Due before the tutorial after Study Break, monday November 5th.

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- Please **STAPLE** your work together before submitting!
  It's painful for Aonghus to mark assignments on loose sheets.
  And it's quite unfair to ask him to staple your pages for you!
- Problems titled [**SELF**] are for your own practice and will not be marked.
- If any calculations are required to obtain your answers, please show them. Your work will be marked for your reasoning/calculations as well as for giving the correct final answer.

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1. (a) [**3 pts.**] Explain the technique of using mathematical induction to prove a statement for all positive integers.

   (b) [**8 pts.**] Use mathematical induction to prove that

   $$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

   for all positive integers n.

2. Find the derivatives of the following functions from first principles, i.e., using the definition $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

   (a) [**5 pts.**] $f(x) = 5x^2 - 2$

   (b) [**8 pts.**] $f(x) = 2\sqrt{x}$
   Comment: This is harder. You might be able to get some inspiration by searching google for "Derivative by first principles, square root", or something similar. However, please do not copy and paste directly; write out your own solution.

   (c) [**SELF**] $f(x) = 3x^2 - 2x$

   (d) [**SELF**] $f(x) = x + \dfrac{1}{\sqrt{x}}$

3. For a general function $f(x)$, we cannot expect $f(a+b)$ to be equal to $f(a)+f(b)$, nor can we expect $f(ab)$ to be equal to $f(a)f(b)$.

   These might be true for some functions or for specific values of a and b, but not in general for any function $f(x)$ for all values of a and b.

   (a) **[2 pts.]** Consider $g(x) = x^2$. Show that $g(a+b) \neq g(a) + g(b)$ in general.

   (b) **[2 pts.]** Consider $g(x) = 2x^2$. Is $g(ab)$ equal to $g(a)g(b)$?

   (c) **[3 pts.]** Consider $h(x) = \ln x$. Is $h(ab)$ equal to $h(a)h(b)$?

   (d) **[SELF]** For $f(x) = \sqrt{x}$, examine whether $f(a+b) = f(a) + f(b)$. Also, examine whether $f(ab) = f(a)f(b)$.

   (e) **[SELF]** For $f(x) = x^2 + x$, examine whether $f(a+b) = f(a) + f(b)$ in general. Also, examine whether $f(ab) = f(a)f(b)$.

4. Find the derivatives of the following functions using the rules for derivatives. Please remember to show your calculations.

   (a) **[3 pts.]** $f(x) = -3/x$

   (b) **[4 pts.]** $f(x) = 2x^3 + \dfrac{4}{\sqrt{x}}$

   (c) **[4 pts.]** $f(x) = \dfrac{x + 2x^2}{\sqrt{x}}$

   (d) **[4 pts.]** $f(x) = 3x^2 + \sqrt{x^3}$

   (e) **[SELF]** $f(x) = 2x^{3/2} - 14x$

   (f) **[SELF]** $f(x) = 2\sqrt{x} + \dfrac{1}{3}x^3$

5. (a) **[4 pts.]** At which value or values of x does the function $f(x) = 3 + 4x - x^2$ have zero slope?

   (b) **[SELF]** At which value or values of x does the slope of the function $f(x) = \frac{1}{2}x^3 - 6x$ disappear?