**Electricity and Magnetism 2 and Statistical Thermodynamics (MP232)**
**Assignment 1**

*Please hand in your solutions no later than Monday, February 22. Late assignments will not be accepted. If you have questions about this assignment,*
*please ask you tutor,*
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*or your lecturer,*
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## Ex. 1.1 Motion of a particle in constant electric and magnetic fields

A particle of charge $q$ is moving through a constant magnetic field $\mathbf{B}$. We choose the $z$-axis along the magnetic field, so that we have $\mathbf{B} = (0, 0, B)$, with $B = |\mathbf{B}|$. The particle has velocity $\mathbf{v} = (v_x, v_y, v_z)$.

a. Assume that the only force acting on the particle is due to the magnetic field. Show that we must have

$$\begin{aligned} \frac{dv_x}{dt} &= \frac{q}{m} B v_y \\ \frac{dv_y}{dt} &= -\frac{q}{m} B v_x \\ \frac{dv_z}{dt} &= 0 \end{aligned}$$

b. Find the general solution for the velocity $\mathbf{v}(t)$ from the equations in part **a**. From there, find the general solution for the position $\mathbf{r}(t)$ of the particle.

Describe the motion of the particle for the following sets of initial positions and velocities. Give the centre and radius of the orbit where applicable.

c. $\mathbf{v}(t = 0) = (u, 0, 0)$ and $\mathbf{r}(t = 0) = (0, 0, 0)$.

d. $\mathbf{v}(t = 0) = (0, u, 0)$ and $\mathbf{r}(t = 0) = (0, 0, 0)$.

e. $\mathbf{v}(t = 0) = (0, 0, u)$ and $\mathbf{r}(t = 0) = (x, y, 0)$.

f. $\mathbf{v}(t = 0) = (u, 0, w)$ and $\mathbf{r}(t = 0) = (0, 0, 0)$. What is the angle $\theta(t)$ that the orbit makes with the magnetic field in this case?

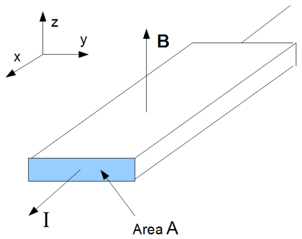
We now add an electric field $\mathbf{E}$ in addition to the magnetic field and orthogonal to it, say in the $y$-direction: $\mathbf{E} = (0, E, 0)$ with $E = |\mathbf{E}|$.

g. Derive the equations of motion for the new situation and compare with part **a.** If the particle is initially stationary at the origin ($\mathbf{r} = (0, 0, 0)$), describe qualitatively how it will move away from there. How does the motion depend on the charge $q$?

We now remove the electric field in the $y$ direction and instead add an electric field $\mathbf{E}$ parallel to the magnetic field, in the $z$-direction: $\mathbf{E} = (0, 0, E)$.

h. Derive the equations of motion for the new situation and give the general solution for the velocity $\mathbf{v}(t)$. Give an expression for the angle $\theta(t)$ that the orbit makes with the magnetic field in this case.

## Ex. 1.1 The Hall effect




Charged particles of charge $q$ move through a conducting slab of material, called a *Hall bar*, giving rise to a current $I$ in the $x$-direction.

a. Assume that

   - the Hall bar has a constant cross sectional area $A$ (orthogonal to the $x$-direction)
   - there is a constant density $\rho$ of moving particles per unit volume
   - the particles move with (average) velocity $v$ in the $x$ direction.

   Now argue that we must have $I = qA\rho v$

We now apply a magnetic field in the direction ortogonal to the current, say $\mathbf{B} = (0, 0, B)$. This gives rise to a Lorentz force on the particles which carry the current and causes their trajectories to bend in the $y$-direction. As a result, the density of charge carriers is raised one side of the Hall bar and lowered on the other side.

b. You are given that the current $I$ flows in the positive $x$-direction. Now suppose that $q$ is positive. On which side of the Hall bar will $\rho$ be raised/lowered? And if $q$ is negative?

If the magnetic field is kept in place, an equilibrium charge density profile is reached; the “excess” of charge carriers on one side and the “shortage” of charge carriers on the other side cause an electric field $\mathbf{E}_{Hall} = (0, E_{Hall}, 0)$ whose action on the particles compensates for the force due to the magnetic field.

c. Show that, in the equilibrium situation, $E_{Hall} = \frac{IB}{qA\rho}$

d. Many simple devices for measuring the magnetic field are based on the Hall effect[1]. Explain how you could measure the magnetic field somewhere in space using a Hall bar with known shape and material properties.

e. In some semiconductors, the current is carried by positively charged “holes” of charge $q = e$ (rather than electrons of charge $q = -e$). Can you tell this from the observed Hall field $\mathbf{E}_{Hall}$?

f. We have made a number of extremely crude assumptions and simplifications in this exercise, mostly implicitly. Give at least two examples of complications that should be taken into account in a more thorough treatment of the Hall effect.

[1] Usually one does not measure the electric field but rather the voltage $V_{Hall}$ or the resistance $R_{Hall}$ over the width of the bar, but this does not matter for the principle