Due before tutorial, monday November 26th.

If any calculations are required to obtain your answers, please show them.

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1. **[5 pts.]** Consider the integral

$$\int_0^3 (x+1)\,dx\,.$$

Sketch a plot of the integrand (the function being integrated) and show (with shading or colouring) the area represented by the above definite integral. Find the integral by geometrically calculating the area.

2. For each of the following integrals, sketch a plot of the integrand and shade the area represented by the definite integral. Find the integral by geometrically calculating the area.

(a) **[3 pts.]** $\displaystyle\int_{-1}^{3} |x|\,dx$

(b) **[3 pts.]** $\displaystyle\int_{-2}^{4} \frac{|x|}{x}\,dx$

(c) **[3 pts.]** $\displaystyle\int_{2}^{7} \frac{|x|+x}{x}\,dx$

(d) **[3 pts.]** $\displaystyle\int_{-3}^{3} \frac{x^3}{2}\,dx$

(e) **[4 pts.]** $\displaystyle\int_{2}^{4} x\,dx$

(f) **[SELF]** $\displaystyle\int_{-2}^{0} \frac{|x|}{x}\,dx$

(g) **[SELF]** $\displaystyle\int_{-2}^{-1} \frac{|x|+x}{x}\,dx$

3. Calculate the following definite integrals using the rule $\displaystyle\int x^n dx = \frac{x^{n+1}}{n+1}$.

(a) **[SELF]** $\displaystyle\int_{0}^{3} (x+1)\,dx$

(b) **[4 pts.]** $\displaystyle\int_{-1}^{1} 4x^3\,dx$

(c) **[5 pts.]** $\displaystyle\int_{0}^{4} \sqrt{x}\,dx$

(d) **[4 pts.]** $\displaystyle\int_{-2}^{-1} (4x-3)\,dx$

4. Remember that if $G'(x) = f(x)$, then $\int f(x)dx \;=\; G(x) + C$. Here C is an arbitrary constant of integration. Use this to prove that

   (a) **[SELF]** $\displaystyle\int \ln(x)\,dx \;=\; x\ln(x) - x + C$

   (b) **[4 pts.]** $\displaystyle\int (x^3 + 3x^2)e^x\,dx \;=\; x^3e^x + C$

   (c) **[4 pts.]** $\displaystyle\int dx\,x\ln(x) \;=\; \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C$

5. **[2 pts.]** Consider the linear equation

$$3x_1 - x_2 = -1$$

   How many solutions does this equation have? (Zero, one, infinity?) Explain why.

6. Given the system of equations

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 2 \\ -x_1 + 3x_2 - x_3 &= -2 \\ 3x_1 + x_2 - 3x_3 &= -2 \end{aligned}$$

   (a) **[3 pts.]** check whether $(x_1, x_2, x_3) = (0, 2, 2)$ is a solution.

   (b) **[3 pts.]** check whether $(x_1, x_2, x_3) = (2, 1, 3)$ is a solution.

   (c) **[SELF]** check whether $(x_1, x_2, x_3) = (4, -1, 2)$ is a solution.