

Due before tutorial, monday December 3rd.

Problems titled [**SELF**] are for your own practice and will not be marked.

If any calculations are required to obtain your answers, please show them.

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1. Consider the linear equation

$$x_1 - 2x_2 = 2$$

- (a) [**4 pts.**] Plot the line (on the x_1 - x_2 plane) which represents all the possible solutions of this equation.
- (b) [**3 pts.**] Show that $(x_1, x_2) = (4, 1)$ is a solution of this equation.
- (c) [**3 pts.**] If $(x_1, x_2) = (0, \alpha)$ is a solution of this equation, find α .
- (d) [**SELF**] Write down three additional solutions of the equation, other than the two discussed already.

2. Matrix inverses.

- (a) [**4 pts.**] Find out and explain whether the matrices

$$\begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$$

are inverse of one another.

- (b) [**5 pts.**] The inverse matrix of $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$ is the matrix $\begin{pmatrix} 2 & \alpha \\ 1 & 2 \end{pmatrix}$. Determine the value of α .

- (c) [**7 pts.**] The matrices

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 3 & -1 & -4 \\ 4 & -1 & -6 \\ -3 & \beta & \gamma \end{pmatrix}$$

are inverse of one another. Determine the values of β and γ .

- (d) [**SELF**] Show that the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is its own inverse.

3. Given the matrices

$$L = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 3 & 2 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} \alpha & \beta \\ x & y \end{pmatrix}$$

Find out and explain which of the following quantities are meaningful, and which are not. If meaningful, calculate the quantity.

- (a) [**SELF**] $MN + L$
- (b) [**5 pts.**] NML
- (c) [**4 pts.**] $ML - 2NM$

4. Linear systems of equations as matrices.

- (a) [**SELF**] Write the following linear system of equations as a matrix equation:

$$\begin{aligned} x + 3y &= -2 \\ -2x + 4y &= 5 \end{aligned}$$

- (b) [**4 pts.**] Write the following linear system of equations as a matrix equation:

$$\begin{aligned} 4x + y - 3z &= 2 \\ -2x + 2y - z &= -3 \end{aligned}$$

Note that the number of equations is not the same as the number of variables, so the coefficient matrix is not square.

5. Consider the system of equations

$$\begin{aligned} x - 2y + z &= 3 \\ x - y + z &= -2 \\ -x - y &= 3 \end{aligned}$$

- (a) [**6 pts.**] Write down the coefficient matrix A . Show that

$$B = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ -2 & 3 & 1 \end{pmatrix}$$

is the inverse of the coefficient matrix.

- (b) [**5 pts.**] Solve the system of equations using the inverse of the coefficient matrix.