MP201 – Vector Calculus & Fourier Analysis

Problem Set 3

Due by 5pm on Friday, 13 October 2017

(Please write your name and tutorial day on the front of your assignment.)

1. Let $f(x,y,z)$ and $g(x,y,z)$ be two scalar fields. As you probably know, the product rule for partial derivatives is

$$\frac{\partial}{\partial x}(fg) \quad = \quad \left(\frac{\partial f}{\partial x}\right) g + f\left(\frac{\partial g}{\partial x}\right),$$

   and similarly for $\partial/\partial y$ and $\partial/\partial z$. Using this rule, prove that

$$\vec{\nabla}(fg) \quad = \quad \left(\vec{\nabla} f\right) g + f\left(\vec{\nabla} g\right).$$

2. Suppose we have the three scalar fields

$$A(x,y,z) = x^3 yz + 2, \quad B(x,y,z) = y\sin(2x)\cos(3z), \quad C(x,y,z) = e^{-x^2-2y^2-5z^2}.$$

   (a) Calculate the gradients of each of these functions.

   (b) Given the three vectors

$$\vec{u} = \hat{\jmath}. \quad \vec{v} = \hat{\imath} - \hat{\jmath} + \hat{k}, \quad \vec{w} = 2\hat{\jmath} - 15\hat{k}$$

   find the directional derivatives $\mathrm{D}_{\vec{u}}A$, $\mathrm{D}_{\vec{v}}B$ and $\mathrm{D}_{\vec{w}}C$.

3. Suppose $\phi(x,y,z)$ is a scalar field and $f(u)$ is a function of a single variable u. The chain rule for partial differentiation says that

$$\frac{\partial}{\partial x}\left[f\left(\phi(x,y,z)\right)\right] \quad = \quad f'\left(\phi(x,y,z)\right)\frac{\partial \phi}{\partial x},$$

   and similarly for $\partial/\partial y$ and $\partial/\partial z$. Show that if $V(r)$ is a potential energy function that depends only on the length of the postion vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then the force due to this potential is

$$\vec{F} \quad = \quad -V'(r)\hat{r}.$$

   (Potentials of this form are very important in physics, as you will see next semester in MP204 and afterwards if you continue in theoretical physics...)

4. Find unit normal vectors for the following surfaces at the given points:

   (a) $z = \sqrt{9 - 4x^2 - y^2}$ at the point $(1,-1,2)$,

   (b) $x\sin y\sin z = 1$ at the point $(2,\pi/4,\pi/4)$.