MATHEMATICAL METHODS: PROBLEM SET 4

November 1^{st}, 2011

** **1.** Show that the Fourier series of

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{4}{\pi}\left[\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \cdots\right]$$

** **2.** Evaluate the Fourier series above at $x = \pi/2$ and deduce the celebrated result that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \cdots$$

** **3.** Write down the formula for the Fourier expansion of a function $f(x)$ on the interval $[-\pi, \pi]$ *and* the formula for its expansion on a *general* interval $[a, b]$. Give the formula for the coefficients a_n and b_n in both cases. Check that when $a = -\pi$ and $b = \pi$ the second set of formulae agree with the first set. Don't prove anything just use your notes.

*** **4.** Let $f(x)$ be a periodic function on the interval $[-\pi, \pi]$ with Fourier series

$$f(x) = \sum_0^\infty a_n \cos(nx) + \sum_1^\infty b_n \sin(nx).$$

Given that the first k derivatives of f exist find a bound on the growth of the Fourier coefficients a_n and b_n for large n.

** **5.** Let $f(x)$ be a periodic function on the interval $[-\pi, \pi]$ with Fourier series

$$f(x) = \sum_0^\infty a_n \cos(nx) + \sum_1^\infty b_n \sin(nx).$$

If $f(-x) = -f(x)$ show that

$$f(x) = \sum_1^\infty b_n \sin(nx)$$

and if $f(-x) = f(x)$ show that

$$f(x) = \sum_0^\infty a_n \cos(nx)$$

** **6.** The periodic function $\phi(x)$ satisfies $\phi(-x) = -\phi(x)$ and is defined by

$$\phi(x) = \phi(x + 2\pi), \qquad \phi(x) = \begin{cases} -(\pi + x)/2, & -\pi \le x < 0 \\ 0, & x = 0 \\ (\pi - x)/2, & 0 < x \le \pi \end{cases}$$

Hence show that it has the Fourier series expansion

$$\phi(x) = \sum_1^\infty \frac{\sin(nx)}{n}$$

** **7.**

Explain briefly, in informal terms, what the Gibbs phenomenon is. Recall that this is to do with the Fourier series of a function with a discontinuity.

*** **8.** The function $\phi(x)$ has the Fourier series expansion

$$\phi(x) = \sum_1^\infty \frac{\sin(nx)}{n},$$

find a formula for its partial sums $S_n(x)$.

*** **9.** Use the result of the previous question to compute the Gibbs interval G for this function.