MATHEMATICAL PHYSICS

SEMESTER 2, MAY EXAM

2019–2020

MP204

Electricity and Magnetism

Prof. P. Coles, Dr. M. Haque, and Prof. D. A. Johnston

Answer **ALL FOUR** questions

Exam Duration: 2 hours

You have three hours to complete this examination. This includes the time taken to download the question paper, answer the question, scan your solutions, and uploading the resulting PDF to the Moodle Examination page E:MP204. You should not spend more than two hours doing the examination itself as this may mean that you do not have time to submit your work.

This is an unsupervised examination. You may consult your notes or textbooks as you wish, but you must show all your working to get full credit for correct answers. You must not collude with other students. Submission of work through your Moodle account will be taken as an assertion that the work was done by yourself.

If you encounter any technical problems with the download or upload please contact the MP204 lecturer (Masud Haque) at

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or email

masud.haque@mu.ie

or by messaging the lecturer on Moodle or on MS Teams.

In case Dr. Haque is unreachable for some reason, Prof. Coles will be on standby for urgent questions: Peter.Coles@mu.ie

The deadline for submitting your answers is **17:30** on Saturday 16th May 2020. Unless you have been granted an adjustment, a late submission will be accepted only at the discretion of the Examiner(s) if you have not notified the Examiner that you have encountered problems.

1. (a) A positive electric charge, q, is placed at the origin $(0, 0, 0)$.
Also, a negative charge, $-2\sqrt{2}q = -\sqrt{8}q$, is placed at the point $(L, 0, 0)$ on the x-axis.
Find the electric field created at the point $(0, L, 0)$ on the y axis.
Here L and q are positive constants.

[9 marks]

(b) An electromagnetic field is described by the scalar potential V and vector potential $\mathbf{A}$, given by

$$V = \alpha c^2 x t \, , \qquad \mathbf{A} = \alpha \left(c^2 t^2 - y^2 \right) \hat{\imath}$$

where α is a positive constant, and $c = 1/\sqrt{\mu_0 \epsilon_0}$ is the speed of light.
Find the electric and magnetic fields.

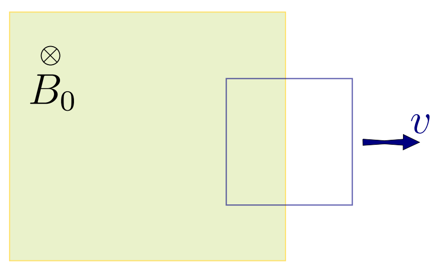
[8 marks]

(c) See figure below. The shaded region supports a uniform magnetic field of strength B_0, pointing perpendicularly into the plane of the paper.

The magnetic field is zero outside this region.

A square-shaped loop moves rightward with speed v.

Each side of the loop has length L_0,



When the loop is completely inside the shaded region, what is the magnitude of the EMF induced in the loop?
When the loop is partially inside and partially outside the shaded region (as shown), what is the magnitude of the induced EMF?

[8 marks]

2. Consider the static electric field:

$$\mathbf{E} = 2\lambda y\hat{\imath} - 3\lambda y\hat{\jmath} + 4\lambda y\hat{k} = \lambda y(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})\,.$$

Here λ is a positive constant.

(a) Find the electric potential difference V_{QP} between the point Q at $(0, 3L, 0)$ and the point P at $(0, L, 0)$.

Find the electric potential difference V_{RP} between the point R at $(4L, L, 0)$ and the point P at $(0, L, 0)$.

Here L is a positive constant.

[14 marks]

(b) Consider the rectangular surface Σ_1 lying in the x-y plane, with three corners at points P, Q, R, and the fourth corner at point S at $(4L, 3L, 0)$. Find the magnitude of the electric flux through this surface.

[11 marks]

3. (a) A charged sphere has radius a_0. The charge density is

$$\rho = \begin{cases} \rho_0 \left(1 - \dfrac{r}{a_0}\right) & \text{for } r < a_0 \\ 0 & \text{for } r > a_0 \end{cases}$$

where r is the distance from the center of the sphere.

Consider the closed surface Σ_2, completely enclosing the sphere. Find the electric flux through this surface.

Find the electric field <u>inside</u> the sphere as a function of r. If you use a Gaussian surface, describe it clearly. If you use any arguments/results based on symmetry, state them clearly.

[19 marks]

(b) The magnetic field in some region is uniform: $\mathbf{B} = B_0\hat{k}$. At time $t = 0$, a charged particle (mass m, charge q) is at the origin and has velocity

$$\mathbf{v} = \alpha\hat{\imath} + w\hat{k}$$

What is the magnitude and direction of the force exerted at $t = 0$ on the particle ?

Explain how the x, y and z components of the velocity change with time.

[6 marks]

4. Steady current I flows through an infinitely long straight wire placed parallel to the z axis, from $z = -\infty$ to $z = +\infty$. The wire runs through the point $(b, 0, 0)$. Here b is a positive constant.

   Reminder: An infinitely long straight wire produces a magnetic field of strength $\mu_0 I/(2\pi d)$ at a point at distance d from the wire.

   (a) Find the magnitude of the magnetic field created at the **origin** due to the current-carrying wire. Find also the x-, y-, and z- components of the magnetic field created at the origin.

   [5 marks]

   (b) At the point $M(4b, 4b, 0)$ in the x-y plane, find the magnitude of the magnetic field created by the current. Find also the x-, y-, and z-components of the magnetic field at this point.

   [13 marks]

   (c) Now consider the point $N(4b, 4b, -7b)$, not in the x-y plane. Find the x-, y-, and z- components of the magnetic field at this point.

   [3 marks]

   (d) Consider the circular surface Σ_3 lying in the x-y plane, having radius b and centered at $(0, -b, 0)$. What is the magnetic flux through this surface? Explain, or show your calculations.

   [4 marks]

Posssibly useful Equations

- Electrostatics: $\mathbf{E} = -\nabla V$; $V_{PQ} = -\int_Q^P \mathbf{E}\cdot d\mathbf{l}$

  Electric potential at $\mathbf{r}$ due to a point charge q_1 at $\mathbf{r_1}$: $V = \frac{q_1}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}-\mathbf{r_1}|}$

  Gauss' law: $\oint_\Sigma \mathbf{E}\cdot d\mathbf{S} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

- Magnetostatics: Ampere's law: $\int_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enclosed}}$

  Biot-Savart law: $d\mathbf{B} = \left(\frac{\mu_0 I}{4\pi}\right)\frac{d\mathbf{l}'\times\left(\widehat{\mathbf{r}-\mathbf{r}'}\right)}{|\mathbf{r}-\mathbf{r}'|^2}$

- Force on a charge: $\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$

  Magnetic force on a current element: $d\mathbf{F} = I d\mathbf{l}\times\mathbf{B}$

- Fields from potentials: $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$, $\mathbf{B} = \nabla\times\mathbf{A}$

- The continuity equation: $\nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0$

- Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$ or $\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}\int_\Sigma \mathbf{B}\cdot d\mathbf{S}$

- Maxwell's Equations:

  ① $\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$ ② $\nabla\cdot\mathbf{B} = 0$

  ③ $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$

  ④ $\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} = \mu_0\left(\mathbf{J} + \mathbf{J_D}\right)$

- Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$ Speed of light: $c = 1/\sqrt{\mu_0\epsilon_0}$

  Energy density of electromagnetic fields: $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$