Thermodynamics (MP460) Assignment 1

Please hand in your solutions no later than Monday, October 3, 10:05 pm. Late assignments will not be accepted. If you have questions about this assignment, please ask your lecturer, Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics

Ex. 1.1: Ideal gases

In an ideal gas, pressure p, volume V, mass m and temperature T in Kelvin satisfy the *Ideal Gas Law*:

$$pV = nRT \tag{1}$$

Here $R = 8.31\, J/(Kmol)$ is the gas constant and n is the number of mol of gas.

a. If we keep the temperature and the amount of gas constant, what happens to the volume if we increase the pressure by a factor α, that is $p \to \alpha p$? Why can we not expect this behavior to be true for arbitrarily large α in a real gas?

b. If we keep the temperature and the volume of gas constant, what happens to the pressure if we increase the amount of gas by a factor α, that is $n \to \alpha n$? Why can we not expect this behavior to be true for arbitrarily large α in a real gas?

c. If we keep the volume and the amount of gas constant, what happens to the pressure if we increase the temperature from $100°C$ to $200°C$?

d. Sketch some isotherms of the ideal gas in a (p, V)-diagram

e. Sketch some isochores of the ideal gas in a (p, T)-diagram

f. Sketch some isobars of the ideal gas in a (V, T)-diagram. Sketch some isotherms and isochores in this diagram as well.

g. Derive a formula for the density $\rho = \frac{m_{gas}}{V_{gas}}$ of the gas from the ideal gas law. Use this to calculate the density of dry air at a temperature of $20°C$ and at atmospheric pressure (approximately 100 kPa). You may use that, for dry air, the mass per mol is $M_{mol} \approx 29\, g$.

h. An oven contains 20 Litres of air (assumed to be an ideal gas). It can communicate freely with the surrounding air, which is at atmospheric pressure. The oven is heated from $20°C$ to $200°C$. What is the mass of the air that leaves the oven in this process? Assuming that all the air outside the oven (including the air that came from the oven) has a temperature of $20°C$, what is the volume of the displaced air? What is the total work performed in displacing this amount of air?

i. An alternative way to write the ideal gas law is

$$pV = Nk_BT. \tag{2}$$

Here N is the number of particles that make up the quantity of gas and k_B is Boltzmann's constant. Given that the the number of particles in one mol is Avogadro's number $N_A = 6.0221 \times 10^{23}$, calculate the value of k_B.

Ex. 1.2: Less ideal gases

In a slightly less ideal gas, pressure p, volume V, mass m and temperature T in Kelvin satisfy the following equation of state,

$$p(V - b) = Nk_BT \tag{3}$$

a. Give the units of b. Show that, for given pressure and temperature, the volume of this gas is larger than that of an ideal gas

b. Show that, in an isothermal expansion at temperature T_0, from an initial volume V_i to a final volume V_f, this gas performs a total amount of work given by

$$L = Nk_BT_0 \log(\frac{V_f - b}{V_i - b}). \tag{4}$$

c. Consider a cycle where the gas is first isotermically expanded as above, at temperature T_0, then cooled at constant volume, to temperature $T_1 < T_0$, then isothermically compressed to the initial volume V_i and finally warmed up at constant volume to the initial temperature T_0. Draw a sketch of this cycle in a (p, V)-diagram. Also draw a sketch of this cycle in a (p, T)-diagram. Finally, calculate the total work done by the gas in this cycle.

d. Now look at a gas with equation of state given by

$$(p + \frac{b}{V^2})V = Nk_BT. \tag{5}$$

Find the units of b. For a given volume and temperature, show that this gas has a lower pressure than an ideal gas.

e. Answer question **c.** also for this gas.