Due on Monday, February 22nd.

If pictures are needed/relevant, please provide them with your solutions.

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1. **Potential from Field.**

   The electric field in some region is $\mathbf{E} = E_0\hat{\imath}$. We will consider the electric potential difference, $V_B - V_A$, between two points A and B on the x-y plane, with coordinates $(0, 3L)$ and $(4L, 0)$ respectively. Ignore the z- coordinate for this problem. Here E_0 and L are positive constants.

   Recall: $V_B - V_A = -\int \mathbf{E} \cdot d\mathbf{l}$, with the line integral proceeding along any curve starting at point A and ending at point B.

   (a) [**1 pt.**] Sketch the situation, showing the x and y axis, the two points A and B, and the direction of the electric field.

   (b) [**1 pt.**] Consider the straight line pointing from A to B. Any element $d\mathbf{l}$ on this line is at the same angle with the direction of the electric field. Calling this angle θ, we obtain $\mathbf{E} \cdot d\mathbf{l} = E_0|d\mathbf{l}|\cos\theta$. Calculate $\cos\theta$ based on your drawing.

   (c) [**4 pts.**] Calculate $V_B - V_A = -\int \mathbf{E} \cdot d\mathbf{l}$, integrating along the straight line pointing from A to B. The value of $\cos\theta$, calculated above, might be helpful.

   (d) [**4 pts.**] Consider now a path consisting of two segments: a line from A to point C at $(4L, 3L)$, and a line from point C to point B. Draw the situation. Only one of these (AC or CB) segments contribute to the line integral for $V_B - V_A$. Which one? Calculate $V_B - V_A$ by integrating along this segment.

2. **Thin ring of charge.**

   A circular ring of radius R lies in the x-y plane with its center at the origin. The ring carries a uniformly distributed postive charge Q.

   (a) [**10 pts.**] Find the electric field due to the charged ring at the point $(0, 0, z_0)$ on the z-axis.

   (b) [**5 pts.**] Sketch plots of each component of the electric field as a function of z_0.

3. **Infinite plane of uniform electric charge.**

The x-y plane ($z = 0$) is covered uniformly with electric charge, with surface charge density σ (i.e., any area A of the plane carries charge σA). Assume σ to be positive. We will calculate the electric field at some point P away from the plane. Without loss of generality, we can take the point to be (0, 0, z_0), i.e., a point on the z-axis.

(a) [**3 pts.**] By symmetry, what should be the direction of the electric field $\mathbf{E}$ at the point P? What should be the direction of the electric field at any other point with positive z-coordinate? What about points with negative z-coordinate?

(b) [**1+1 pts.**] Consider an infinitely long strip of the charged plane along the x-direction, running from $x = -\infty$ to $x = +\infty$, parallel to the x-axis. Let the strip be infinitesimally thin, located between $y = u$ and $y = u + du$.

Draw this situation in two ways, showing (1) A top view of the xy plane, and (2) a three-dimensional view also showing the z-axis and the point P.

(c) [**4 pts.**] What is the linear charge density of the strip?

(d) [**7 pts.**] Recall from the previous week: an infinite thin rod of charge, if carrying linear charge density λ, produces a field

$$\frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

at a point which is at radial distance r from the rod. In the present geometry, what is the magnitude and direction of the element of field ($d\mathbf{E}$) produced at P by our infinitesimal strip of charge?

Hint: a picture of the yz plane, showing the strip position and the point P, is useful, and is expected.

(e) [**3 pts.**] Find the z-component of $d\mathbf{E}$.

(f) [**5 pts.**] We now want to use the principle of superposition to obtain the net field at P. This means summing over all infinitesimal strips to cover the complete plane of charge, i.e., integrating your expression from $u = -\infty$ to $u = \infty$. It might help to remember that

$$\int \frac{dv}{v^2+1} = \tan^{-1} v \qquad \text{and} \qquad \lim_{v\to\pm\infty} \tan^{-1} v = \pm\frac{\pi}{2}$$

Note that you only need to deal with the $z-$component; the other components are zero by symmetry.

(g) [**1 pt.**] What is the dependence of the net field on z_0?