## * Measurement of spin-$\frac{1}{2}$ object (Example for postulates 4, 3)

# Measure $S_z$ :- result is either $+\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$ because eigenvalues of $\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are $\pm \hbar/2$.

# If state is $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, normalized, measurement can give

$+\frac{\hbar}{2}$, with probability $|c_1|^2$

$-\frac{\hbar}{2}$, with probability $|c_2|^2$

# If $S_z$ measured to be $+\frac{\hbar}{2}$, system immediately after measurement will be in state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

If $S_z$ measured to be $-\frac{\hbar}{2}$, system after measurement will be in state $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Eigenvector (of $\hat{S}_z$) corresponding to $-\frac{\hbar}{2}$ is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

# $\hat{S}_x = \frac{\hbar}{2}\sigma_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Eigenvalue, eigenvector

$+\hbar/2 \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$-\hbar/2 \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} +1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

upto phase

# Measure $S_x \rightarrow$ result either $+\hbar/2$ or $-\hbar/2$. If $+\hbar/2$, state after measurement is $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$.