

MP201 – Vector Calculus & Fourier Analysis

Problem Set 2

Due by 5pm on Friday, 6 October 2017

(Please write your name and tutorial day on the front of your assignment.)

1. For any three vectors \vec{A} , \vec{B} and \vec{C} , prove that

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = \vec{A} \cdot (\vec{B} \times \vec{C}).$$

2. Consider three vectors \vec{a} , \vec{b} and \vec{c} .

- (a) Show that if any two of these vectors are parallel, then their scalar triple product is zero.
- (b) Show that if all three vectors are coplanar (i.e. they all lie in the same plane), then their scalar triple product is zero.

3. Suppose that \vec{u} and \vec{v} are two vectors which are perpendicular to one another. Show that

$$\vec{u} \times (\vec{u} \times \vec{v}) = -u^2 \vec{v}.$$

4. A plane may be defined in the following way: if \hat{n} is a unit vector perpendicular to the plane, and \vec{r}_0 is the position vector of a point in the plane, then all points in the plane have position vectors \vec{r} which satisfy $\hat{n} \cdot \vec{r} = a$, where a is the dot product of \hat{n} and \vec{r}_0 .

Show that every point in the plane just defined must satisfy

$$[\hat{n} \times (\hat{n} \times \vec{r})] \cdot \vec{r} = a^2 - r^2.$$