Due on Monday, March 8th.

If pictures are needed/relevant, please provide them with your solutions.

– – – – – – – – – – – – – – ⋆ – – – – – – – – – – – – –

The questions marked [**SELF**] are for yourself — will not be marked — and need not be submitted.

However: skip them at your own risk! You are expected to work through [**SELF**]-questions as well, in order to learn exam-relevant material.

– – – – – – – – – – – – – – ⋆ – – – – – – – – – – – – –

1. The electrostatic field is $\mathbf{E} = \alpha x\hat{\imath} + \alpha y\hat{\jmath}$ in some region of space. Here α is a positive constant.

   (a) [**SELF**] Show that this satisfies the requirement of zero curl.

   (b) [**2 pts.**] Using Maxwell's first equation, find the electric charge density ρ in this region of space. How does ρ vary in space in this region?

   (c) [**SELF**] Calculate the difference of electric potential between the points $(L, 5L, -L)$ and $(L, 5L, L)$. Here L is a positive constant.

   (d) [**5 pts.**] Calculate the difference of electric potential between the points $(0, 5L, 0)$ and $(2L, 5L, 0)$.

2. A charged sphere has radius a_0 and volume charge density $\rho = \rho_0(r/a_0)$, where r is the distance from the center of the sphere.

   (a) [**6 pts.**] What is the total electric flux through any closed surface surrounding the sphere?

   Hint: Use Gauss' law (Gauss' dielectric flux theorem). You will need to first calculate the total charge on the sphere.

   (b) [**4 pts.**] Using Gauss' law, find the electric field outside the sphere.

   (c) [**8 pts.**] Using Gauss' law, find the electric field inside the sphere.

(d) **[3 pts.]** Plot the magnitude of the electric field as a function of r. The plot should extend from $r = 0$ to $r = 2a_0$.

(e) **[7 pts.]** Calculate the electric potential as a function of r, both outside the sphere and inside the sphere. You can take $r = \infty$ as the reference, i.e., the potential at infinite distance can be taken to be zero.

3. Recall that the current through a surface (e.g., through a cross-section of a current-carrying wire) is the surface integral of the current density.

   (a) **[SELF]** A current I is uniformly distributed over the cross-section of a wire. The cross-section is circular, with radius R. Find the current density J, as a function of I and R.

   (b) **[7 pts.]** Now consider instead the situation that the current density is larger near the outer regoin of the wire cross-section than at the center: $J = kr$, where r is the radial distance from the center of the wire. Find the current I, as a function of k and R. You will probably need to do a two-dimensional integral in polar coordinates. (In two-dimensional polar coordinates, the area element is $dS = rdrd\theta$.)

4. A copper conductor has 8.5×10^{28} electrons per cubic meter available to carry current, i.e., the carrier density is $8.5 \times 10^{28}\text{m}^{-3}$. The drift velocity is found to be 3×10^{-6}m/s.

   (a) **[0 pts.]** Look up and report the charge of an electron in SI units.

   (b) **[3 pts.]** Calculate the current density J.

5. **[5 pts.]** The current density in some region is found to be

$$\mathbf{J} \;=\; \frac{K_0 y}{\sqrt{x^2+y^2}}\hat{i} \;-\; \frac{K_0 x}{\sqrt{x^2+y^2}}\hat{j}$$

   Use the continuity equation to show that the charge density in this region does not change with time.