \epsilon x}$$

Also angular shifts:

$$\phi = \hat{\phi} + \Omega_\phi^{\text{EVH}} t, \qquad \psi_i = \hat{\psi}_i + \Omega_{\psi_i}^{\text{EVH}} t.$$

- χ: geometric circle whose corresponding angular momentum vanishes in the EVH limit:

$$\boxed{a_\chi = 0 \Rightarrow J_\chi = 0}$$

The Near Horizon Limit of EVH Black Holes

- Take $\epsilon \to 0$:
- **Near Horizon Geometry:**

$$\boxed{ds^2 = \Gamma_1(\theta) ds^2_{AdS_3} + \Gamma_2(\theta) ds^2_{M_7}},$$

where

$$\boxed{ds^2_{AdS_3} = -\frac{x^2}{\ell_3^2} d\tau^2 + \frac{\ell_3^2 dx^2}{x^2} + x^2 d\tilde{\chi}^2}, \text{ and}$$

- $ds^2_{M_7}$ is some compact space.
- warping factors: Γ_1, Γ_2.
- Locally $AdS_3 \times M_7$.
- AdS_3 radius is function of EVH parameters:
  $\ell_3 = \ell_3(a_\phi, a_q, a_m = a_m(a_\phi, a_q))$

The Near Horizon Limit of EVH Black Holes

Near Horizon Geometry of EVH Black Hole:

$$ds^2_{AdS_3} = -\frac{x^2}{\ell_3^2}d\tau^2 + \frac{\ell_3^2 dx^2}{x^2} + x^2 d\tilde{\chi}^2$$

Pinching AdS_3:

- AdS_3 circle $\hat{\chi}$:
- $\chi = \frac{\hat{\chi}}{\epsilon} \Rightarrow \hat{\chi} = \hat{\chi} + 2\pi\epsilon$: Vanishing Periodicity. Locally AdS_3 structure is a **pinching** $\mathbf{AdS_3}$.

EVH Black Hole

- EVH Black Hole: Point on EVH surface
- Near Horizon Geometry: pinching AdS_3

Given a generic EVH point, one can decompose the space of deformations into *tangential* and *orthogonal*.

- Tangential deformations: take us from one EVH black hole to a different one on the EVH hyperplane.
- Orthogonal deformations: excitations of an EVH black hole $\Rightarrow$ *near-EVH black holes*.
- **Near-EVH black holes** $A_{\rm BH}, T_{\rm H} \sim \epsilon << 1 \Rightarrow$:

$$\boxed{A_{\rm BH} \sim T_{\rm H} \sim \epsilon \Rightarrow r_+ \sim \epsilon\,, \quad a_\chi \sim \epsilon^2}$$

Near-EVH Black Holes

Rotating near-EVH configuration:

$$\boxed{r_+ : 0 \to \epsilon x; \qquad a_\chi : 0 \to \epsilon^2 \hat{a}_\chi; \qquad a_m : a_m(a_\phi, a_q) \to a_m + \epsilon^2 M}$$

- physical excitations of rotating EVH black holes are described by deformation parameters $(\hat{a}_\chi, M)$.
- The horizon is small and non-zero in this case; from the horizon equation we have $r_\pm^2 = \epsilon^2 x_\pm^2$ where

$$x_\pm^2 \sim \frac{r_\pm^2}{\epsilon^2} = x_\pm^2(\hat{a}_\chi, M).$$

Near Horizon Geometry of Near-EVH Black Holes

- Near Horizon Limit: same as for EVH case.
- **Near horizon geometry:** $\boxed{ds^2 = \Gamma_1 ds^2_{BTZ} + \Gamma_2 ds^2_{M_7}}$,

  where $ds^2_{M_7}$ is as for the EVH case, and

$$ds^2_{BTZ} = -\frac{(x^2 - x_+^2)(x^2 - x_-^2)}{\ell_3^2 x^2} d\tau^2 + \frac{\ell_3^2 x^2 dx^2}{(x^2 - x_+^2)(x^2 - x_-^2)} + x^2 \left(d\tilde{\chi} - \frac{x_+ x_-}{\ell_3 x^2} d\tau \right)^2$$

- $\hat{\chi} \sim \hat{\chi} + 2\pi\epsilon$: **pinching BTZ black hole.**
- Near-EVH limit: **NH pinching AdS_3 excited to NH pinching BTZ**

Near-EVH Near Horizon BTZ Black Holes

- Compactify 10d type IIB supergravity action to 3d: $\frac{1}{G_3} \sim N^2$
- M_{BTZ}, J_{BTZ} calculated by integrating over *pinching* S^1 at infinity $\Rightarrow$ extra ϵ.
- BTZ Temperature, Mass, Angular Momentum, Angular Velocity:

$$T_{\text{BTZ}} \equiv \frac{x_+^2 - x_-^2}{2\pi x_+ \ell_3^2} \sim \frac{T_{\text{H}}}{\epsilon}, \qquad \ell_3 M_{\text{BTZ}} = \frac{x_+^2 + x_-^2}{8\ell_3 G_3}\epsilon =\sim N^2\epsilon,$$

$$J_{\text{BTZ}} = \frac{x_+ x_-}{4\ell_3 G_3}\epsilon \sim N^2\epsilon, \qquad \Omega_{\text{BTZ}} = \frac{x_+ x_-}{\ell_3 x_+^2}$$

BTZ Entropy Captures near-EVH limit of asymptotically AdS_5 black hole entropy:

$$\boxed{S_{\text{BTZ}} \equiv \frac{2\pi\epsilon \cdot x_+}{4G_3} = S_{\text{BH}}}$$

- **EVH black hole $\xrightarrow{\text{Near Horizon}}$ Pinching AdS_3**
- **Near EVH black hole $\xrightarrow{\text{Near Horizon}}$ Pinching BTZ black hole**
- **10d entropy is given by BTZ entropy**

CFT_2 Quantities

- AdS_3/CFT_2: Pinching $AdS_3 \Rightarrow$ dual CFT_2
- Brown Henneaux Central Charge on *regular* cylinder: $c_L = c_R = c$:

$$c = \epsilon c_p = \frac{3\ell_3}{2G_3}\epsilon = f(\text{EVH parameters})\ N^2\epsilon$$

- The central charge is a function of EVH parameters. **Defines CFT but not excitations.**
- $c \sim N^2\epsilon$ is kept fixed for large N:

$$\boxed{N^2\epsilon = \text{fixed}}$$

- $N =$ number of branes in string theory on $AdS_5 \times S^5$
- $N =$ rank of SU(N) gauge group in dual 4d N=4 SYM
- $N =$ flux through S^5

CFT_2 Quantities

- Excitations:

$$L_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} - J_{\text{BTZ}}) \sim f(\hat{a}_\chi, M) N^2 \epsilon$$
$$\bar{L}_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) \sim f(\hat{a}_\chi, M) N^2 \epsilon$$

- CFT excitations depend on orthogonal deformations from EVH surface $\hat{a}_\chi, M$.
- In large N limit ($N^2 \epsilon \sim 1$) the CFT excitations are finite.
- Cardy's formula:

$$S_{\text{CFT}} = 2\pi \sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} + 2\pi \sqrt{\frac{\bar{c}}{6}\left(\bar{L}_0 - \frac{\bar{c}}{24}\right)} = S_{BH}$$

CFT_2 Quantities

- *finite* central charge in IR 2d CFT: *large N* limit:

$$\boxed{N^2\epsilon = \text{fixed}}$$

  1. entropy $S_{\rm BH} \sim N^2\epsilon$ finite in this limit
  2. $M_{\rm BTZ}, J_{\rm BTZ} \sim N^2\epsilon$ also finite in this limit
  3. $c_L = c_R,\, L_0,\, \bar{L}_0,\, \mathrm{S}_{Cardy} \sim N^2\epsilon$ also finite in this limit
- EVH point ($r_\pm = 0, a_\chi = 0; a_m = a_m(a_\phi, q)$) determines the IR 2d CFT central charge and vacuum structure
- Near-EVH point ($r_\pm = \epsilon x_\pm, a_\chi = \epsilon^2 \hat{a}_\chi; a_m = a_m(a_\phi, q) + \epsilon^2 M$) encodes *finite* excitations.

First law of thermodynamics, IR vs. UV, 3d vs. 5d

- **First Law of thermodynamics:**

$$T_{\rm H}\, dS_{\rm BH} = dE - \Omega_\phi\, dJ_\phi - \Omega_{\chi_1}\, dJ_{\chi_1} - \Omega_{\chi_2}\, dJ_{\chi_2} - 2\Omega_1\, dJ_1$$

- Expand all quantities in ϵ.
- For a fixed point in parameter space, physical variations belong to the subspace of orthogonal deformations to the EVH hyperplane, leaving the EVH point fixed.
- eg: $E = E^0 + \epsilon^2 E^{(2)}(\hat{a}_\chi, M)$. Then $dE = 0 + \epsilon^2 dE^{(2)}$.

1st Law of Thermodynamics

$$T_{\rm H} dS_{\rm BH} = dE - \Omega_\phi \, dJ_\phi - \Omega_{\chi_1} \, dJ_{\chi_1} - \Omega_{\chi_2} \, dJ_{\chi_2} - 2\Omega_1 \, dJ_1$$

$\Downarrow$ Near-EVH Expansion

$$T_{\rm BTZ} dS_{\rm BTZ} = dM_{\rm BTZ} - \Omega_{\rm BTZ} dJ_{\rm BTZ}$$

The UV 1st law reduces in the near-EVH approximation to an IR 1st law for BTZ black hole.

This Result is Universal for Any Near-EVH Black Hole in Any Background!

Upcoming Paper: Extremal Black Holes and First Law of Thermodynamics: MJ, M.M.Sheikh-Jabbari, Joan Simón, Hossein Yavartanoo

- 10d dimensional black hole has dual description in terms of $\mathcal{N} = 4$ SYM on boundary of AdS_5.
- NH limit of AdS_5 black hole $\leftrightarrow$ low energy limit of dual CFT_4
- CFT_4 dual to asymptotically AdS_5 black hole = UV CFT
- Low energy/Near Horizon limit of CFT_4 = IR CFT
- Relate quantum numbers of IR theory to those of NH CFT_2

- Fourier decomposition of massless scalar field in background:

$$\Psi \sim e^{-i(\omega t + m_\phi \phi + m_{a_1}\phi_1 + m_{a_2}\phi_2 + \sum_i m_i \psi_i)}$$

- Isometries of $AdS_5 \times S^5 \Rightarrow$ UV quantum numbers of scalar field: eigenvalues of operators

$$\Delta_{\mathrm{UV}} = \ell E = i\ell\partial_t, \qquad J_{1,2} = -i\partial_{\phi_1,\phi_2} \qquad J_i = -\ell Q_i = -i\partial_{\psi_i}.$$

- Isometries of NH $AdS_3 \times M_7 \Rightarrow$ IR quantum numbers of scalar field: eigenvalues of operators

$$\Delta_{\mathrm{IR}} = i\ell_3\partial_\tau, \qquad J_{\tilde\chi} = -i\partial_{\tilde\chi}$$

IR-UV charge mapping

Charges Have a Near-EVH Expansion:

$$\boxed{Z = Z_{EVH} + \epsilon^p Z^{(p)}}, \qquad \text{where}$$

- Z_{EVH} is the value at the EVH point.
- $Z^{(p)}$ are the near-EVH excitations.
- Also recall near horizon limit:

$$\boxed{t \sim \frac{\tau}{\epsilon}, \qquad \chi \sim \frac{\tilde{\chi}}{\epsilon}, \qquad r \sim \epsilon x}$$

Also angular shifts:

$$\phi = \tilde{\phi} + \Omega_{\phi}^{\text{EVH}} t, \qquad \psi_i = \tilde{\psi}_i + \Omega_{\psi_i}^{\text{EVH}} t.$$

IR-UV charge mapping

Use chain rule to express IR charges in terms of UV ones.

- **In the IR limit:**

$$
\begin{aligned}
J_{\tilde{\chi}} &= -i\frac{\partial}{\partial\tilde{\chi}} = -i\frac{\partial\chi}{\partial\tilde{\chi}}\frac{\partial}{\partial\chi} \\
&= J_{\mathrm{BTZ}} \\
&= L_0 - \bar{L}_0
\end{aligned}
$$

In the Large N limit:

- Spin of scalar probe in NH region is given by spin of state in 2d CFT.

IR conformal dimension $\Delta_{\rm IR}$

$$
\begin{aligned}
\Delta_{\rm IR} &\equiv i\ell_3 \frac{\partial}{\partial \tau} \\
&= i\ell_3 \left(\frac{\partial t}{\partial \tau}\frac{\partial}{\partial t} + \frac{\partial \phi}{\partial \tau}\frac{\partial}{\partial \phi} + \frac{\partial \psi_i}{\partial \tau}\frac{\partial}{\partial \psi_i} \right) \\
&= \Delta^0_{IR}(\text{EVH parameters}) + M_{\rm BTZ} \\
&= \text{"ground state energy"} + L_0 + \bar{L}_0 - \frac{c}{12}
\end{aligned}
$$

- **Conformal dimension of scalar probe in NH region is given by scaling dimension of state in 2d CFT.**

Rotating Near-EVH Limit

- UV charges $\xrightarrow{\text{Near-EVH NH}}$ IR charges
- IR quantum **given by CFT_2 quantum numbers**
- Suggests that **near-EVH sector in UV 4d dual is sector described by IR 2d dual.**
- Near horizon information given by 2d CFT:
  evidence for EVH/CFT_2 Correspondence.

EVH/CFT_2 Correspondence

Near Horizon of EVH black hole described by 2d CFT

Check of EVH/CFT proposal

$$\boxed{c_{\text{EVH AdS}_3}\Big|_{\text{extremal}} = c_{\text{Kerr/CFT}}\Big|_{\text{near-EVH}}}:$$

chiral CFT Kerr/CFT proposal=chiral limit of the CFT_2 in EVH/CFT correspondence.

Summary

1. Near horizon limit of an EVH black hole: local AdS_3 factor.
2. Near horizon limit of a near-EVH black hole: local BTZ
3. Entropy of asymptotically AdS_5 black hole = entropy of BTZ black hole
4. First law of thermodynamics reduces $\rightarrow$ first law for a btz black hole. This result appears to be universal to any black hole, in any dimension and in any spacetime
5. BTZ excitations $\Rightarrow$ CFT central charge and generators.
6. Near-EVH limit: no allowed dynamics in the CFT unless large N limit all black hole and CFT quantities kept finite.
7. Scalar probe analysis $\Rightarrow$ 2d CFT describes low energy sector of N=4 SYM dual theory.
8. Our results are consistent with the Kerr/CFT correspondence.
9. This is all evidence in favour of the proposal that EVH black holes can be described by 2d CFTs, or the EVH/CFT