Magnetic Catalysis in compact spaces

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Outline

1. Magnetic Catalysis
   - Essence
   - Magnetic catalysis in the NJL model
2. AdS/CFT correspondence
   - Statement of the correspondence
   - Adding flavours
3. Magnetic Catalysis on S^3
   - Flavours on S^3
   - Magnetic catalysis and Phase diagram
   - Meson Spectrum

Essence

- Broadly defined as an enhancement of dynamical symmetry breaking by an external magnetic field.
- Lowest Landau level. Dimensional reduction $D \to D - 2$ of the dynamics of fermion pairing.
- Intuitively: opposite charges and opposite spins $\to$ same magnetic moments $\to$ the pair is stabilized.

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Magnetic catalysis in the NJL model

- Consider the NJL model [Miranski et al. hep-ph/9412257]:
  $\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m_0)\Psi + \frac{G}{2}\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma_5\Psi)^2\right]$,

  $D_\mu = \partial_\mu + iBx_2\delta_{\mu 1}$ and m_0 is the bare mass.
- This model possesses an $U(1)_L \times U(1)_R$ chiral symmetry. A non-zero condensate $\langle\bar{\Psi}\Psi\rangle$ would break it down to $U(1)_V$.
- To zeroth order in G:

$$\langle\bar{\Psi}\Psi\rangle = -\mathrm{tr}[S(x,x)] = -\frac{4m_0}{(2\pi)^4}\int d^4k \int_{1/\Lambda^2}^{\infty} \frac{ds}{s}\, e^{-s\left(m_0^2 + k_{||}^2 + k_\perp^2 \frac{\tanh(eBs)}{eBs}\right)}$$

$$= -\frac{m_0}{(2\pi)^2}\left[\Lambda^2 + eB\log\left(\frac{eB}{\pi m_0^2}\right) - m_0^2\log\left(\frac{\Lambda^2}{2eB}\right) + O\left(\frac{m_0^4}{2eb}\right)\right]$$

- Note that to zeroth order in G $\langle\bar{\Psi}\Psi\rangle \to 0$ as $m_0 \to 0$

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Magnetic catalysis in the NJL model

- Gap equation at weak coupling:

$$m = G\,\mathrm{tr}[S(x,x)] \ ,$$

where m is the dynamically generated mass.

- To leading order in $G \ll 1$ the gap equation reads:

$$m \approx G\frac{m}{(2\pi)^2}\left[\Lambda^2 + eB\log\left(\frac{eB}{m^2}\right)\right]$$

- The nontrivial (and nonperturbative) solution is:

$$m^2 = \frac{eB}{\pi}\exp\left(\frac{\Lambda^2}{eB}\right)\exp\left(-\frac{4\pi^2}{GeB}\right) \quad \text{for any} \quad G \ll 1$$

- At $B = 0$ the gap equation should be re-derived:

$$m \approx G\frac{m}{(2\pi)^2}\left[\Lambda^2 - m^2\log\frac{\Lambda^2}{m^2}\right]$$

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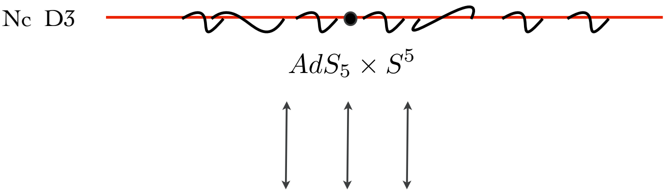

$$\mathcal{N} = 4 \; SU(N_c) \; \text{SUSY YM}$$

- Gubser-Klebanov-Polyakov-Witten formula:

$$\langle e^{\int d^dx \phi_0(x) \langle \mathcal{O}(x) \rangle} \rangle_{\text{CFT}} = \mathcal{Z}_{\text{string}}[\phi_0(x)]$$

Type IIB String Theory on

$AdS_5 \times S^5$

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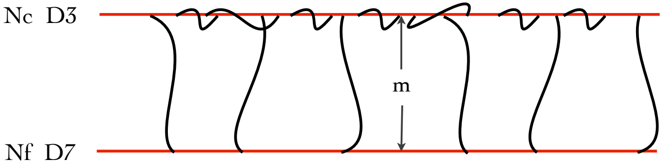
Adding flavours

Generalizing the correspondence



	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	·	·	·	·	·	·
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- Adding N_f massive $\mathcal{N} = 2$ Hypermultiplets:

$$m_q \int d^2\theta \, \tilde{Q} \, Q \to \text{SYM} \quad \text{with} \quad m_q = m/2\pi\alpha'$$

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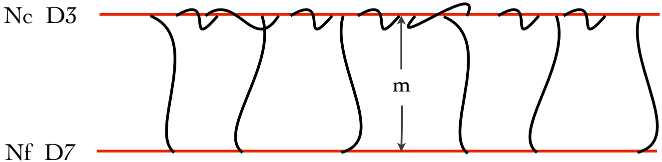

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Flavours on S^3

- We consider D7–brane probes on global $\mathrm{AdS}_5 \times S^5$, describing flavoured $\mathcal{N} = 2$ SYM on a S^3.
- The metric of global $\mathrm{AdS}_5 \times S^5$ can be written as:

$$ds^2 = -\frac{u^2}{R^2}\left(1+\frac{R^2}{4u^2}\right)^2 d\tau^2 + u^2\left(1-\frac{R^2}{4u^2}\right)^2 d\Omega_3^2 + \frac{R^2}{u^2}\left(du^2 + u^2 d\Omega_5^2\right) .$$

where $u \geq R/2$ and the metric has a conformal $\mathbb{R}^6$ part.

- S^5 is parametrize as:

$$d\Omega_5^2 = d\theta^2 + \cos\theta^2\, d\tilde{\Omega}_3^2 + \sin\theta^2 d\phi^2$$

- We introduce probe D7-brane extended along the AdS_5 directions and $\tilde{\Omega}_3$ with the ansatz $\theta = \theta(u)$, $\phi = \mathrm{const}$.
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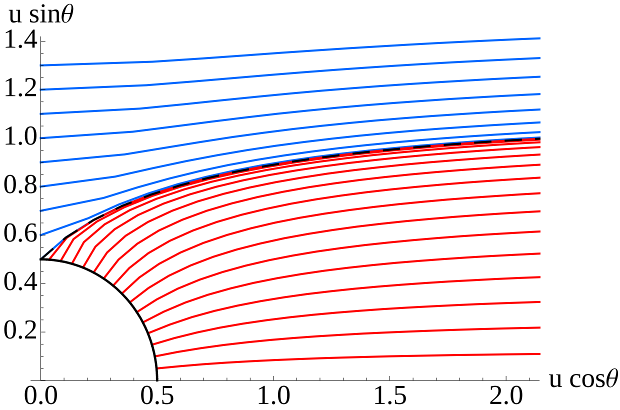

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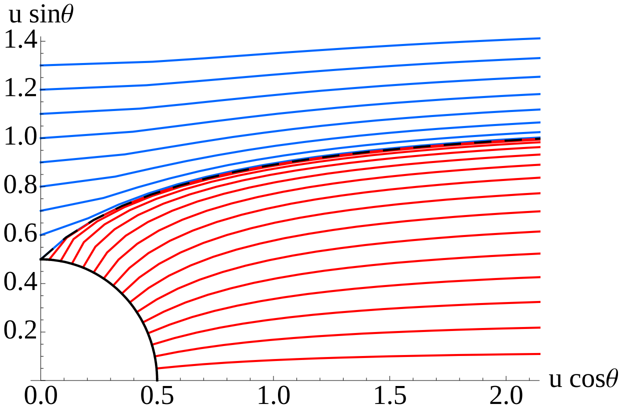

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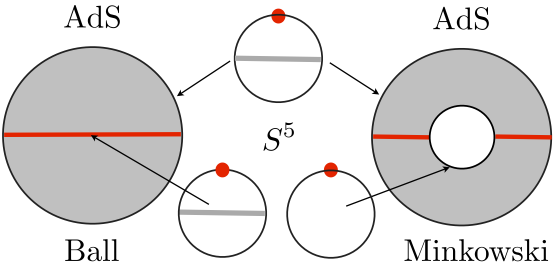
Static test charges (A. O'Bannon et. al. arXiv: 0906.4959)

Consider a pair of quark and anti-quark. The flux tube connecting the quarks is described by an open string probing the geometry.



- For Minkowski embeddings the string snaps corresponding to Gribov confinement
- The screening length $L_s \propto 1/m_q$. Ball embeddings correspond to $L_s > \pi R_3$ and the string never snaps.

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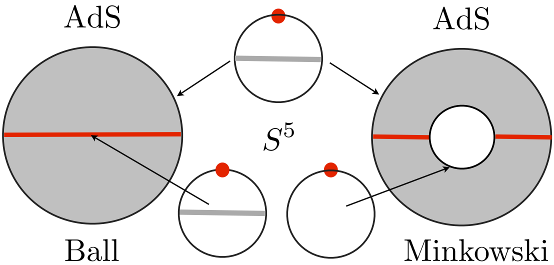

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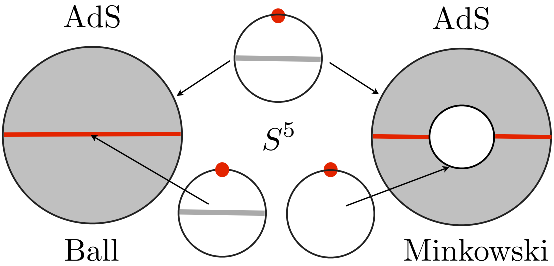

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AdS/CFT Dictionary

- The asymptotic separation $m = \lim_{u\to\infty} u \sin\theta(u)$ is associated with the bare quark mass $m_q = m/2\pi\alpha'$.
- The AdS/CFT dictionary relates the asymptotics of $\theta(u)$ to $\langle\bar\Psi\Psi\rangle$:

$$\sin\theta = \frac{m}{u} + \frac{c_1}{u^3} - \frac{m}{2u^2}\log u + \dots\ ,$$

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- We introduce the dimensionless quantities:

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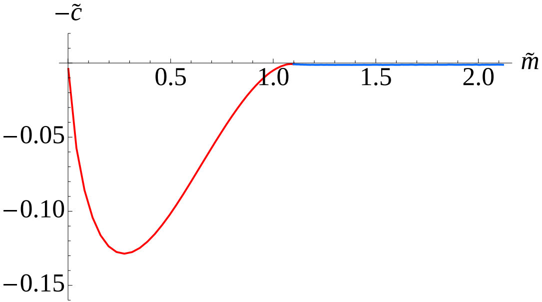

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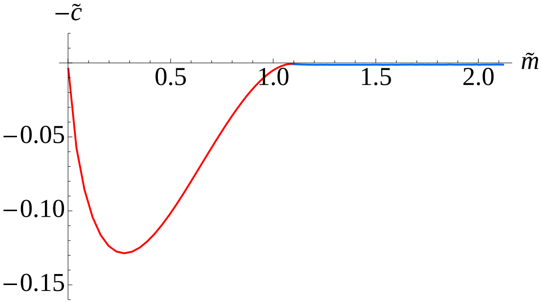

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Introducing magnetic field

In order to introduce magnetic field we consider a pure gauge B-field, along two of the directions of S^3.

- Writing the metric on S^3 in local tetrads:

$$d\Omega_3^2 = e_{(1)}^2 + e_{(2)}^2 + e_{(3)}^2 \ .$$

- Tetrads are take to be:

$$e_{(1)} = R\, d\theta_1 \ , \quad e_{(2)} = R\, \sin\theta_1\, d\phi_1 \ , \quad e_{(3)} = R\, \cos\theta_1\, d\psi_1 \ .$$

- A natural choice for the B-field is:

$$B = H\, e_{(1)} \wedge e_{(2)} \ .$$

- The DBI lagrangian is modified to:

$$\mathcal{L} \propto u \cos^3\theta_3 \left(1 - \frac{R^4}{16u^4}\right) \sqrt{u^4 \left(1 - \frac{R^2}{4u^2}\right)^4 + H^2 R^4} \sqrt{1 + u^2 \theta_3'(u)^2}$$

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Introducing magnetic field

In order to introduce magnetic field we consider a pure gauge B-field, along two of the directions of S^3.

- Writing the metric on S^3 in local tetrads:

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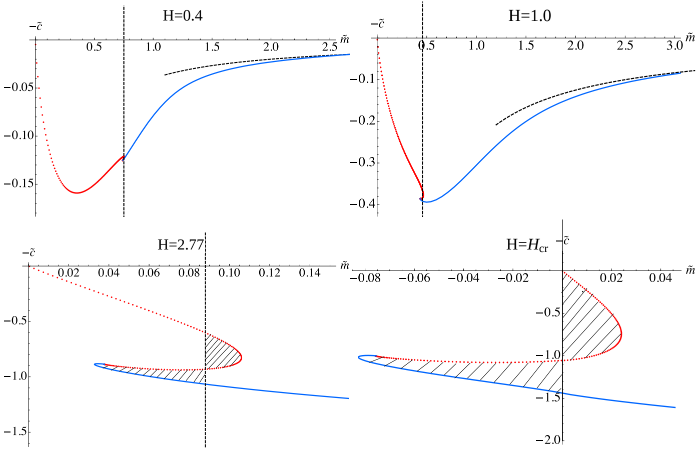

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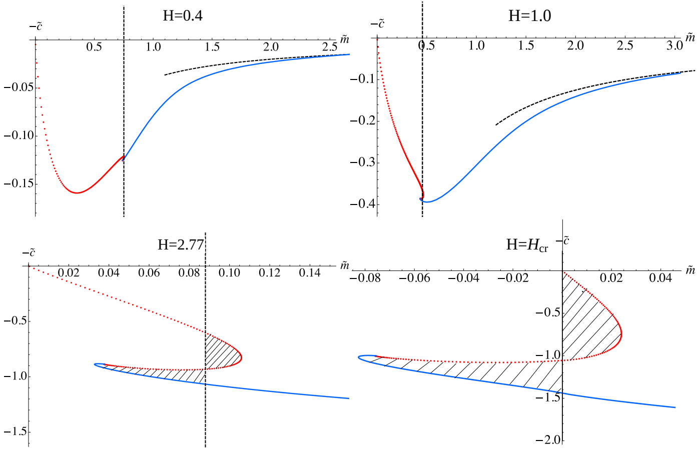

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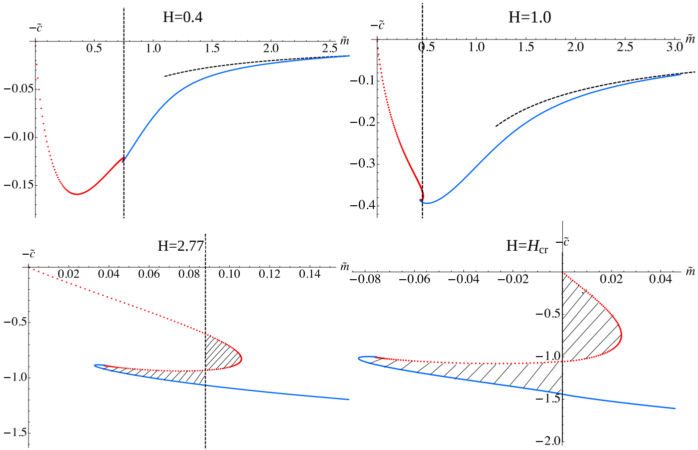

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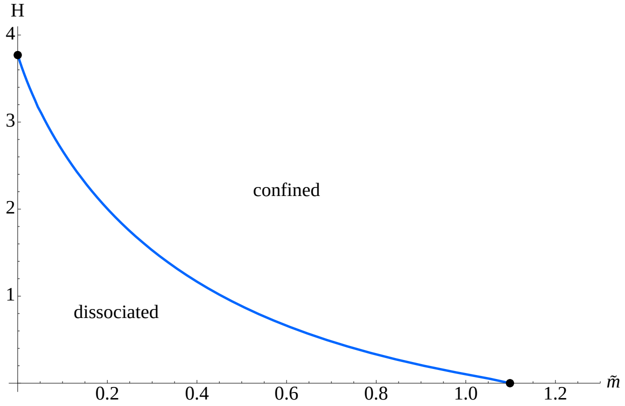

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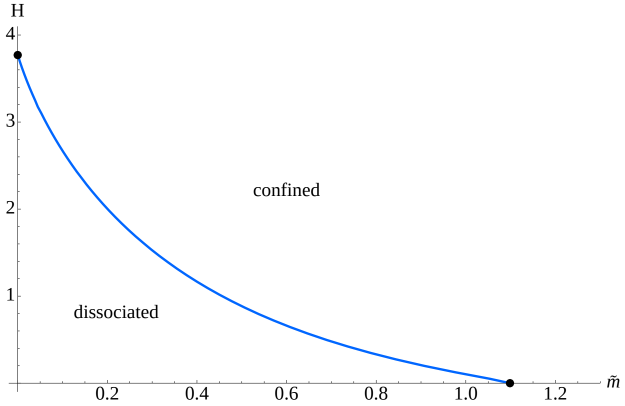

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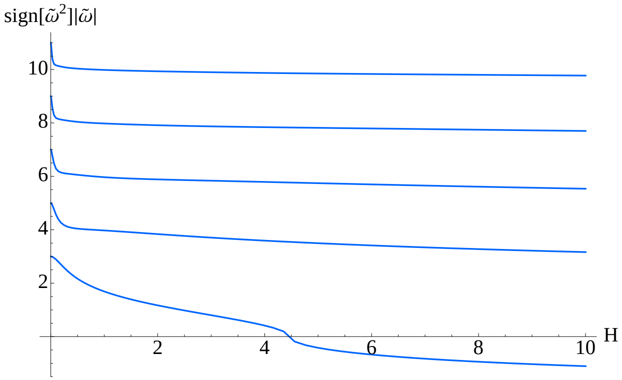
Meson Spectrum-Fluctuations along θ

By studying the spectrum of semiclassical fluctuations of the probe branes we can extract the meson spectrum of the theory.



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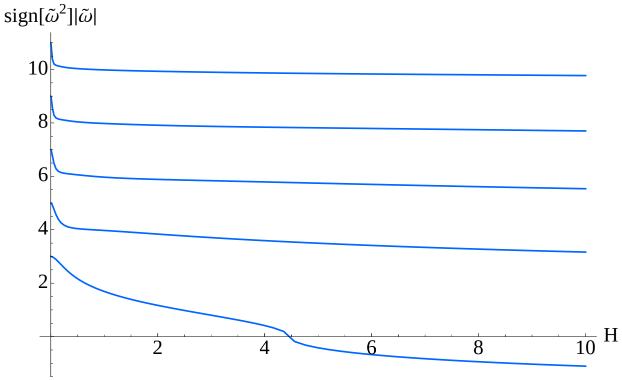

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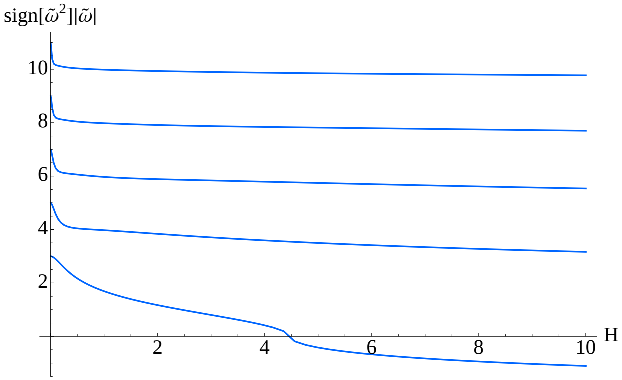

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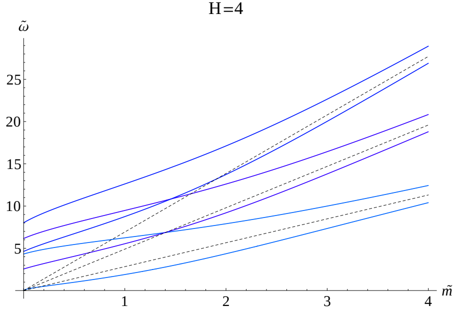

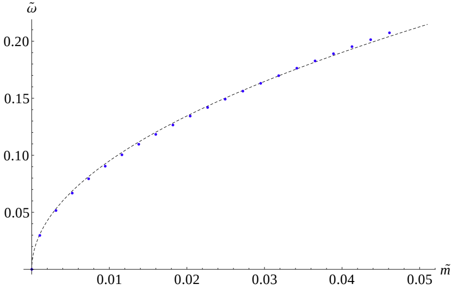

The plot of the lowest lying meson state for small bare mass exhibits the GMOR behaviour characteristic of a Goldstone boson. This corresponds to the broken $U(1)_A$ symmetry and is an analog of the η' meson.

Summary

- We introduced magnetic field to flavoured $\mathcal{N} = 2$ SYM on S^3.
- Casimir energy competes with magnetic catalysis.
- The phase diagram has a first order confinement / deconfinement phase transition ending on a third order one (at $H = 0$).
- The meson spectrum features GMOR relation.

- Outlook
  - Going beyond the quenched approximation.
  - Studying defect field theories in the same context.