Thermodynamics (MP460) Assignment 4

Please hand in your solutions no later than Monday, October 24, 10:05 am. Late assignments will not be accepted. If you have questions about this assignment, please ask your lecturer, Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics

Ex. 4.1: Adiabatic expansion and the atmosphere

A gas has the following equation of state,

$$p(V - nb) = nRT \tag{1}$$

Here $R = 8.31\, J/(Kmol)$ is the gas constant, n is the number of moles of gas and b is the minimal molar volume of the gas. The energy of this gas is given by

$$U = c_V nT + W \tag{2}$$

where c_V and W are constants (c_V is the specific heat per mol at constant volume).

a. Show that the specific heat of the gas at constant pressure is equal to $c_V + R$.

b. Derive a formula relating volume and temperature on the adiabatic curves of this gas. Also give a formula relating pressure and temperature.

c. Assume the equation of state above as an approximate description for air. Argue that, in the atmosphere, we must have $\frac{dp}{dh} = -(\frac{gM_{mol}p}{bp+RT})$, where M_{mol} is the molar mass of air ($M_{mol} \approx 29g$).

d. Find an expression for $\frac{dT}{dh}$, in terms of p, T c_V, g, M_{mol} and b. Assume the temperature difference at different h is caused by the adiabatic expansion of rising air.

e. Estimate the temperature drop per vertical km in the atmosphere near the surface of the earth at $20°C$. Take b to be the molar volume of liquid air, which is approximately 33 cc. Take $c_v = \frac{5}{2}R$. Compare the result to the estimate in the book by Fermi.