Statistical Mechanics (MP461)

If you have questions about these exercises, please ask your lecturer,
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I recommend Ex. 5.1 below and Ex. 1.46 and 5.14 in Schroeder's book.

Ex. 5.1 Deriving and Using the Clausius-Clapeyron equation

a. Give a derivation of the Maxwell relation $\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$

b. Now consider a liquid in coexistence with its vapor at constant pressure p and temperature T. The infinitesimal amount of heat needed to evaporate a quantity of liquid of infinitesimal mass dm is given by $dQ = \lambda\, dm$, where λ is the latent heat of evaporation. The specific volumes (volumes per unit mass) of the liquid and of the vapor are v_{liquid} and v_{vapor}.
Derive Clapeyron's equation for the pressure in this system,

$$\left(\frac{dp}{dT}\right)_V = \frac{\lambda}{(v_{vapor} - v_{liquid})T}$$

c. Assume that v_{liquid} is negligible compared to v_{vapor} and assume that the vapor satisfies the equation of state $pV = nR(T - \alpha^2/T)$, where n is the number of *mol* of the gas and α is a constant. Show that in this case the vapor pressure is given by

$$p = C\left(\frac{T-\alpha}{T+\alpha}\right)^{\frac{\lambda M_{mol}}{2\alpha R}}.$$

Here M_{mol} is the molar mass of the substance and C is a constant.
Hint: you may use that $\int \frac{1}{z^2-1}\, dz = \frac{1}{2}\log(z-1) - \frac{1}{2}\log(z+1) + c$, *where* c *is a constant of integration.*