Thermodynamics (MP460) Assignment 2

Please hand in your solutions no later than Monday, October 10, 10:05 am. Late assignments will not be accepted. If you have questions about this assignment, please ask your lecturer, Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics

Ex. 2.1, 2.2 and 2.3

Ex. 2.1 is exercise 1 on page 10 of the book by Fermi. Please give the answer in SI units.
Ex. 2.2 is Exercise 4 on page 10 of the book by Fermi. Please give the answer in SI units.
Ex. 2.3 is Exercise 1 on page 28 of the book by Fermi. Please give the answer in SI units.

Ex. 2.4: Some properties of the van der Waals gas

The van der Waals equation is an equation of state for a fluid (this may be a gas or a liquid), which, for many fluids is a better approximation than the ideal gas law. The equation takes the following form,

$$(p + \frac{n^2 a}{V^2})(V - nb) = nRT \tag{1}$$

Here $R = 8.31\, J/(Kmol)$ is the gas constant, n is the number of moles of fluid and a and b are parameters which depend on the type of fluid. For air, we have $a = 0.136 Jm^3/mol^2$ and $b = 3.64 \times 10^{-5} m^3/mol$. These values are optimized to make the equation of state work for gaseous air. The mass of a mol of air is approximately $29\, g$.

a. Derive a formula for the pressure of the fluid in terms of the density, temperature and molar mass, and the constants a,b and R.

b. Calculate the pressure for air at temperature $20°C$ and density $1.19\, kg/m^3$. Compare to the result of exercise **1.1g**

c. Show that if we take the pressure to infinity at constant temperature, the volume approaches nb. Similarly, show that if we take the temperature to zero at constant pressure, the volume approaches nb. Give a rough estimate of the density of liquid air (the actual density is $870\, kg/m^3$)

d. Assume that $V = nb + \epsilon$, with $\epsilon << nb$. Show that we have, approximately, $\epsilon = \frac{nRT}{p + \frac{a}{b^2}}$. In this approximation, what volume does the fluid take when p is taken to zero at constant temperature?

e. If T is too large, we cannot trust the approximation to ϵ calculated in part **d.** (why not?). With the values of a and b given for air, calculate the temperature below which, even if $p = 0$, we have $\frac{\epsilon}{nb} \leq \frac{1}{10}$ (using the approximation of part **d.**).

f. Calculate the work performed by a van der Waals gas with parameters a and b when it expands isothermally from an initial volume V_i to a final volume V_f.

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g. Consider the following cycle for a van der Waals gas.

- The gas starts in a state A at pressure p_A and volume V_A and is then heated at constant volume, from its initial temperature T_A to a final temperature T_B. We call the state of the gas at this point B.
- The gas is now allowed to expand isothermally to a state C at volume V_C.
- The gas is then cooled down at constant volume to a state D which is back at the original temperature T_A.
- Finally, the gas is isothermally compressed until it is back in the state A.

Draw a sketch of this cycle in a (p, V)-diagram, indicating the points A,B,C and D. Calculate the total amount of work performed by the gas and the total amount of heat absorbed by the gas in this cycle.