NUI MAYNOOTH

Ollscoil na hÉireann Má Nuad

OLLSCOIL NA hÉIREANN MÁ NUAD

THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

MATHEMATICAL PHYSICS

Year 2

Autumn Repeat Examination

2007–2008

Electromagnetic Theory

MP232

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Time allowed: $1\frac{1}{2}$ hours

Answer **two** questions

All questions carry equal marks

1. Write down Maxwell's equations in free space and use them to show that both **E** and **B** travel as transverse waves with **E** perpendicular to **B**.

An electromagnetic wave has an electric field given by

$$\mathbf{E} = E_0 \cos(2\pi\nu t) \sin(\frac{2\pi z}{\lambda})\,\mathbf{i}$$

where λ and ν are constants. Use Maxwell's equations to calculate its associated electric field **B**; hence calculate its associated Poynting vector **S**.

2. Write down Maxwell's equations in the presence of a current density **J**. If u is the energy density given by

$$u = \frac{\epsilon_0}{2}(\mathbf{E}^2 + c^2\mathbf{B}^2)$$

use these equations to show that u satisfies

$$\frac{\partial u}{\partial t} + \nabla.\mathbf{S} = -\mathbf{E}.\mathbf{J},$$

where **S** is Poynting's vector: $\mathbf{S} = \epsilon_0 c^2 \mathbf{E} \times \mathbf{B}$.

3.

Derive the Maxwell-Boltzmann distribution function in the form

$$f(v^2) = A\exp[-Bv^2]$$

where A and B are constants. Given that $A = N(m/2\pi kT)^{3/2}$ calculate the constant B. *Note:* $\int_{-\infty}^{\infty} \exp[-ax^2]\,dx = (\pi/a)^{1/2}$.

4. Define the term mean free path; hence derive a formula for the mean free path λ of a molecule in a perfect gas.

Use this formula and the pair equations

$$\begin{aligned}\mathbf{J} &= \sigma\mathbf{E} \\ &= Ne\mathbf{v}\end{aligned}$$

to obtain a formula for the conductivity σ of wire carrying a current density **J** (e is the electric charge, **v** the drift velocity and N the number of charges/unit vol.).