Statistical Mechanics (MP461) - Spring 2012 - Assignment 2

Please hand in your solutions no later than Tuesday February 28, at the start of the lecture.
If you have questions about this assignment, please ask your lecturer,
Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics.

Interacting paramagnets

A two-state paramagnet is composed of N elementary magnetic dipoles with dipole moments $\pm\mu$ in the z-direction. The total magnetization M is $M = (N_\uparrow - N_\downarrow)\mu$, where $N_\uparrow$ is the number of dipoles with positive (upward pointing) dipole moment, $N_\downarrow$ is the number of dipoles with negative (downward pointing) dipole moment and $N_\uparrow + N_\downarrow = N$. if we apply a magnetic field of strength B in the positive z-direction, the energy $U(B)$ of the paramagnet is given by $U = -MB$.

a. What is the total number of microstates of this paramagnet? What is the number of microstates with given magnetisation $M = M_0$?

b. What is the lowest energy state of the parmagnet if $B > 0$? What if $B < 0$? What is the entropy of a macrostate with given energy $U = U_0$?

Suppose we have two two-state paramagnets composed of N_1 and N_2 elementary dipoles of the same strength μ.

c. What is the total number of microstates of this system of two paramagnets? What is the number of microstates in which the first paramagnet has magnetisation $M = M_1$ and the second has magnetisation $M = M_2$?

d. If the first paramagnet is in a macrostate A with entropy $S(A)$ and the second is in a macrostate B with entropy $S(B)$, what is the entropy of the two paramagnets combined?

In any state of the combined system, the total energy U equals $U_1 + U_2$, where U_1 and U_2 are the energies of the individual paramagnets. We assume the two paramagnets are exposed to the same magnetic field B, which fixes the total energy of the system U.

d. Consider the case where $N_1 = 5$ and $N_2 = 3$. Calculate the total number of microstates with $U = -2B\mu$. If $U = -2B\mu$, what are the possible values of M_1 and U_1? How many microstate are there for each of the possible values of M_1 ?

e. Now consider the case where both N_1 and N_2 are large. Show that the entropy of the system is given, to good approximation, by

$$S_{total} = -N_1(x\log(x) + (1-x)\log(1-x)) - N_2(y\log(y) + (1-y)\log(1-y)),$$

where $x = \frac{1}{2} - \frac{U_1}{2N_1\mu B}$ and $y = \frac{1}{2} - \frac{U_2}{2N_2\mu B}$.

f. Show that, for the given total energy U, the entropy S_{total} is maximized when $U_1 = \frac{N_1}{N}U$ and $U_2 = \frac{N_2}{N}U$.

Additional recommended exercises *from the book by Schroeder:*
Ex. ***2.17***,***2.18***,***2.19*** *and* ***2.22***. *These do not have to be handed in.*