Due on Monday, March 1st.

If pictures are needed/relevant, please provide them with your solutions.

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1. (a) **[4 pts.]** Using cartesian coordinates, show that the curl of the graidient of a scalar field is zero, i.e., $\nabla \times (\nabla f) = 0$ for any scalar field $f(x, y, z)$.

   [Note: this means: a vector field that can be expressed as a gradient of a scalar field, must have zero curl. Thus the static electric field, which is the gradient of $-V$, must have zero curl: $\nabla \times \mathbf{E} = 0$.]

   (b) **[3 pts.]** Show that the vector field $\mathbf{G} = y\hat{\imath} - x\hat{\jmath}$ cannot be a static electric field.

2. The electric potential in some region of space is given by

$$V(x, y, z) = \frac{C}{\sqrt{x^2 + y^2 + z^2}}$$

or, in spherical coordinates, $V(r) = C/r$. Here C is a positive constant.

   (a) **[4 pts.]** Calculate an expression for the electric field in this region, using cartesian coordinates.

   (b) **[2 pts.]** Noting that $x\hat{\imath} + y\hat{\imath} + z\hat{k} = \mathbf{r} = r\hat{r}$, express the expression for $\mathbf{E}$ in spherical coordinates. Here $\hat{r}$ is the radial unit vector of spherical coordinates, sometimes also denoted as $\mathbf{e}_r$ or $\hat{e}_r$.

   (c) **[4 pts.]** Look up and report the expression for the gradient operator in spherical coordinates. Re-derive the expression for $\mathbf{E}$, now directly in spherical coordinates.

   (d) **[5 pts.]** Look up and report the expression for the Laplacian operator in spherical coordinates. By calculating the Laplacian of V, find out the charge density ρ in this region.

   (e) **[4 pts.]** What kind of charge arrangement generates a potential like the one given? Explain whether your calculated ρ is consistent with the expected charge arrangement.

3. **Metallic sphere.** Metals are very good electric conducors. An ideal conductor conducts charges perfectly well. In a static situation, the electric potential is constant everywhere in the metal. All electric charges are forced to sit on the surface of the metal. (Otherwise the charges would move until this static situation is reached.)

   We consider a solid metallic sphere of radius R, carrying a total charge Q.

   (a) [**3 pts.**] What is the surface charge density on the surface of the sphere?

   (b) [**5 pts.**] What is the electric field at distance $r > R$ from the center of the sphere? Use Gauss' dielectric flux theorem.

   Your result should be the same as what you would get if all the charge was concentrated at the center of the sphere, instead of being spread out on the surface.

   (c) [**3 pts.**] What is the electric field at distance $r < R$ from the center, i.e., within the sphere? (We've mentioned already that the potential is constant at all points of the sphere.)

   (d) [**3 pts.**] What is the electric potential outside the sphere, $r > R$? Use the fact that the electric field is identical to a simpler situation.

   (e) [**4 pts.**] Sketch a plot of the electric potential V as a function of the distance r from the center of the sphere. Make sure to clearly mark the $r = R$ point.

4. The electric field in some region is $\mathbf{E} = E_0\hat{\imath} + 2E_0\hat{\jmath}$.

   (a) [**2 pts.**] A planar surface of area 2α lies in the x-y plane, i.e., the normal to the surface is in the z direction. Find the magnitude of the flux of the electric field through this surface.

   (b) [**4 pts.**] A planar surface of area 6α lies parallel to the y-z plane. Find the magnitude of the flux of the electric field through this surface.