Electricity and Magnetism 2 and Statistical Thermodynamics (MP232)
Assignment 3

Please hand in your solutions no later than Tuesday, March 30, at the start of the 11am lecture. Late assignments will not be accepted. If you have questions about this assignment, please ask your tutor,
Glen Burrella, (`glen-dot-burella-at-nuim-dot-ie`), Office 1.1, Mathematical Physics,
or your lecturer,
Joost Slingerland, (`joost-at-thphys-dot-nuim-dot-ie`), Office 1.7D, Mathematical Physics.

Ex. 3.1: Wave equations and such

a. From Maxwell's equations, derive that the magnetic field satisfies the following equation

$$\nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = -\mu_0 \nabla \times \mathbf{J}$$

b. Inside a homogeneously charged insulator that rotates around the z-axis at angular velocity ω, the current density has the form $\mathbf{J} = \rho\,\omega(y, -x, 0)$, where ρ is the constant charge density in the object. Assuming this form of $\mathbf{J}$ and assuming that $\mathbf{B}$ does not depend on x and y, show that the equation from part **a.** reduces to

$$\frac{\partial^2 \mathbf{B}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = 2\mu_0 \rho\,\omega \hat{\mathbf{z}}$$

We see that that, for B_x and B_y, we have simply the wave equations

$$\frac{\partial^2 B_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 B_x}{\partial t^2} = 0 \qquad \frac{\partial^2 B_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2} = 0,$$

that is, the same equations as for the magnetic field in vacuum. To solve the wave equation for B_x (or B_y), we make a change of variables.

c. Define $u = z + ct$ and $v = z - ct$. Show that, for any function f

$$\frac{\partial^2 f}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 4\frac{\partial^2 f}{\partial u \partial v}$$

d. Show that $B_x(z,t) = f_+(z+ct) + f_-(z-ct)$, for some functions f_+ and f_-.

e. Use the Maxwell equation for $\nabla \cdot \mathbf{B}$ to simplify the equation B_z and then solve for B_z. Using the solution, explain why the situation described in this problem cannot continue indefinitely.

Ex. 3.2: Plane waves

An electromagnetic wave has an electric field given by

$$\mathbf{E} = E_0 \cos(kx - \omega t)\,\hat{z}$$

a. Write the frequency ν, the wavelength λ, and the velocity c of this wave in terms of ω and k.

b. Calculate the frequency of a wave with $\lambda = 510nm$ (green light). Also calculate the wavelength of a wave with frequency $88.2MHz$ (RTÉ radio 1 FM).

c. Calculate the **B**-field of this wave by direct use of the Maxwell equations

d. Calculate the energy density u and Poynting vector **S** of this wave.