Thermodynamics (MP460) Assignment 4

Please hand in your solutions no later than Monday, November 7, 10:05 am. Late assignments will not be accepted. If you have questions about this assignment, please ask your lecturer, Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics

Ex. 5.1: The Otto Cycle

An internal combustion engine (e.g. a car engine) has an idealized description as a heat engine. A single cycle of such an engine consists of four steps.

1. A mixture of air and fuel vapor, at initial temperature T_1 is adiabatically compressed (in a cylinder) from an initial volume V_1 to a final volume $V_2 < V_1$.
2. The mixture is ignited, which raises its temperature to T_2, at constant volume. We can think of T_2 as the temperature of the hot reservoir (even though there is no hot reservoir, strictly speaking). We also ignore the chemical changes in the gas.
3. The mixture is allowed to expand adiabatically until it reaches the intial volume V_1
4. Finally, the gas is replaced by a new fuel/air mixture at the initial temperature T_1 and the same volume V_1. This can be modeled as if the system was simply cooling down at constant volume, in contact with a cold reservoir at temperature T_1.

This cycle is called the Otto cycle, after Nikolaus August Otto (1832-1891), inventor of the four stroke engine.

a. Draw the idealized cycle in a (p, V) diagram.

b. Assume that throughout the process, the gas in the cylinder always has a constant specific heat c_V and energy $U = c_V nT$. Calculate the change in temperature in each of the four steps of the cycle in terms of T_1, T_2, V_1 and V_2.

c. Express the efficiency of this cycle in terms of the compression ratio $\frac{V_1}{V_2}$

Ex. 5.2: Performance of refrigerators

Consider an idealized refrigerator, let's call it a cooling engine. This engine is in contact with two heat reservoirs at temperatures T_1 and T_2, with $T_2 > T_1$. In a single cycle, the engine absorbs heat Q_1 from the reservoir at temperature T_1 and it also releases heat Q_2 to the reservoir at temperature T_2. In the process, the engine does work $L = Q_1 - Q_2$, or rather, work $-L$ has to be done on the engine to operate it for a cycle.

a. Show that $Q_2 \geq Q_1$.
Hint: use Kelvin's formulation of the second law.

We define the *coefficient of performance* ξ of the engine as follows,

$$\xi = \frac{Q_1}{-L}$$

b. Show that $\xi > 0$

c. Consider two cooling engines E and E', both operating between the same temperatures T_1 and T_2, with $T_2 > T_1$. Call the associated amounts of heat exchanged with the two reservoirs Q_1, Q_2 and Q'_1, Q'_2. Assume that E is reversible. Show that we have $\xi_E \geq \xi_{E'}$ and if both E and E' are reversible, we have $\xi_E = \xi_{E'}$

d. Show that the maximal value ξ_{max} of the coefficient of performance for a cooling engine working between temperatures T_1 and T_2 (with $T_2 > T_1$) is given by

$$\xi_{max} = \frac{1}{\frac{T_2}{T_1} - 1}$$