Electricity and Magnetism 2 and Statistical Thermodynamics (MP232)
Assignment 4

Please hand in your solutions no later than Tuesday, April 20, at the start of the 11am lecture. Late assignments will not be accepted. If you have questions about this assignment, please ask your tutor,
Glen Burrella, (`glen-dot-burella-at-nuim-dot-ie`), Office 1.1, Mathematical Physics,
or your lecturer,
Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics.

Ex. 4.1: The ideal gas law

In an ideal gas, pressure p, volume V and temperature T in Kelvin satisfy the *Ideal Gas Law.* We give this law below in three different forms:

$$pV = Nk_BT \tag{1}$$
$$pV = nRT \tag{2}$$
$$pV = \frac{m}{M_{mol}}RT \tag{3}$$

In the first line, N is the number of particles that make up the quantity of gas and $k_B \approx 1.38 \times 10^{-23}$ is Boltzmann's constant.
In the second line, n is the number of mol of gas and $R \approx 8.31\,J/(Kmol)$ is the gas constant.
In the third line, m is the mass of the quantity of gas and M_{mol} is the mass of one mol of gas.
The number of particles in one mol of gas is Avogadro's number $N_A \approx 6.0221 \times 10^{23}$.
For dry air, we have, $M_{mol} \approx 29\,g/mol$.

a. Check that the three forms of the ideal gas law given above are all equivalent to each other.

b. If we keep the temperature and the amount of gas constant, what happens to the volume if we increase the pressure by a factor α, that is $p \to \alpha p$? Why can we not expect this behavior to be true for arbitrarily large α in a real gas?

c. If we keep the temperature and the volume of gas constant, what happens to the pressure if we increase the amount of gas by a factor α, that is $m \to \alpha m$? Why can we not expect this behavior to be true for arbitrarily large α in a real gas?

d. If we keep the volume and the amount of gas constant, what happens to the pressure if we increase the temperature from $100\,K$ to $200\,K$?

e. If we keep the volume and the amount of gas constant, what happens to the pressure if we increase the temperature from $100°C$ to $200°C$?

f. Derive a formula for the density ρ of the gas from the ideal gas law (the density is the mass per unit volume). Use this to calculate the density of dry air at a temperature of $20°C$ and at atmospheric pressure (approximately 100 kPa).

Ex. 4.2: Maxwell-Boltzmann velocity distribution

For a gas of N particles moving in three dimensions, the Maxwell-Boltzmann velocity distribution function f is of the form $f(\mathbf{v}) = Ce^{-\frac{mv^2}{2kT}}$, where $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ and C is a constant (independent of v)

a. Show that the constant C must be given by $C = N\sqrt{(\frac{m}{2\pi kT})^3}$.
Hint: use that $\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$

b. What is the most likely value of the velocity $\mathbf{v}$?

c. What is the most likely value of the speed v?
Hint: there are many different velocities corresponding to the same speed. The most likely speed is the speed v for which there are the most particles with speeds between v and $v + dv$, for fixed, infinitesimally small dv.

d. What is the most likely value of the kinetic energy $\frac{1}{2}mv^2$ of a single particle? Does it correspond to the kinetic energy of a particle at the most likely speed?