Due on Monday, March 22nd, after the Study Break.

If pictures are needed/relevant, please provide them with your solutions.

This problem set is long, as appropriate for the Study Break.

The questions marked [**SELF**] are for yourself and need not be submitted.

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1. Two metallic plates are placed parallel and close to each other. They each have area A. They carry opposite charges $+Q$ and $-Q$. Such an arrangement is known as a *capacitor*.

   (a) [**4 pts.**] The plates are large enough that, if we are considering the region between the two plates (far from the plate edges), we can regard each charged plate as an infinite sheet of charge. Show that the magnitude of the electric field in the region between the plates is $E = Q/(A\epsilon_0)$.

   (b) [**3 pts.**] The *capacitance* of a capacitor is the charge on a plate divided by the potential difference between the two plates ($C = Q/V$). If the distance between the plates is d, find the capacitance of the arrangement.

2. An electron (charge$\approx 1.602 \times 10^{-19}$ C; mass$\approx 9.109 \times 10^{-31}$ kg) moving at speed 200 m/s finds itself in a region of constant magnetic field. The magnetic field points perpendicular to the electron velocity and has strength 10^{-3} Tesla.

   (a) [**2 pts.**] Find the magnitude of the magnetic force experienced by the electron.

   (b) [**3 pts.**] The magnetic force causes cyclotron motion, so that the electron moves around a circular trajectory. Find the radius of the circular trajectory.

   (c) [**SELF**] How long does it take for the electron to complete one cycle? (i.e., what is the period of the cyclotron motion?)

3. Steady current I flows through an infinitely long straight wire placed along the z azis, from $z = -\infty$ to $z = +\infty$ through the origin.

   Reminder: An infinitely long straight wire produces a magnetic field of strength $\mu_0 I/(2\pi d)$ at a point at distance d from the wire.

   (a) **[3 pts.]** Sketch a top view of the x-y plane, and show the point P with coordinates $(-4L, -L, 0)$ and the point Q with coordinates $(3L, 2L, 0)$. For each of these points, draw the direction of the magnetic field produced by the steady current I.

   (b) **[8 pts.]** Calculate the magnitudes of the magnetic field created at those two points (P and Q).

   (c) **[10 pts.]** At the point $Q(3L, 2L, 0)$, find the x-, y-, and z- components of the magnetic field. (Sketching a top view of the x-y plane might help.)

   (d) **[3 pts.]** Now consider a point $(3L, 2L, z_0)$, not necessarily on the x-y plane. What is the magnitude of the magnetic field at this point? What are the x-, y-, and z- components of the magnetic field at this point?

4. A particle having mass m and charge q is subjected to uniform electric and magnetic fields, both pointing in the z direction: $\mathbf{E} = E_0\hat{k}$, $\mathbf{B} = B_0\hat{k}$.

   At time $t = 0$, the particle is at the origin and has velocity $\mathbf{v} = v\hat{\imath}$.

   (a) **[7 pts.]** At which times does the particle pass through the z axis?

   (b) **[7 pts.]** What are the z values where the trajectory meets the z axis?

5. A particle with charge Q and mass m is placed in a region with uniform electric field in the y-direction, $\mathbf{E} = E\hat{\jmath}$, and uniform magnetic field in the z direction, $\mathbf{B} = B\hat{k}$. The particle position coordinates are x, y, z, and its velocity components are denoted as v_x, v_y, v_z.

   (a) **[5 pts.]** Calculate the force $\mathbf{F}$. Write out separately the expressions for its components, F_x, F_y, F_z. (These expressions will involve the quantities Q, B, E, v_x, and v_y.)

   (b) **[4 pts.]** Using your results for F_x, F_y and Newton's 2nd law, $\mathbf{F} = m\frac{d\mathbf{v}}{dt}$, calculate expressions for $\dfrac{dv_x}{dt}$ and $\dfrac{dv_y}{dt}$. You should thus obtain coupled first-order differential equations for $v_x(t)$ and $v_y(t)$.

   (c) **[4 pts.]** If the particle starts at rest, i.e., if the initial conditions are $v_x(0) = v_y(0) = 0$, then the solutions to the above equations are of the form

   $$v_x = \frac{E}{B} - \frac{E}{B}\cos(\omega t) \qquad v_y = \frac{E}{B}\sin(\omega t) \tag{1}$$

   Substituting these into your first differential equation, find an expression for ω. Substituting these into your second differential equation, show that you find the same expression for ω.

   (d) **[6 pts.]** The equations (1) are themselves differential equations for $x(t)$ and $y(t)$, since $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$. Assume that the particle starts at the origin, so that the initial conditions are $x(0) = y(0) = 0$. Find the solutions for $x(t)$ and $y(t)$.

   (e) **[3 pts.]** Sketch $x(t)$ as a function of t and $y(t)$ as a function of t. Sketch the trajectiory of the particle in the x-y plane.

6. Two infinite straight wires are placed parallel to each other, at a distance of 5 meters from each other. Each carries a current of 0.2 Amperes.

   (a) **[SELF]** Look up and report the value of μ_0 (permeability of free space) in SI units. Also calculate μ_0/π.

   (b) **[5 pts.]** If the currents in the two wires are in the same direction, calculate the magnitude of the magnetic field at a point halfway between the two wires.

   (c) **[5 pts.]** If the currents in the two wires are in opposite directions, calculate the magnitude of the magnetic field at a point halfway between the two wires.

7. **[5 pts.]** The magnetic field in some region is given by

$$\mathbf{B} = \frac{\mu_0}{\pi}\left(\frac{Cy}{x^2+y^2}\hat{\imath} + \frac{2x}{x^2+y^2}\hat{\jmath}\right) .$$

   Use one of Maxwell's equations to determine the constant C.

8. (a) **[5 pts.]** A long straight wire carrying steady current 0.4 Amperes is placed on the x-y plane, passes through the origin, and makes an angle of $30^\circ = \pi/6$ with the y-axis. The region is subjected to a uniform magnetic field of $\mathbf{B} = (10^{-2}T)\hat{\jmath}$, pointing in the y-direction. What is the force experienced by a segment of the wire of length 0.8m? Give both the magnitude and the direction of the force.

   (b) **[8 pts.]** A long straight wire carrying steady current I_1 runs along the y axis: The current flows from $y = -\infty$ to $y = +\infty$. The region is subjected to a magnetic field of $\mathbf{B} = \lambda y^2 \hat{k}$, pointing in the z-direction. Find the force experienced by the segment of the wire between $y = L$ and $y = 2L$.

   (Note that the magnetic field is not uniform. Here λ is a positive constant.)