MP463
Problem Set 6
Time-dependent perturbation theory
Quantum theory of the system of identical particles.

1. Consider a physical system with the Hamiltonian H_0 whose eigenvalue equation is $H_0|\,\phi_n\rangle = E_n|\,\phi_n\rangle$. At time $t=0$ a time-dependent perturbation $W(t)$ is applied, so the new Hamiltonian is obtained

$$H(t) = H_0 + \hat{W}(t) = H_0 + \lambda W(t) \tag{1}$$

where $\lambda \ll 1$ and $W(t=0) = 0$. Using the time-dependent perturbation theory, we can approximate the state $|\,\psi(t)\rangle$ of the system at a later time $t > 0$. We express it as a linear combination of the eigenstates of H_0

$$|\,\psi(t)\rangle = \sum_n c_n(t)|\,\phi_n\rangle \tag{2}$$

where $c_n(t) = \langle\phi_n|\psi(t)\rangle$, and for small perturbation we can write it as

$$c_n(t) = b_n(t)e^{-iE_nt/\hbar} \tag{3}$$

where $b_n(t)$ is a slowly varying function of time. Substituted into the Schrödinger equation it gives

$$i\hbar\frac{d}{dt}b_n(t) = \lambda\sum_k e^{i\omega_{nk}t}W_{nk}(t)b_k(t) \tag{4}$$

where we have introduced the Bohr frequency $\omega_{nk} = (E_n - E_k)/\hbar$. Assume that the initial state of the system is $|\,\psi(t=0)\rangle = |\,\phi_i\rangle$. Using the time-dependent perturbation theory, derive the state $|\,\psi(t)\rangle$ of the system at a later time $t > 0$ to the first order.

2. Symmetrize and antisymmetrize a state of three particles.