Condensed Matter Theory (MP473) Assignment 3

Please hand in your solutions no later than Tuesday, March 10, 11:05 pm. If you have questions about this assignment, please ask your lecturer,

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Ex. 3.1: Fermions interacting on a circle - Hartree Fock approximation

Consider a system consisting of a large number N of spinless interacting fermions in a large one-dimensional box of length L with periodic boundary conditions. The particles interact via a delta-function potential, and so the Hamiltonian is

$$H = \sum_k Ak^2 c_k^\dagger c_k + \frac{V}{2L} \sum_{k,k',q} c_{k-q}^\dagger c_{k'+q}^\dagger c_{k'} c_k$$

with A and V constants. The sums proceed over all permitted values of k, k' and q, so the terms with $q = 0$ are included.

(a) Calculate the energy E_0 of the ground state of the non-interacting system (in terms of A, N and L) and show that $E_0 = \frac{1}{3} N\epsilon_F$.

(b) Calculate the energy of the ground state of the interacting system, to first order in perturbation theory.

(c) State in physical terms why the answer you obtained to part (b) must be an exact solution of the problem.
Hint: the fact that we used a delta function potential is important

(d) Check that, with an appropriate value for A, the Hamiltonian H acting on particles with coordinates x_i and momenta p_i is indeed given by

$$H = \sum_i \frac{p_i^2}{2m} + \frac{V}{2} \sum_{i,j} \delta(x_i - x_j)$$

(e) Answer question (b) again but now with the potential V replaced by $V_q = Vq^{2l}$ (this has to go inside the sum over q now). You should find a nonzero result (if $V \neq 0$) which is extensive and depends on the density $\frac{N}{L}$

Ex. 3.2: Dimensional analysis of Jellium

(a) Consider the Hamiltonian for the Jellium nodel of a metal given in the book by Fetter and Walecka, pg. 25, formula (3.19). Show explicitly how this transforms into the form given in formula (3.24) on the same page.

(b) The book by Fetter an Walecka uses Gaussian or cgs (centimetre-gram-second) units. Look up Gaussian units, for example on Wikipedia (`https://en.wikipedia.org/wiki/Gaussian_units`).
Find out if and how formulas (3.19), (3.24) and (3.37) would change when converted from Gaussian to SI units.

Ex. 3.3: White Dwarfs and Jellium

Revisit exercise 7.23 in the book *An Introduction to Thermal Physics* by Daniel V. Schroeder. This exercise describes a white dwarf star in terms of a gas of free electrons without interactions. If you have not done this problem before, please do it now (parts (a)–(g)). In addition do the following part (h)

(h) Assume that the electrons are weakly interacting, instead of free, and the electrons in the neutron star are described by the Jellium model. This means in particular that their kinetic and interaction energy is given by formula (3.37) in the book by Fetter and Walecka.

  (i) Express the electron interaction energy in terms of the mass and radius of the star (and various constants, like the proton mass, the electron charge etc.)

  (ii) Discuss the consequences of this new term in the energy for the equilibrium radius of the star, in particular for a 1 solar mass dwarf (as in part (d)).

  (iii) Finally calculate the value of the parameter r_s in the Jellium nodel for this star.